Comment on "Quantum teleportation of an arbitrary two-qubit state and its relation to multipartite entanglement"

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The multipartite state in the Rigolin's protocol [Phys. Rev. A **71**, 032303 (2005)] for teleporting an arbitrary two-qubit state is just a product state of N Einstein-Podolsky-Rosen pairs in essence, not a genuine multipartite entangled state, and this protocol in principle is equivalent to the Yang-Guo protocol [Chin. Phys. Lett. **17**, 162 (2000)].

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In a recent paper [1], Rigolin proposed a protocol (called Rigolin's protocol for short below) for teleporting an arbitrary two-qubit state $|\phi\rangle_{X_1X_2}=a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$ with a four-particle generalized Bell state and a four-particle joint measurement. Here $|a|^2+|b|^2+|c|^2+|d|^2=1$. In this Comment, we want to show that Rigolin's protocol [1] is equivalent to the Yang-Guo protocol [2] for teleporting an arbitrary multipartite state as the multipartite state in the Rigolin's protocol is just a product state of N Einstein-Podolsky-Rosen (EPR) pairs in essence, not a genuine multipartite entangled state.

In Ref. [1], Rigolin defined 16 generalized Bell states $\{|g_i\rangle\}$ for describing a four-particle quantum system and divided them into four groups.

Group 1:

$$|g_1\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0000\rangle+|0101\rangle+|1010\rangle+|1111\rangle),$$

$$|g_2\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0000\rangle+|0101\rangle-|1010\rangle-|1111\rangle),$$

$$|g_3\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0000\rangle-|0101\rangle+|1010\rangle-|1111\rangle),$$

$$|g_4\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0000\rangle-|0101\rangle-|1010\rangle+|1111\rangle), \quad (1)$$

Group 2:

$$|g_5\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0001\rangle+|0100\rangle+|1011\rangle+|1110\rangle),$$

$$|g_6\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0001\rangle+|0100\rangle-|1011\rangle-|1110\rangle),$$

$$|g_7\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0001\rangle-|0100\rangle+|1011\rangle-|1110\rangle),$$

$$|g_8\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0001\rangle-|0100\rangle-|1011\rangle+|1110\rangle), \quad (2)$$

Group 3:

$$|g_9\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0010\rangle+|0111\rangle+|1000\rangle+|1101\rangle),$$

$$|g_{10}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0010\rangle+|0111\rangle-|1000\rangle-|1101\rangle),$$

$$|g_{11}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0010\rangle-|0111\rangle+|1000\rangle-|1101\rangle),$$

$$|g_{12}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0010\rangle-|0111\rangle-|1000\rangle+|1101\rangle), \quad (3)$$

Group 4:

$$|g_{13}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0011\rangle+|0110\rangle+|1001\rangle+|1100\rangle),$$

$$|g_{14}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0011\rangle+|0110\rangle-|1001\rangle-|1100\rangle),$$

$$|g_{15}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0011\rangle-|0110\rangle+|1001\rangle-|1100\rangle),$$

$$|g_{16}\rangle_{A_1A_2B_1B_2}=\frac{1}{2}(|0011\rangle-|0110\rangle-|1001\rangle+|1100\rangle). \quad (4)$$

Rigolin assumed [1] that the two parties, Alice and Bob, previously shared a four-particle quantum system $A_1A_2B_1B_2$ in the generalized Bell state $|g_1\rangle$, and here the two qubits A_1A_2 belong to Alice and the other two particles B_1B_2 are transmitted to Bob. Hence the initial joint state is [1]

$$
\begin{aligned}
|\Phi\rangle &\equiv |\phi\rangle_{X_1X_2} \otimes |g_1\rangle_{A_1A_2B_1B_2} \\
&= \frac{a}{2}\{|000000\rangle + |000101\rangle + |001010\rangle + |001111\rangle\} \\
&\quad + \frac{b}{2}\{|010000\rangle + |0100101\rangle + |011010\rangle + |011111\rangle\} \\
&\quad + \frac{c}{2}\{|100000\rangle + |100101\rangle + |101010\rangle + |101111\rangle\} \\
&\quad + \frac{d}{2}\{|110000\rangle + |110101\rangle + |111010\rangle + |111111\rangle\}. \quad (5)
\end{aligned}
$$

Alice makes a four-particle joint measurement on the four particles $X_1X_2A_1A_2$ with the basis $\{|g_i\rangle\}$, and she obtains one of the 16 generalized Bell states with equal probabilities. With the result of the measurement, Bob can take two local unitary operations $U_i^1 \otimes U_j^2$ on his two particles B_1B_2 to recover the original state $|\phi\rangle_{X_1X_2}$, where $U_i^1, U_j^2 \in \{I, \sigma_z, \sigma_x, i\sigma_y = \sigma_z\sigma_x\}$ and σ_i are the usual Pauli matrices and I is the identity matrix.

