Decoherence induced by a composite environment

Fernando C. Lombardo* and Paula I. Villar†

*Departamento de Física, FCEyN UBA, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Pabellón I,
1428 Buenos Aires, Argentina*

We study the decoherence induced by the environment over a composite quantum system, comprising two coupled subsystems A and B, which may be a harmonic or an upside-down oscillator. We analyze the case in which the B subsystem is in direct interaction with a thermal bath, while the other remains isolated from the huge reservoir. We compare the results concerning the decoherence suffered by the A subsystem.

PACS number(s): 03.65.Yz, 05.40.−a

Decoherence is the process by which most pure states evolve into mixtures due to the interaction with an environment [1]. The very notion of a quantum open system implies the appearance of dissipation and decoherence as a ubiquitous phenomenon and plays important roles in different branches of physics [2,3]. Often times, a large system, consisting of two or a few subsystems interacting with their environment (thermal bath comprising a large number of degrees of freedom), can be adequately described as a composite system. Examples include electron transfer in solution [4], a large biological molecule, vibrational relaxation of molecules in solution, and excitons in semiconductors coupled to acoustic or optical phonon modes. Quantum processes in condensed phases are usually studied by focusing on a small subset of degrees of freedom and considering the other degrees of freedom as a bath.

In this article, we analyze the decoherence induced in a composite quantum system, in which an observer can distinguish between two different subsystems, one of them coupled to an external environment. Our composite system is formed by a subsystem A coupled to a subsystem B which is also bilinearly coupled to an external environment $\mathcal{E}$. The coupling to this external environment is only through subsystem B. Subsystem A remains isolated from $\mathcal{E}$ except for the information delivered by B through a bilinearly coupling between subsystems A and B. We will consider the thermal bath to be at high temperature and will work in the underdamped limit.

In order to investigate this problem, we mainly consider a simple model where subsystem A is represented by a harmonic oscillator and subsystem B is an upside-down one. The main motivation for studying this model is to deepen and enlighten previous analysis of decoherence induced by chaotic environments. The upside-down oscillator has recently been used to model a chaotic environment which induces decoherence on the system [5]. Even though it is an oversimplified model for a chaotic environment, it displays exponential sensitivity to perturbations, which is crucial in order to analyze chaotic evolutions. In this context, we shall consider two different cases. First, we consider the case

where the chaotic degree of freedom is part of the environment (i.e., an unstable system B) and is directly coupled to an external reservoir $\mathcal{E}$ and to another subsystem A with different bare frequency. Secondly, we consider the case where subsystem A is unstable and directly coupled to a harmonic oscillator (subsystem B) which, in turn, is coupled to an external bath $\mathcal{E}$. These are an extension of previous works done in [5] and [6,7] for the first and second case, respectively. In both situations, we will estimate the decoherence time, which is the usual scale after which classicality emerges.

The analysis is completed by the inclusion of the other two different possibilities for the quantum composite system, i.e., a composite system constituted by a subsystem A coupled to subsystem B, both harmonic oscillators, and a composite system formed by subsystem A coupled to subsystem B, both inverted oscillators. As in the other two cases mentioned above, subsystem B is also coupled to an external reservoir $\mathcal{E}$. All in all, we have four different composite systems to analyze. For each and every situation, we study the dynamics of the subsystem A. Not only did we study the influence of "its" environment (formed by subsystem B and $\mathcal{E}$) at low and high temperature, but also in the absence of the external reservoir $\mathcal{E}$. Each case develops a different dynamics, being possible, in some cases, to find a quantum open system described using mixed quantum-classical dynamics [8–10] (part of the composite system completely decohered and the other did not).

The total AB$\mathcal{E}$ classical action is $S[x,q,Q]=S_A[x]+S_B[q]+S_\mathcal{E}[Q]+S_{AB}[x,q]+S_{B\mathcal{E}}[q,Q]$. In the spirit of the quantum Brownian motion (QBM) paradigm, the environment is taken to be a set of N independent harmonic oscillators with frequencies $\widetilde{\omega}_n$, masses m_n, and coordinates $\hat{Q}_n$. Subsystem B consists of a single oscillator (upside-down or harmonic, depending on the case considered) with bare mass M_B, frequency Ω, and coordinate operator $\hat{q}$. The interaction between subsystem B and the thermal environment is assumed to be bilinear $q(s)Q_n(s)$. For simplicity, we assume an Ohmic environment, with the spectral density $I_\mathcal{E}(\omega)=2M\gamma_0\widetilde{\omega}e^{-\widetilde{\omega}^2/\Lambda^2}$, where Λ is a physical cutoff, related to the maximum frequency present in the environment. Finally, we consider subsystem A consisting of a single oscillator (again, this oscillator can be an upside-down or harmonic one) with coordinate operator $\hat{x}$ and frequency ω. We suppose that sub-

---

*Electronic address: lombardo@df.uba.ar
†Electronic address: paula@df.uba.ar

system A is bilinearly coupled to subsystem B by the interaction term $\lambda x(s)q(s)$.

