Generation of entangled states in cavity QED

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We propose a scheme to generate four Bell states and a four-atom entangled cluster state in a thermal cavity. The photon-number-dependent parts in the effective Hamiltonian are canceled with the assistance of a strong classical field. The cavity field is only virtually excited; no quantum information will be transferred from the atoms to the cavity and thus the scheme is insensitive to the cavity decay and the thermal field. The scheme can also be used to generate the cluster state for the trapped ions.

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Quantum entanglement is the basic resource of quantum information. Quantum entanglement not only gives the possibility for test of quantum mechanics against local hidden theory [1–3], but also has practical applications in quantum teleportation [4], quantum dense coding [5], and quantum cryptography [6]. Most of the research in quantum information processing is based on entanglement of two particles. Entangled states for two particles have been realized for atoms in cavity QED [7–9], ions in a trap [10] and photons [11].

In contrast to two-particle entangled states, multiparticle entangled states also play an important role, such as GHZ states, which not only can provide much stronger refutations of local realism and reveal a contradiction with local hidden variable theory from a single set of measurements, but also are useful in quantum information processing. In Ref. [12], Briegel *et al.* introduced a class of entangled states, i.e., the cluster states. The cluster states can be regarded as a resource for GHZ states and are more immune to decoherence than GHZ states. On the other hand, cluster states have been shown to constitute a universal resource for quantum computation. The proof of Bell's theorem without the inequality was given for cluster states, and Bell inequality is maximally violated by the four-qubit cluster state and is not violated by the four-qubit GHZ state. Recently Zou *et al.* [13] proposed a scheme for generation of polarization entangled cluster state. The success probability of obtaining photons in such a state is 0.25. As one of the possible candidates for engineering quantum entanglement, the cavity quantum electrodynamics system always has many applications in quantum information processing. This is due to the fact that cold and localized atoms are not only the important resource of entanglement, but also well suited for storing quantum information in long-lived internal states. Thus how to prepare a multiparticle entangled state in cavity QED has abstracted much attention. The generation of the GHZ state of three particles has been demonstrated experimentally in high-Q cavities [14]. Fidio and Vogel [15] proposed a scheme for preparing a W state of three trapped atoms in leaky cavities. Guo *et al.* [16] have proposed a scheme to generate the multiparticle entangled states for atoms interacting dispersively with a vacuum cavity. Zou *et al.* [17] presented a scheme to generate the W states, GHZ states, and cluster states of four distant atoms trapped separately in leaky cavities with the certain probability. Here we propose a scheme to generate four Bell states and a cluster state of four atoms by the atom-cavity field interaction. The distinct advantage of the scheme is that during the operation, the cavity is only virtually excited and thus the effective decoherence time of the cavity is greatly prolonged. During the passage of the atoms through the cavity field, a strong classical field is accompanied so that the photon-number-dependent parts are canceled. Thus this scheme is insensitive to both the cavity decay and the thermal field. In addition in the scheme four Bell states and the entangled cluster state can be generated by one step with the success probability 1.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. In the rotating-wave approximation, the Hamiltonian for the system is [18,19]

$$H=\omega_0 S_z+\omega_a a^\dagger a+\sum_{j=1}^{2}\left[g(a^\dagger S_j^- + aS_j^+)+\Omega(S_j^+e^{-i\omega t}+S_j^- e^{i\omega t})\right], \tag{1}$$

where $S_z=\frac{1}{2}\Sigma_{j=1,2}(|e_j\rangle\langle e_j|-|g_j\rangle\langle g_j|$, $S_j^+=|e_j\rangle\langle g_j|$, $S_j^-=|g_j\rangle\langle e_j|$, and $|e_j\rangle,|g_j\rangle$ are the excited and ground states of the jth atom, respectively. $a^\dagger,a$ are the creation and annihilation operators for the cavity mode, and g is the atom-cavity coupling strength, Ω is the Rabi frequency, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Assuming $\omega_0=\omega$, in the interaction picture, the interaction Hamiltonian is

$$H_I=\Omega\sum_{j=1,2}(S_j^+ + S_j^-)+g\sum_{j=1}^{2}(e^{-i\delta t}a^\dagger S_j^- + e^{i\delta t}aS_j^+), \tag{2}$$

where δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω_a.

