Breakdown of the Born approximation in laser phase-noise to amplitude-noise conversion

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When a vapor absorbs laser light, the field's intrinsic phase fluctuations (PM) induce variations in the atoms' absorption cross section, which in turn yield fluctuations in the transmitted light intensity. In the Born approximation, the observed amplitude noise (AM) arises from a single scattering of the input field. Here, we show that the Born approximation breaks down for optically thick vapors and that the observed AM is influenced by scatterings of the "medium-perturbed" field. With importance for spectroscopy, we find that the multiple field-atom scatterings *reduce* the PM-to-AM conversion efficiency.

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When a single-mode field passes through a resonant vapor, the act of photon absorption causes the field to develop excess amplitude noise [1,2]. Briefly, the field's intrinsic phase fluctuations (PM) induce variations in the atoms' or molecules' absorption cross section, and these in turn yield amplitude variations (AM) in the field [3]. This PM-to-AM conversion process is a fundamental characteristic of the field-atom interaction: (1) the process acts through fluctuations in the field-created atomic superposition state, and (2) all fields exhibit some degree of phase (i.e., frequency) variability. Therefore, in *any* experiment pertaining to the field-atom interaction the phenomenon is operative and can have important consequences (e.g., an additional source of noise in quantum nondemolition measurements). Though PM-to-AM conversion has found application as a novel "noise spectroscopy" [4], it is generally problematic for precision spectroscopy and metrology [5], and at present is not fully understood.

Theoretical descriptions of PM-to-AM conversion typically invoke some form of Born approximation: "…the wave is expressed as the sum of the incident wave and a diffracted secondary wave, [where] the scattering of the secondary wave is neglected" [6]. Consequently, the laser's transmitted intensity variations arise from what amounts to a single scattering of the incident stochastic field with the medium. The question we address here concerns the validity of this approximation, especially in the relatively important situation of optically thick vapors where a laser's amplitude noise can build non-negligibly as it propagates through the resonant medium [7]. While atoms located near the entrance to an optically thick vapor will interact with a field only exhibiting the laser's intrinsic phase variations, atoms located near the exit will interact with a modified field that suffers both phase and amplitude noise. In such a situation the field experiences multiple scatterings with the medium prior to detection, and the validity of the Born approximation must be called into question. At issue in this work is the manner, the significance, and under what conditions these multiple field-atom scatterings influence the intensity noise of a laser after it has propagated through a resonant medium.

Our experimental arrangement is relatively simple and illustrated in Fig. 1. The output from a transverse junction stripe (TJS) single-mode diode laser, with a linewidth of approximately 20 MHz, was collimated into a ~0.8-cm-diam beam and attenuated with neutral density filters (ranging from 1.9 to 2.4) before passing through a cylindrical glass resonance cell (L=3.9 cm and $2R$=2.2 cm) containing ^{87}Rb and 3 torr N_2. This laser had a measured relative intensity noise of −123 dB and therefore corresponded to a nearly ideal phase diffusion field (PDF). Without neutral density filters in the beam path, the laser intensity entering the cell was 560 μW/cm^2. The laser was tuned to the D_1 transition of Rb at 795.0 nm, and the transmitted intensity was detected with a Si photodiode. For a given resonance cell temperature we performed a number of "experimental runs" corresponding to different average light intensity levels. For a single run, we measured the average transmitted intensity $\langle I\rangle$ and intensity noise δI_{rms} as functions of laser detuning, effected by varying the diode laser's injection current i [8]. (On resonance i/i_{th}=1.76, where i_{th} is the laser's threshold current.) Here, δI_{rms} is the root-mean-square noise in a 1-Hz bandwidth at a Fourier frequency of 212 Hz, which we measured with a spectrum analyzer. With $\langle I_0\rangle$ the average light intensity transmitted by the resonance cell in the absence of the vapor (i.e., the laser tuned off resonance), and $\langle I_{res}\rangle$ the average light intensity transmitted by the vapor with the laser tuned to the (F'=2)–(F=1) transition as indicated in Fig. 2, we estimated the vapor's optical depth $(N\langle\sigma\rangle)^{-1}$ for a given light intensity and cell temperature using

$$N\langle\sigma\rangle = \frac{1}{L}\ln\left(\frac{\langle I_0\rangle}{\langle I_{res}\rangle}\right).$$