The dynamical properties of interest can be computed from the density matrix of the system at time t. However, if we want to know how the decoherence process is for the subsystem A, we have to trace over all the degrees of freedom that belong to a composite environment. We can assume that our new problem is a subsystem A and a subsystem B which are coupled through an effective interaction defined by

$$S_{\text{eff}}^{\text{int}}(x,q,x',q') = S_{\text{AB}}(x,q) - S_{\text{AB}}(x',q')$$
$$- 2M_B\gamma_0 \int_0^t ds\Delta q(s)\dot{\Sigma}q(s)$$
$$+ i\frac{2M_B\gamma_0 k_B T}{\hbar}\int_0^t ds[\Delta q(s)]^2, \quad (1)$$

where the last two terms are the usual influence action for the QBM problem, in the environmental high-T limit [11–14].

After integrating out the external bath, the information about subsystem A is encoded in the reduced density matrix. That is to say, we have yet to trace over the degrees of freedom of subsystem B. This final reduced density matrix satisfies a master equation which can be presented, as usual, as [12,15]

$$i\hbar\dot{\rho}_r(x,x';t) = \left(-\frac{\hbar^2}{2M_A}\left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2}\right]\right.$$
$$+ \frac{1}{2}M_A\Omega^2(x^2 - x'^2)\right)\rho_r(x,x';t)$$
$$+ \frac{1}{2}M_A\delta\Omega^2(t)(x^2 - x'^2)\rho_r(x,x';t)$$
$$- i\hbar\Gamma(t)(x-x')\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right]\rho_r(x,x';t)$$
$$- iM_A\mathcal{D}(t)(x-x')^2\rho_r(x,x';t)$$
$$- \hbar\Gamma(t)f(t)(x-x')\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial x'}\right]\rho_r(x,x';t),$$
$$(2)$$

where $\mathcal{D}(t)(x-x')^2$ is the new diffusion term, which produces the decay of the off-diagonal elements. For simplicity, we omitted the subindex f to indicate the final configuration x_f. The total diffusion coefficient is given by

$$\mathcal{D}(t) = \frac{2\gamma_0 k_B T}{\hbar\Omega^2}\lambda^2\int_0^t ds\Delta q_{\text{cl}}(s)\dot{\Delta}q_{\text{cl}}(s)$$
$$+ \frac{\lambda^2\sigma}{32\hbar}\int_0^t ds\tilde{\nu}(t-s)\Delta x_{\text{cl}}(s), \quad (3)$$

where $\Delta q_{\text{cl}}(s) = q_{\text{cl}} - q'_{\text{cl}}$ is built from the solution of $\ddot{q}(s) - \Omega^2 q(s) = (\lambda/M_B)x(s)$ (assuming subsystem B is an upside-down oscillator [case (a)]). After imposing initial and final conditions $q(s=0)=q_0$ and $q(s=t)=q_f$, respectively, we write the complete classical solution as

$$q_{\text{cl}}(s) = q_0\frac{\sinh[\Omega(t-s)]}{\sinh(\Omega t)} + q_f\frac{\sinh(\Omega s)}{\sinh(\Omega t)}$$
$$- \frac{\lambda}{M_B\Omega}\frac{\sinh(\Omega s)}{\sinh(\Omega t)}\int_0^t x(u)\sinh[\Omega(s-u)]du$$
$$+ \frac{\lambda}{M_B\Omega}\int_0^s x(u)\sinh[\Omega(s-u)]du. \quad (4)$$

The kernel $\tilde{\nu}$ is the new noise kernel (product of the interaction between subsystem A and B), and it is given by $\tilde{\nu}(s_1-s_2)=(\lambda^2\sigma/32\hbar)\cosh[\Omega(s_1-s_2)]$. σ is the width of the initial wave packet, used to describe the initial state of subsystem B. It is important to note that $\Delta q_{\text{cl}}(s)$ is the solution of the coupled system, and the new noise kernel is not the usual T-dependent noise kernel of the QBM problem.