For the new atomic basis $|\pm_j\rangle=(|g_j\rangle\pm|e_j\rangle)/\sqrt{2}$ [18,19], we can rewrite H_I as

$$H_I = \sum_{j=1,2} \left\{ g\left[e^{-i\delta t} a^\dagger \left(\sigma_{z,j} + \frac{1}{2}\sigma_j^- - \frac{1}{2}\sigma_j^+ \right) + e^{i\delta t} a \left(\sigma_{z,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^- \right) \right] + 2\Omega \sigma_{z,j} \right\}, \tag{3}$$

where $\sigma_{z,j}=1/2\,(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|)$, $\sigma_j^+ = |+_j\rangle\langle -_j|$, and $\sigma_j^- = |-_j\rangle\langle +_j|$.

The time evolution of this system is described by Schrodinger's equation,

$$i\frac{d|\psi(t)\rangle}{dt} = H_I|\psi(t)\rangle. \tag{4}$$

By performing the unitary transformation

$$|\psi(t)\rangle = e^{-iH_0 t}|\psi'(t)\rangle, \tag{5}$$

where

$$H_0 = 2\Omega \sum_{j=1,2} \sigma_{z,j}, \tag{6}$$

then we have

$$i\frac{d|\psi'(t)\rangle}{dt} = H_I'|\psi'(t)\rangle, \tag{7}$$

where

$$H_I' = \sum_{j=1,2} g\left[e^{-i\delta t} a^\dagger \left(\sigma_{z,j} + \frac{1}{2}\sigma_j^- e^{-i\Omega t} - \frac{1}{2}\sigma_j^+ e^{i\Omega t} \right) + e^{i\delta t} a \left(\sigma_{z,j} + \frac{1}{2}\sigma_j^+ e^{i\Omega t} - \frac{1}{2}\sigma_j^- e^{-i\Omega t} \right) \right]. \tag{8}$$

Assume that in the strong driving regime $\Omega \gg \delta, g$, we can realize a rotating-wave approximation and eliminate the terms oscillating fast. Then H_I' is [18]

$$H_I' = \frac{1}{2}\sum_{j=1,2} g(e^{-i\delta t}a^+ + e^{i\delta t}a)(S_j^+ + S_j^-). \tag{9}$$

In the case $\delta \gg \mathrm{g}$, there is no energy exchange between the atomic system and the cavity. The resonant transitions are $|e_j g_k n\rangle \leftrightarrow |g_j e_k n\rangle$ and $|e_j e_k n\rangle \leftrightarrow |g_j g_k n\rangle$. The transition $|e_j g_k n\rangle \leftrightarrow |g_j e_k n\rangle$ is mediated by $|g_j g_k n\pm 1\rangle$ and $|e_j e_k n \pm 1\rangle$. The contributions of $|g_j g_k n \pm 1\rangle$ are equal to those of $|e_j e_k n \pm 1\rangle$. The corresponding Rabi frequency is given by $g^2/(2\delta)$. Since the transition paths interfere destructively, the Rabi frequency is independent of the photon number of the cavity mode [20–22]. The Rabi frequency for $|e_j e_k n\rangle \leftrightarrow |g_j g_k n\rangle$, mediated by $|g_j e_k n \pm 1\rangle$ and $|e_j g_k n \pm 1\rangle$, is also $g^2/(2\delta)$. The Stark shifts for the state $|e_j\rangle$ and $|g_j\rangle$ are both equal to $g^2/(4\delta)$. The strong classical field induces the terms $g(e^{-i\delta t}a^\dagger S_j^- + e^{i\delta t} a S_j^+)$, which result in the photon-number-dependent Stark shifts negative to those induced by $g(e^{-i\delta t}a^\dagger S_j^+ + e^{i\delta t} a S_j^-)$. Thus the photon-number-dependent Stark shifts are also canceled. Then in the interaction picture, the effective interaction Hamiltonian reads [20–22]