Here, N is the number density of absorbing alkali-metal atoms and $\langle\sigma\rangle$ is the average absorption cross section [i.e.,

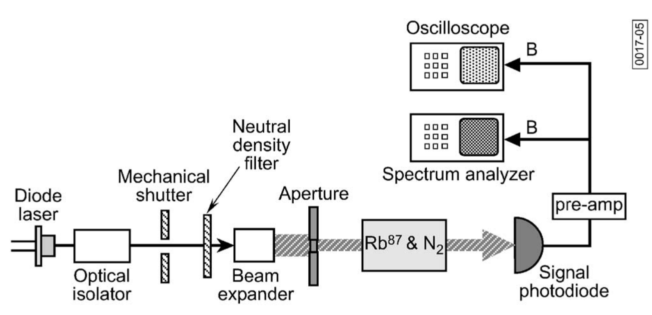

FIG. 1. Experimental arrangement.

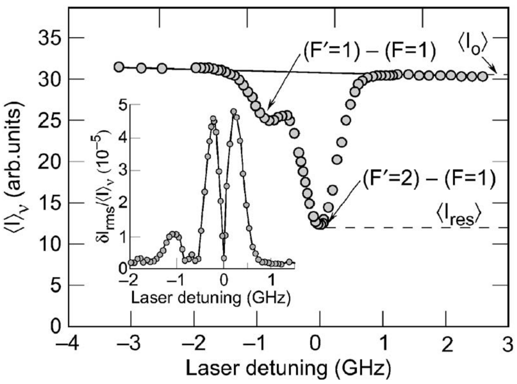

FIG. 2. Transmitted light intensity as a function of laser detuning with a resonance cell temperature of 41.1 °C and a 2.2 neutral density filter in the laser beam path. The laser was tuned to the ^{87}Rb D_1 transition; specifically, the $5\,^2P_{1/2}(F'=1)$-$5\,^2S_{1/2}(F=1)$ and $5\,^2P_{1/2}(F'=2)$-$5\,^2S_{1/2}(F=1)$ transitions. The inset shows the measured relative intensity noise (RIN) of the transmitted light as a function of laser detuning. Note that the significant decrease in RIN on resonance attests to the near ideal PDF nature of our laser.

$\sigma(t)=\langle\sigma\rangle+\delta\sigma(t)]$. To account for optical pumping effects, which could modify N, $(N\langle\sigma\rangle)^{-1}$ was extrapolated to zero light intensity, and this yielded our measure of optical depth for a given resonance cell temperature. (We note that linear extrapolation is valid under our conditions of weak optical pumping: the collisional relaxation rate in our 3-torr N_2 cell was $\sim 880\ \text{sec}^{-1}$ and our maximum optical pumping rate was $\sim 35\ \text{sec}^{-1}$.) We found that our optically determined values of $N\langle\sigma\rangle$ were linear with the N inferred from resonance cell temperature measurements using Killian's formula ($r^2=0.985$) [9], and these yielded $\langle\sigma\rangle=2.4\times10^{-12}\ \text{cm}^2$. In what follows, we employ the optically determined values of $N\langle\sigma\rangle$ as our measure of alkali-metal number density.