At this stage, we assume that our subsystem A is a harmonic oscillator (being possible to obtain the solution for an upside-down oscillator by just replacing ω for $i\omega$). If we ask for initial and final conditions of the form $x(s=0)=x_0$ and $x(s=t)=x_f$, the classical solution of the free equation is

$$x_{\text{cl}}(s) = x_0\frac{\sin[\omega(t-s)]}{\sin(\omega t)} + x_f\frac{\sin(\omega s)}{\sin(\omega t)}.$$

After integrating out all the degrees of freedom corresponding to the external hot environment Q_n, and the coordinates q belonging to the subsystem B, we obtained the diffusive terms that induce decoherence on subsystem A. Therefore, we numerically integrated the diffusive terms in time, in order to plot the decoherence factor $\Gamma(t)=\exp\{-\int_0^t\mathcal{D}(s)ds\}$. We will consider four different situations: *Case (a): Harmonic Oscillator+Upside-Down Oscillator+$\mathcal{E}$.* This is the generalization of the toy model considered in Ref. [5] where they did not consider the interaction of subsystem B (upside-down oscillator) with an external environment. It is easy to find results of [5] just by setting $\gamma_0=0$ in our results. Case (a) is the situation in which a Brownian particle (in a harmonic potential) suffers decoherence from an environment with one (or more) chaotic degree of freedom. *Case (b): Upside-Down Oscillator+Harmonic Oscillator+$\mathcal{E}$.* It represents the possibility of studying decoherence induced on an unstable system (toy model for a chaotic subsystem) by a completely harmonic environment [6,7,16,17]. We will see that this is the most decoherent system among all four cases studied in this paper. *Case (c): Harmonic Oscillator+Harmonic Oscillator+$\mathcal{E}$.* For completeness, we also consider the case of two harmonic oscillators coupled together and one of them coupled to an external environment. *Case (d): Upside-Down Oscillator+Upside-Down Oscillator+$\mathcal{E}$.* We will see that this case is the most sensitive to external perturbations (both subsystems are unstable) when there is no external environment, thus decoherence is much more effective than in the other cases. In particular, it is interesting to note that this case decoheres long before the others when there is no thermal environment ($\gamma_0=0$).
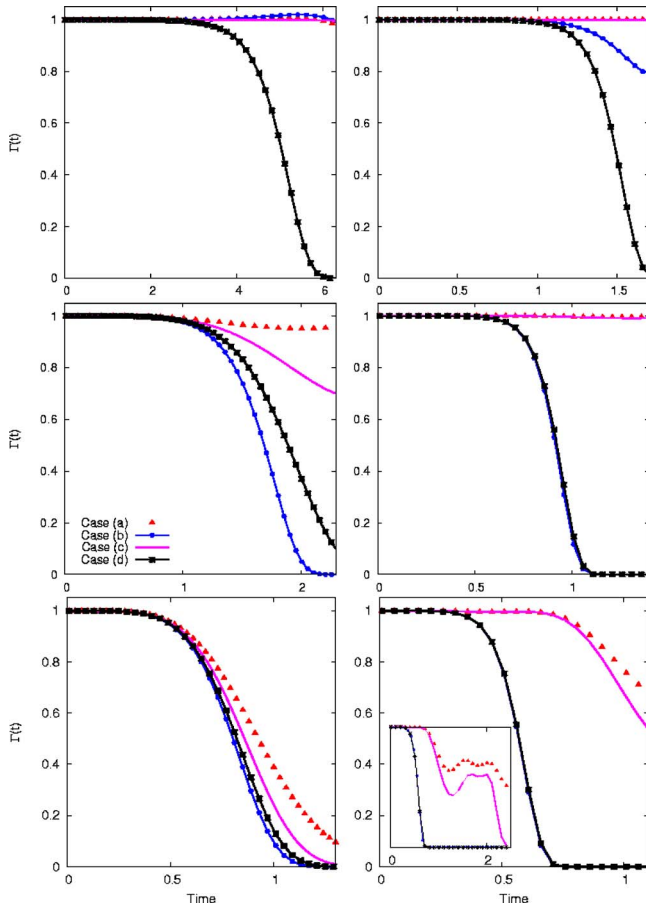
FIG. 1. (Color online) Decoherence factor $\Gamma(t)$. Isolated composite system decoheres first for the (d) case (figures on the top). Plots in the middle and bottom are the decoherence factors for $\gamma_0 k_B T = 1$ and 100, respectively. Case (b) is more decoherent. On the left, both frequencies of the subsystems A and B are of the same order of magnitude ($\omega \approx \Omega$) and on the right we show the case $\omega > \Omega$.

