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Simultaneous cooling and trapping of atoms by a single cavity-field mode

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The optical dipole trap scheme is generalized by considering the laser as a dynamical cavity field mode. For a large enough finesse, the far-detuned trapping field itself exerts a significant friction force on the atom. Unlike in free-space far-off resonance dipole traps, here the trapping is stable in the steady state. The scheme in principle applies to arbitrary linearly polarizable particles, e.g., molecules.

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Far-off-resonance dipole traps (FORT) are commonly used for long-time capturing and localization of neutral atoms by laser light fields [1]. The basic idea behind tuning the laser frequency ω_L very far below the atomic resonance ω_A resides in the fact that the depth of the trap potential and the spontaneous photon scattering rate scale differently with the detuning $\Delta_A=\omega_L-\omega_A$. The former is proportional to Ω^2/Δ_A while the latter to Ω^2/Δ_A^2 where Ω is the Rabi frequency of the atom-laser coupling. As a consequence, deep traps can be formed at a reduced level of the recoil noise generated by spontaneous emissions in the large detuning limit. This scheme of almost conservative trapping preserves the coherence of the atomic gross motion. This is vital for all-optical Bose condensation [2], and also for processing quantum information with neutral atoms carrying a q-bit in their internal degree of freedom. Furthermore, because of the large detuning, the trapping potential captures simultaneously atoms in different Zeeman sublevels of the ground state, an important element of the condensation of Cs [3], or different isotopes to create fermion-boson mixtures of Yb [4].

Nevertheless, in steady state the atoms would not be localized at the antinodes of the field in a FORT because Doppler cooling is inefficient in the large Δ_A limit. Long trapping times are obtained only for an initially cold ensemble because the relaxation process, i.e. spontaneous photon scattering is very slow. Its rate is usually so low that, in fact, the trapping time is limited rather by technical noise in the experiments [5].

In this Communication we revisit the FORT scheme in a generalized form. The square of the Rabi frequency, Ω^2, is a key parameter, which is determined by the local energy density of the electromagnetic field at the atomic position. However, a given energy density can be provided in a variable manner since it depends not only on the number of photons but also on, roughly speaking, the "volume" of a photon. Consider now that the field is enclosed in a Fabry-Perot resonator where the photons have a well defined mode volume. The shorter the cavity length, the smaller number of photons are needed to reach a given local field density. Simply, photons are "recycled" by mirror reflections and hit the atom more frequently in a unit of time. The opposite limit of infinite mode volume and infinite number of photons, keeping their ratio constant, corresponds to the standard optical lattice of a free standing-wave laser field.

We will show that, keeping the Rabi frequency Ω invariant but reducing the mode volume, the atomic motion is dramatically influenced. The main effect is that a strong localization and stable trapping of atoms *in steady state* becomes possible, at a very small degree of perturbation of the atomic ground state. Let's emphasize that the strong-coupling regime of cavity QED is not needed. Even for a moderate coupling between the cavity photons and the atomic dipole, the trapping time gets longer by many orders of magnitude than it would be in a free-space FORT with the same Rabi frequency. The effect relies on the correlations between the atomic motion and the field which yield, on top of the far-off resonance trapping, a cavity-cooling mechanism [6,7], experimentally demonstrated recently [8]. In this generalization of FORT the necessary and sufficient cooling is provided by the trapping field itself without the need for additional near-resonant fields [9,10], or for a magic wavelength [11,12].

For the actual discussion we consider a single two-level atom moving in one dimension along the cosine mode of a cavity. In accordance with the large detuning limit, i.e. Δ_A is much larger than the atomic linewidth γ (HWHM), the atom is expected to behave as a linearly polarizable particle and we can disregard its internal structure and dynamics. By this assumption, we completely eliminate the friction force responsible for Doppler cooling and for the finite steady-state temperature of the atom in a standing-wave laser field. The linear atomic polarization, $\mathbf{P}=\varepsilon_0\chi\mathbf{E}$, is expressed via the complex susceptibility

$$\chi=\chi'-i\chi''=-\frac{|\langle g|\mathbf{d}|e\rangle|^2}{\hbar\varepsilon_0(\Delta_A+i\gamma)}, \tag{1}$$

where $\mathbf{d}$ is the atomic dipole moment operator, and its matrix element is taken between the ground and excited states, $|g\rangle$ and $|e\rangle$, respectively.