In essence, each of the 16 generalized Bell states shown in Eqs. (1)–(4) is just a product state of two EPR pairs, not a genuine multipartite entangled state, because

Group 1′:

$$
\begin{aligned}
|g_1\rangle_{A_1A_2B_1B_2} &= |\phi^+\rangle_{A_1B_1}|\phi^+\rangle_{A_2B_2}, \\
|g_2\rangle_{A_1A_2B_1B_2} &= |\phi^-\rangle_{A_1B_1}|\phi^+\rangle_{A_2B_2}, \\
|g_3\rangle_{A_1A_2B_1B_2} &= |\phi^+\rangle_{A_1B_1}|\phi^-\rangle_{A_2B_2}, \\
|g_4\rangle_{A_1A_2B_1B_2} &= |\phi^-\rangle_{A_1B_1}|\phi^-\rangle_{A_2B_2}, \quad (6)
\end{aligned}
$$

Group 2′:

$$
\begin{aligned}
|g_5\rangle_{A_1A_2B_1B_2} &= |\phi^+\rangle_{A_1B_1}|\psi^+\rangle_{A_2B_2}, \\
|g_6\rangle_{A_1A_2B_1B_2} &= |\phi^-\rangle_{A_1B_1}|\psi^+\rangle_{A_2B_2}, \\
|g_7\rangle_{A_1A_2B_1B_2} &= |\phi^+\rangle_{A_1B_1}|\psi^-\rangle_{A_2B_2}, \\
|g_8\rangle_{A_1A_2B_1B_2} &= |\phi^-\rangle_{A_1B_1}|\psi^-\rangle_{A_2B_2}, \quad (7)
\end{aligned}
$$

Group 3′:

$$
\begin{aligned}
|g_9\rangle_{A_1A_2B_1B_2} &= |\psi^+\rangle_{A_1B_1}|\phi^+\rangle_{A_2B_2}, \\
|g_{10}\rangle_{A_1A_2B_1B_2} &= |\psi^-\rangle_{A_1B_1}|\phi^+\rangle_{A_2B_2}, \\
|g_{11}\rangle_{A_1A_2B_1B_2} &= |\psi^+\rangle_{A_1B_1}|\phi^-\rangle_{A_2B_2}, \\
|g_{12}\rangle_{A_1A_2B_1B_2} &= |\psi^-\rangle_{A_1B_1}|\phi^-\rangle_{A_2B_2}, \quad (8)
\end{aligned}
$$

Group 4′:

$$
\begin{aligned}
|g_{13}\rangle_{A_1A_2B_1B_2} &= |\psi^+\rangle_{A_1B_1}|\psi^+\rangle_{A_2B_2}, \\
|g_{14}\rangle_{A_1A_2B_1B_2} &= |\psi^-\rangle_{A_1B_1}|\psi^+\rangle_{A_2B_2}, \\
|g_{15}\rangle_{A_1A_2B_1B_2} &= |\psi^+\rangle_{A_1B_1}|\psi^-\rangle_{A_2B_2}, \\
|g_{16}\rangle_{A_1A_2B_1B_2} &= |\psi^-\rangle_{A_1B_1}|\psi^-\rangle_{A_2B_2}, \quad (9)
\end{aligned}
$$

where

$$
|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle),
$$

$$
|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle) \quad (10)
$$

are the four Bell states for two-particle quantum system. That is, Alice and Bob share two EPR pairs $|\phi^+\rangle_{A_1B_1}$ and $|\phi^+\rangle_{A_2B_2}$ in the Rigolin's protocol [1] only, which is the same as that in the Yang-Guo protocol [2]. Also, the joint measurement with basis $\{|g_i\rangle\}$ on particles $X_1X_2A_1A_2$ can be considered as two Bell-state measurements on the particles X_1A_1 and X_2A_2, respectively. So the Rigolin's protocol is equivalent to the Yang-Guo protocol for teleporting an arbitrary two-qubit state. This conclusion can also be generalized to the case for teleporting an arbitrary multipartite state as each of the $2N$-particle generalized Bell states in the Rigolin's protocol is a product state of N two-particle Bell states.

In a word, the Rigolin's protocol [1] is equivalent to the Yang-Guo protocol [2] for teleporting an arbitrary multipartite state in principle even though the demonstration of the quantum teleportaton with those two protocols may be different in the aspect of technology.

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[1] G. Rigolin, Phys. Rev. A **71**, 032303 (2005).

[2] C. P. Yang and G. C. Guo, Chin. Phys. Lett. **17**, 162 (2000).