In the limit of small deviations from the Born approximation (and weak fields), it is valid to employ the Beer-Lambert law of exponential attenuation and write[10]

$$I(t)=I_0\exp\{-N[\langle\sigma\rangle+\delta\sigma(t)]L\}$$
$$=I_0\exp[-N\langle\sigma\rangle L]\{1-N\delta\sigma(t)L+[N\delta\sigma(t)L]^2/2+\cdots\}.$$

Further, we allow $\delta\sigma(t)$ to correspond to an *arbitrary* random process, subject only to the mild (and physically reasonable) constraint that the $\delta\sigma$ be at all times described by a normal probability distribution (NPD). [For a random walk process, this is equivalent to stating that $\delta\sigma(t)$ follows a normal probability distribution whose mean is zero but whose variance grows linearly in time.] Defining $\langle\delta I_{\text{PA}}^2\rangle$ as the variance of intensity fluctuations solely due to PM-to-AM conversion and given the NPD nature of the $\delta\sigma(t)$, it is relatively straightforward to show that only even powers of $\langle\delta\sigma^n\rangle$ can contribute to $\langle\delta I_{\text{PA}}^2\rangle$. Therefore, ignoring terms of order $(N\delta\sigma_{\text{rms}}L)^3$ and higher, we find that $(\delta I_{\text{PA}}/\langle I\rangle_\nu)=NL\delta\sigma(\nu)_{\text{rms}}$, where δI_{PA} is now the rms value of intensity fluctuations due to PM-to-AM conversion and the expression corresponds to laser detuning ν.

Based on the work of Walser and Zoller [2], $\delta\sigma(\nu)_{\text{rms}}$ will be independent of N for a PDF as long as the Born approximation is valid. Thus, in optically thin situations (i.e., single field-atom scatterings) the relative intensity noise (RIN) of transmitted laser light arising from PM-to-AM conversion will be a linear function of $N\langle\sigma\rangle L$. In particular, as we are interested in the resonance as a whole we integrate the expression for RIN over ν, indicating the frequency averaging with an overbar, and thereby obtain $\overline{(\delta I_{\text{PA}}/\langle I\rangle)}=N\langle\sigma\rangle L(\overline{\delta\sigma_{\text{rms}}}/\langle\sigma\rangle)$; for convenience, we will refer to this expression as PM-to-AM relative intensity noise or PM-to-AM RIN.

To go beyond the Born approximation, we write $\overline{\delta\sigma_{\text{rms}}}=\delta\sigma_0/(1+\zeta\sqrt{N\langle\sigma\rangle L})$. Theoretical justification for this empirical expression is provided by the work of Walser and Zoller [2], where it is shown that for input fields dominated by amplitude noise the transmitted AM varies like $\sqrt{N\langle\sigma\rangle L}$. Therefore, as the Born approximation breaks down and RIN is generated by a field with amplitude variations "picked up" upon propagation through the resonant medium, we should observe a sublinear dependence of PM-to-AM RIN on the alkali-metal density.

In the experiment, the measured rms intensity noise δI_{rms} corresponds to a sum of independent effects: $\delta I_{\text{rms}}^2=\delta I_{\text{PA}}^2+\delta I_{\text{int}}^2+\delta I_{\text{shot}}^2+\delta I_{\text{dark}}^2$. Here, δI_{dark} is the dark noise, δI_{shot} is the shot noise, and δI_{int} is the laser's intrinsic relative intensity noise [11]. For each run we blocked the laser and obtained a measure of δI_{dark}. Then, with the laser unblocked and off resonance we measured $\delta I_{\text{int}}^2+\delta I_{\text{shot}}^2$. For our 73 runs we plotted $(\delta I_{\text{int}}^2+\delta I_{\text{shot}}^2)/\langle I_0\rangle$ as a function $\langle I_0\rangle$; this yielded a straight line with an intercept β, providing a calibration for δI_{shot}, and a slope α, providing a measure of δI_{int}. As indicated above, we averaged $\delta I_{\text{rms}}^2/\langle I\rangle_\nu^2$ over a laser detuning Δ of 1.3 GHz (i.e., from -550 MHz to 750 MHz):

$$\frac{1}{\Delta}\int\frac{\delta I_{\text{PA}}^2}{\langle I\rangle_\nu^2}d\nu=\frac{1}{\Delta}\int\frac{\delta I_{\text{rms}}^2}{\langle I\rangle_\nu^2}d\nu-\alpha-\frac{\beta}{\Delta}\int\frac{d\nu}{\langle I\rangle_\nu}-\frac{\delta I_{\text{dark}}^2}{\Delta}\int\frac{d\nu}{\langle I\rangle_\nu^2}. \tag{1}$$