In contrast to the standard FORT scheme, the laser field does not directly interact with the atom. It pumps a cavity mode which is dynamically coupled to the atom. The cavity resonance frequency ω_C may differ from the laser frequency, thus an additional system parameter, the detuning $\Delta_C=\omega_L-\omega_C$ is introduced.

The fully quantum mechanical model of the system consists of the Master equation

$$\dot{\rho} = \frac{1}{i\hbar}[H,\rho] + \mathcal{L}\rho, \tag{2}$$

with the Hamiltonian and Liouvillean involving the cavity mode operators, a and $a^\dagger$, and the atomic position, x, and momentum, p, operators:

$$H = \frac{p^2}{2\mu} - \hbar(\Delta_C - U_0\cos^2(kx))a^\dagger a - i\hbar\eta(a - a^\dagger), \tag{3a}$$

$$\mathcal{L}\rho = \kappa(2a\rho a^\dagger - [a^\dagger a,\rho]_+) + \Gamma_0\left(2\int_{\mathbf{u}}\cos(kx)ae^{-iku_x x}\rho e^{iku_x x}\cos(kx)a^\dagger - [\cos^2(kx)a^\dagger a,\rho]_+\right). \tag{3b}$$

k, κ, and η is the wave number, the half of the mode line width, and the effective pumping strength for the mode, respectively. According to the above discussion, the atomic internal dynamics is represented only by the light-shift and absorption processes,

$$\hbar U_0 = -\frac{\hbar\omega_C}{V}\chi', \quad \hbar\Gamma_0 = -\frac{\hbar\omega_C}{V}\chi'', \tag{4}$$

where V is the mode volume, hence the prefactor is the energy density at an antinode for one single photon in the cavity. In Eq. (3b) the integrational variable $\mathbf{u}$ is the direction vector of the spontaneously emitted photon. In the same term the factor $e^{iku_x x}$ accounts for the atomic recoil. The wave number of the emitted photon can be taken equal to that of the mode since the detuning is negligible compared to the frequencies.

In the forthcoming discussions we will present solutions of Eq. (2) obtained by two methods. On the one hand, we will use the semiclassical approach introduced in Ref. [13] to describe the system when the photon number is large ($\gtrsim 4$ is sufficient). For the complementary low photon number regime, we adopt the Monte Carlo wave function method to approximate the exact density matrix, details are given in Ref. [14].

The semiclassical equations serve as a good starting point to outline the principal question here. As discussed in detail in Ref. [13] they are derived from Eq. (2) by truncating the partial differential equation for the joint atom-cavity Wigner function to obtain a Fokker-Planck equation and taking the equivalent Langevin equation for the stochastic atomic variables x,p, and complex field amplitude α. The atom moves under the action of the field as

$$\dot{x} = p/m,$$

$$\dot{p} = \hbar k\, U_0|\alpha|^2\sin(2kx) + \xi_p, \tag{5a}$$

where $|\alpha|^2$ is the photon number. One can recognize the local energy density $\hbar\omega_C|\alpha|^2/V$ in the force term. In the following we attempt to keep this constant while varying the mode volume V from large values ($\leftrightarrow$ standard FORT) towards the "cavity FORT" regime, where much fewer photons create the same local field density. We note that, for simplicity, the cavity loss rate κ is assumed to be constant.

The field amplitude α is a dynamical variable, its evolution couples to the atomic position as

$$\dot{\alpha} = (i\Delta_C - \kappa - (iU_0 + \Gamma_0)\cos^2(kx))\alpha + \eta + \xi_\alpha. \tag{5b}$$

With properly setting the pump field η and the detuning Δ_C, the stationary mean α can be adjusted at will for a fixed position of the atom. The coupled equations of motion, on average, can be kept closely invariant under the variation of V. The only point for which this cannot be done exactly is the correlated fluctuations of the atomic position and the amplitude α. The fluctuations stem from the noise terms ξ_p and ξ_α which obey second-order correlation laws:

$$\langle\xi_\alpha^*\xi_\alpha\rangle = \kappa + \Gamma_0\cos^2 kx, \tag{6a}$$

$$\langle\xi_p\xi_\alpha\rangle = -i\hbar k\Gamma_0\cos kx\sin kx, \tag{6b}$$

$$\langle\xi_p\xi_p\rangle = 2\hbar^2k^2\Gamma_0|\alpha|^2(\overline{u^2}\cos^2 kx + \sin^2 kx). \tag{6c}$$

The efficient way to simulate these noise processes is written in Ref. [6].