In order to illustrate the different behaviors, we present all four Γ coefficients for two different situations: both frequencies of the subsystems A and B are of the same order of magnitude ($\omega \approx \Omega$) and when $\omega > \Omega$, as shown in Fig. 1. Both cases are considered in the absence of external environment $\mathcal{E}$ (i.e., $\gamma_0 = 0$) and for low and high values of $\gamma_0 k_B T$.

From the numerical results shown on top of Fig. 1, we can stress that in the absence of a hot bath, the decoherence time is smaller in case (d) than in (b), and both of them decohere long before cases (a) and (c). This is due to the fact that subsystem A, which is the sole coupling to subsystem B, generates noise and dissipation at large scales. Thus, this noise and dissipation is bigger when the subsystem B is an upside-down oscillator [case (d)] than when it is a harmonic oscillator [case (b)]. In this situation ($\gamma_0 = 0$), case (d) is two-fold exponential in time. In the former, the oscillatory dynamics of the A oscillator and the hyperbolic stretching of the B environment proceed largely independently of one another. The B environment induces only minor perturbations in the subsystem A and this subsystem does not disturb the environment. The stretching of the environment (due to be-

ing an inverted oscillator) along its unstable manifold is reflected in the system as diffusion. The same physical process occurs in case (b), with the sole and essential difference that the one stretching along an unstable direction is the subsystem A, while the environment is oscillating. As this stretching results in diffusion, the more stretching the system has, the more diffusion it feels. Case (d) is the best example in this "isolated" model, because both A and B stretch along a direction in the phase space, producing exponential diffusion. This is the reason why it is the most decoherent case. Case (c) is shown for completeness, but it is easily seen that decoherence occurs in a longer time scale (there is no stretching here). Therefore, case (d) decoheres at the time in which all the other examples do not.

As soon as the interaction between B and the thermal environment is switched on, oscillator B dissipates not only on the bath but also on A. This is shown in the middle and at the bottom of Fig. 1. At very high environmental temperatures, there is no difference between cases (b) and (d); both of them decohere in the same temporal scale. The huge reservoir dominates the diffusion coefficient [first term in Eq. (3)]. But they still differ from the cases where there are harmonic oscillators as subsystems A [cases (a) and (c)]. The inset in Fig. 1 (at the right bottom) presents the behavior of the $\Gamma(t)$ factors for these cases for a longer time scale. We can observe that we need to wait longer times for decoherence to be effective in cases (a) or (c) with respect to (b) and (d) even in the highest temperature case. It is possible to observe some recoherent effects in the case c, at the time in which unstable subsystems have fully decohered.

When the final system A is an upside-down oscillator [7,17], an unstable point forms in the center of the phase space with associated stable and unstable directions. These are characterized by Lyapunov coefficients Λ ($\Lambda = 2\omega^2$ in the linear case). The time dependence of the package width in the direction of the momenta is given by $\sigma_p(t) = \sigma_p(t_0) \exp[\Lambda t]$, where $\sigma_p(t_0)$ is the corresponding width at the initial time.

Diffusion effects limit the squeezing of the state on the phase space. The bound on the width of the packets is given by $\sigma_c = \sqrt{2\mathcal{D}_i/\Lambda}$ [3,14] (where i is b or d). There is another scale, t_{max}, corresponding to the time in which decoherence starts to be effective, and after which squeezing becomes of the order of the limiting value. One can estimate the time corresponding to the transition from reversible to irreversible evolution as $t_c = (1/\Lambda)\ln[\sigma_p(0)/\sigma_c]$. Thus, we can use this scale as the typical scale for decoherence, setting $t_D \approx (1/\Lambda)\ln[\sigma_p(t_{max})/\sigma_c]$. Therefore, we obtain $t_D = t_{max} + (1/\Lambda)\ln[\sigma_p(0)/\sigma_c]$. For the same parameters used in Fig. 1, we can numerically estimate decoherence times as $t_{D_b} \sim 7.7$ and $t_{D_d} \sim 6.4$ for the first set of parameters on the left of Fig. 1, where $\gamma_0 = 0$; $t_{D_b} \sim 2.4$ and $t_{D_d} \sim 2.7$ for $\gamma_0 k_B T = 1$, and $t_{D_b} \sim 1.6$ and $t_{D_d} \sim 1.7$ in the high-T case $\gamma_0 k_B T = 100$. For the set in Fig. 1 on the right, we obtain $t_{D_b} \sim 3.0$ and $t_{D_d} \sim 2.7$ for $\gamma_0 = 0$. We also got $t_{D_{b,d}} \sim 0.1$ for $\gamma_0 k_B T = 1$, and $t_{D_{b,d}} \sim 0.6$ in the case $\gamma_0 k_B T = 100$. All these results agree with the decoherence times, defined by the times at which the decoherence factor $\Gamma(t)$ goes to zero, which can be seen in the plots above.