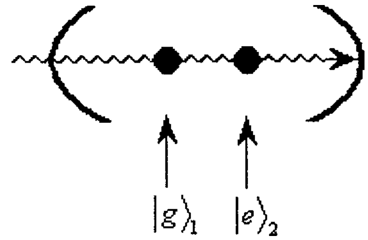

FIG. 1. Two atoms in a single-mode cavity and a classical field shown by a wavy line.

$$H_e = \frac{\chi}{2}\left(\sum_{j=1}^{2} (|e\rangle_{jj}\langle e| + |g\rangle_{jj}\langle g|) + \sum_{i,j=1,i\neq j}^{2} (S_i^+ S_j^- + S_i^+ S_j^+ + \mathrm{H.c.}) \right), \tag{10}$$

where $\chi = g^2/2\delta$. We note that the effective Hamiltonian is independent of the cavity field state, allowing it to be in a thermal state.

Then the evolution operator of the system is given by

$$U(t) = e^{-iH_0 t}e^{-iH_e t}. \tag{11}$$

Assume two atoms are initially in the state $|ge\rangle_{12}$. The two atoms interact simultaneously with a single-mode cavity, at the same time the atoms are driven by a classical field (see Fig. 1).

The state evolution of the system is

$$|\psi\rangle_{12} = e^{-i\chi t}\{\cos(\chi t)[\cos(\Omega t)|g\rangle_1 - i\sin(\Omega t)|e\rangle_1][\cos(\Omega t)|e\rangle_2 - i\sin(\Omega t)|g\rangle_2] - i\sin(\chi t)[\cos(\Omega t)|e\rangle_1 - i\sin(\Omega t)|g\rangle_1][\cos(\Omega t)|g\rangle_2 - i\sin(\Omega t)|e\rangle_2]\}. \tag{12}$$

We choose the interaction time and Rabi frequency appropriately so that $\Omega t = \pi$, $\chi t = \pi/4$. Then we obtain the maximally two-atom entangled state, that is, one of the four Bell states,

$$|\psi\rangle_{12}^- = \frac{1}{\sqrt{2}}(|ge\rangle_{12} - i|eg\rangle_{12}). \tag{13}$$

If we choose $\Omega t = \pi$, $\chi t = 5\pi/4$, another Bell state can be generated,

$$|\psi\rangle_{12}^+ = \frac{1}{\sqrt{2}}(|ge\rangle_{12} + i|eg\rangle_{12}). \tag{14}$$

Consider that two atoms are initially in the state $|gg\rangle_{12}$. Let the two atoms interact simultaneously with a single-mode cavity, and at the same time the atoms is driven by a classical field; the state evolution of the system has

$$|\psi\rangle_{12} = e^{-i\chi t}\{\cos(\chi t)[\cos(\Omega t)|g\rangle_1 - i\sin(\Omega t)|e\rangle_1][\cos(\Omega t)|g\rangle_2 - i\sin(\Omega t)|e\rangle_2] - i\sin(\chi t)[\cos(\Omega t)|e\rangle_1 - i\sin(\Omega t)|g\rangle_1][\cos(\Omega t)|e\rangle_2 - i\sin(\Omega t)|g\rangle_2]\}. \tag{15}$$

With the choice of $\Omega t = \pi$, $\chi t = \pi/4$, we can prepare a Bell state

$$|\phi\rangle_{12}^{-} = \frac{1}{\sqrt{2}}(|gg\rangle_{12} + i|ee\rangle_{12}). \tag{16}$$

If we choose $\Omega t = \pi$, $\chi t = 5\pi/4$ for the interaction transformation (15), we can obtain another Bell state,

$$|\phi\rangle_{12}^{+} = \frac{1}{\sqrt{2}}(|gg\rangle_{12} - i|ee\rangle_{12}). \tag{17}$$