Generally, the corrections to δI_{rms} on the right-hand side (RHS) of Eq. (1) were small. On average the first term on the RHS of Eq. (1) was 1.7×10^{-9}; the second term 4.8×10^{-13}, the third term 3.5×10^{-12}, and the last 1.5×10^{-10}. Again, to account for optical pumping effects, $\overline{(\delta_{\text{PA}}/\langle I\rangle)}$ was extrapolated to zero light intensity for each resonance cell temperature, and the results are shown in Fig. 3. Employing our empirical expression for $\overline{(\delta_{\text{PA}}/\langle I\rangle)}$ we obtain $\delta\sigma_0=9.7\times10^{-17}\ \text{cm}^2$ and $\zeta=0.21$ from a nonlinear least squares fit to the data. Given that $\zeta<1$, so that the nonlinearity in RIN only becomes significant for $N\langle\sigma\rangle L>1$ (a regime where we expect multiple field-atom scatterings to be important), given the linearity between the optically determined values of $N\langle\sigma\rangle$ and the temperature determined values of N (indicating that nonlinearity cannot be associated with a systematic $N\langle\sigma\rangle$ effect), and given that the RIN transitions to a $\sqrt{N\langle\sigma\rangle L}$ dependence as suggested theoretically, we associate the nonlinearity of Fig. 3 with a breakdown of the Born approximation.

Though observation of the Born approximation's violation is important for understanding present theory's limits of reliability, just as important (and perhaps more so) is our discovery that the rate of increase of PM-to-AM RIN slows

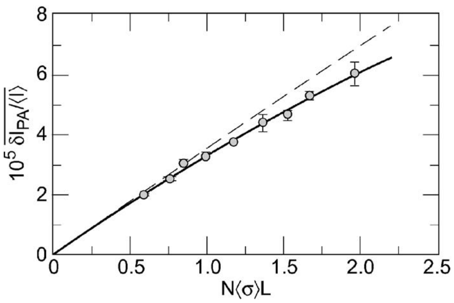

FIG. 3. Laser-detuning-averaged relative intensity noise as a function of the attenuation coefficient, $N\langle\sigma\rangle L$. The dashed line shows the linear trend, while the solid line is a nonlinear least-squares fit to the empirical expression discussed in the text.

down in optically thick vapors. One could easily imagine that as a propagated field picks up amplitude noise in traversing a resonant medium, the combined effects of laser phase noise and amplitude noise would make the detected intensity fluctuations worse, arguing that the results of Walser and Zoller [2] do not provide a good guide to the stochastic-field–atom interaction problem when the Born approximation breaks down. However, our results suggest that the results of Walser and Zoller can (at a minimum) provide intuitive insight into the multiple-scattering regime and therefore that the field's incurred amplitude noise tends to *inhibit* the PM-to-AM conversion process.

Notwithstanding the qualitative agreement between our results and theoretical expectations, a number of questions regarding the stochastic-field–atom interaction problem are raised by the present study. How does ζ depend on the stochastic characteristics of the incident field, in particular the incident field's linewidth? How does the present work's empirical expression for $\overline{\delta\sigma_{\mathrm{rms}}}$ [i.e., $\delta\sigma_0/(1+\zeta\sqrt{N\langle\sigma\rangle L})$] relate to theory? In the regime of multiple scattering, what is the nature of the transmitted field's second-order correlation function, and how does this relate to the incident field's first-order correlation function? Appreciating the fact that most naturally occurring electromagnetic fields are stochastic, answers to these questions will be relevant to our basic understanding of radiative processes.

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