Decoherence times for cases (a) and (c) occur as for the usual harmonic systems. We can estimate them by using the result of the high-temperature limit of the QBM paradigm, i.e., t_D is the solution of $1 \approx L^2 \int_0^{t_D} \mathcal{D}(s)ds$ (we have to take the typical distance L as 2σ, proportional to the dispersion in position of our initial packet). The inset in Fig. 1 on the right shows $\Gamma(t)$ for a longer time scale in order to establish the corresponding hierarchy in the environmental decoherent effectiveness.

In this article we analyzed the decoherence induced by an effective environment. The effective environment was considered to be formed by part of a composite system and an infinite set of harmonic oscillators. The composite system was considered to be any of the four possible combinations made up with a harmonic and an inverted oscillator.

Since a set of harmonic oscillators is a stable system, small perturbations due to the state of the coupled system do not induce exploration of a large volume of the phase space for any oscillator. When one considers an inverted oscillator, it can explore its volume more efficiently when it is perturbed.

We integrated out subsystem B in order to study the effect of having (or not) unstable degrees of freedom into the full environment. Then we analyzed different situations and concluded that cases (b) and (d) are the most efficient (smaller decoherence times) at high temperatures, and (d) is the most diffusive case, when one turns off the thermal bath. There is a clear hierarchy between the different compositions of the composite systems. Those in which oscillator A is unstable [cases (b) and (d)] decohere before those with a harmonic oscillator as the A subsystem [cases (a) and (c)]. At high temperatures of the external environment, it has been shown that cases (b) and (d) have the same decoherence time scale, while composite system (c) loses quantum coherence before case (a).

[1] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).

[2] E. B. Davies, *Quantum Theory of Open Systems* (Academic, London, 1976); B. J. Lindenberg and B. J. West, *The Nonequilibrium Statistical Mechanics of Open System* (VCH, New York, 1990); U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).

[3] J. P. Paz and W. H. Zurek, in *Coherent Matter Waves, 72nd Les Houches Summer School*, edited by R. Kaiser, C. Westbrook, and F. David (Springer-Verlag, Berlin, 2001), pp. 533–614.

[4] O. V. Prezhdo and P. J. Rossky, Phys. Rev. Lett. **81**, 5294 (1998).

[5] R. Blume-Kohout and W. H. Zurek, Phys. Rev. A **68**, 032104 (2003).

[6] F. C. Lombardo, F. D. Mazzitelli, and D. Monteoliva, Phys. Rev. D **62**, 045016 (2000); N. D. Antunes, F. C. Lombardo, and D. Monteoliva, Phys. Rev. E **64**, 066118 (2001).

[7] Hans-Thomas Elze, *Vacuum-Induced Quantum Decoherence and the Entropy Puzzle*, e-print hep-ph/9407377.

[8] M. Toutounji and R. Kapral, Chem. Phys. **268**, 79 (2001); O. V. Prezhdo and V. V. Kisil, Phys. Rev. A **56**, 162 (1997).

[9] K. Shiokawa and R. Kapral, J. Chem. Phys. **117**, 7852 (2002).

[10] A. Venugopalan, Phys. Rev. A **61**, 012102 (1999).

[11] R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).

[12] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992).

[13] H. Grabert, P. Schramm, and G. L. Ingold, Phys. Rep. **168**, 115 (1988).

[14] J. P. Paz, S. Habib, and W. H. Zurek, Phys. Rev. D **47**, 488 (1993).

[15] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **47**, 1576 (1993).

[16] F. C. Lombardo and P. I. Villar, Phys. Lett. A **336**, 16 (2005).

[17] W. H. Zurek and J. P. Paz, Phys. Rev. Lett. **72**, 2508 (1994).