For generating the entangled cluster state for four atoms, we prepare the four atoms initially in the state $|ggee\rangle_{1234}$. Let the atoms 1, 2 interact simultaneously with a single-mode cavity, at the same time the atoms are driven by a classical field, and atoms 3, 4 interact simultaneously with another single-mode cavity, at the same time the atoms 3, 4 are driven by another classical field, the state evolution of the whole system is

$$\begin{aligned}|\psi\rangle_{1234} = e^{-i2\chi\tau}\{&\cos(\chi\tau)[\cos(\Omega\tau)|g\rangle_1 - i\sin(\Omega\tau)|e\rangle_1][\cos(\Omega\tau)\\ &\times|g\rangle_2 - i\sin(\Omega\tau)|e\rangle_2] - i\sin(\chi\tau)[\cos(\Omega\tau)|e\rangle_1\\ &- i\sin(\Omega\tau)|g\rangle_1][\cos(\Omega\tau)|e\rangle_2 - i\sin(\Omega\tau)|g\rangle_2]\}\\ &\times\{\cos(\chi\tau)[\cos(\Omega\tau)|e\rangle_3 - i\sin(\Omega\tau)|g\rangle_3][\cos(\Omega\tau)\\ &\times|e\rangle_4 - i\sin(\Omega\tau)|g\rangle_4] - i\sin(\chi\tau)[\cos(\Omega\tau)|g\rangle_3\\ &- i\sin(\Omega\tau)|e\rangle_3][\cos(\Omega\tau)|g\rangle_4 - i\sin(\Omega\tau)|e\rangle_4]\}. \end{aligned} \tag{18}$$

With the choice of $\Omega\tau = \pi$, $\chi\tau = \pi/4$, we obtain the maximally four-atom entangled state,

$$|\psi\rangle_{1234} = \frac{1}{2}(-i|gggg\rangle + |ggee\rangle + |eegg\rangle - i|eeee\rangle). \tag{19}$$

Using local operation, we can transform the state (19) into the entangled cluster state [12]

$$|\psi\rangle_{1234} = \frac{1}{2}(|gggg\rangle + |ggee\rangle + |eegg\rangle - |eeee\rangle). \tag{20}$$

We note that the idea can also be applied to the ion trap system. We consider that N ions are confined in a linear trap. Then the ions are simultaneously excited with two lasers. In the Lamb-Dicke regime, the interaction Hamiltonian in the interaction picture is [20–22]

$$H_i = i\eta\Omega e^{-i\phi}\sum_{j=1}^{2} S_j^{+}(a^{\dagger}e^{-i\delta t} + ae^{i\delta t}) + \text{H.c.}, \tag{21}$$

where η is the Lamb-Dicke parameter. Here assume that the lasers have the same Rabi frequencies Ω. In the case $\delta \gg \eta\Omega$, there is no energy exchange between the external and internal degrees of freedom. The effective Hamiltonian has same form as Eq. (10), with $\chi = 2\eta^2\Omega^2/\delta$. Thus we can generate the four Bell states and the entangled cluster states of four trapped ions using the procedure similar to that for cavity QED.

Next we give a brief discussion on the experimental matters. For the Rydberg atoms, the radiative time is about $T_r = 3\times 10^{-2}$ s, and the coupling constant is $g = 2\pi\times 24$ kHz [9]. The required atom-cavity-field interaction time is on the order of $T \approx 10^{-4}$ s. Then the time needed to complete the whole procedure is much shorter than T_{r}. Meanwhile it is noted that the atomic state evolution is independent of the cavity field state, thus the cavity decay will not affect the generation of the entangled states.

One of the difficulties for the present scheme is that two atoms are required to be simultaneously sent through a cavity, otherwise there will be an error. Assume that during the generation of the Bell state one atom enters the cavity $0.01t$ sooner than another atom. In the case the fidelity is decreased by $\Delta F \approx \sin^2(0.01\Omega t/2) + \sin^2(0.99\chi t)$. Setting $\Omega = 5\delta$, we have $\Delta F \approx 0.02$.

In conclusion, we have proposed a simple protocol to realize the preparation of the four Bell states and the four-atom entangled cluster states in cavity QED using the interaction of two two-level atoms with a single-mode nonresonant cavity with the assistance of a strong classical driving field. The scheme only involves atom-field interaction with a large detuning and does not require the transfer of quantum information between the atoms and cavity. In addition to the help of a strong classical driving field the photon-number-dependent parts in the evolution operator are canceled. Thus the scheme is insensitive to the thermal field and the cavity decay, and is feasible with current experimental technology.

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