The Rabi frequency, which we want to keep constant, can be expressed in terms of the single-photon Rabi frequency g as $\Omega^2 = g^2|\alpha|^2$. In Fig. 1 the steady-state properties of a single trapped atom are plotted for varying g, representing the variation of the mode volume V via $g \propto 1/\sqrt{V}$. The detuning is set to $\Delta_C = -\kappa + U_0$, and the pumping strength is varied as $\eta \propto 1/g$ to keep Ω^2 constant. One can verify that the mean atomic excitation, i.e. the probability P_e of the two-level system being in the excited state, is also kept constant by this assumption. It is about 0.05 for the whole range of this plot with a small residual variation due to the localization effect, i.e. the atoms are not always at the very antinode of the field where they are maximally coupled to the mode.

The kinetic temperature drops several orders of magnitude for increasing g, finally reaching the limiting temperature about $k_B T \approx \hbar\kappa$. Recall that in free-space Doppler cooling $(g \to 0)k_B T \approx \hbar\Delta_A/2$ for $\Delta_A \gg \gamma$. The temperature reduction can be attributed to the action of the coupled atom-field dynamics. For low enough temperature, trapping becomes possible, which is demonstrated by the localization plot in Fig. 1(b). The measure is the mean spread $\langle(kx/\pi)^2\rangle$ in units of $E \equiv 1/12 \approx 0.08$ which corresponds to the uniform distribution. A mean spread significantly below E expresses that the atom is localized in the vicinity of an antinode. Note that the localization is destroyed again for large values of g where the mean intensity corresponds to only a few photons. Such a minimum behavior would not be possible were the atom moving in a normal potential, because the virial theorem ensures a monotonic connection between the mean kinetic and the potential energy. Here, however, the potential in the Hamiltonian Eq. (3a) is proportional to $a^\dagger a$. Hence the part of the joint atom-cavity wave function which is associated with the 0th photon number state does not feel any potential and spreads freely.

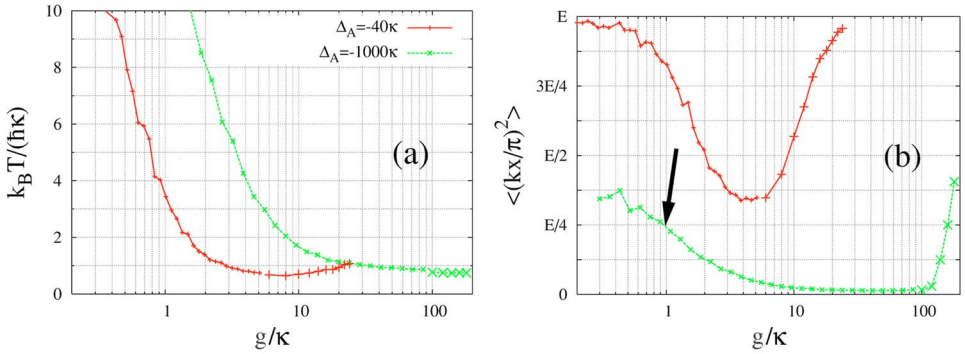

FIG. 1. (Color online) (a) Steady-state kinetic temperature and (b) localization of the atomic center-of-mass motion as a function of the coupling constant to the cavity mode. The localization is given in units of $E \equiv 1/12$ corresponding to a uniform distribution. The two curves were generated for different atomic detunings, $\Delta_A = -40\ \kappa$ (solid line) and $\Delta_A = -1000\ \kappa$ (dashed line), and $\kappa = \gamma/2$ was set for both. The bigger marks (for large g values) originate from the quantum Monte Carlo simulations, the smaller ones from the semiclassical approach. $P_e = 5\%$ on the whole range of g.

The cooling mechanism, responsible for the steady-state localization, can be analytically shown to survive in the large detuning limit, $\Delta_A \gg \gamma$ (see also Ref. [7]). The linear response of the field amplitude to the atomic motion can be calculated from Eq. (5b), which yields a linear friction force with a coefficient

$$\frac{\beta}{2\gamma P_e} = \frac{\hbar k^2}{2m\gamma} 4\sin^2(kx) \times \frac{2g^2(\Delta_C - U_0\cos^2(kx))(\kappa + \Gamma_0\cos^2(kx))}{\left[(\Delta_C - U_0\cos^2(kx))^2 + (\kappa + \Gamma_0\cos^2(kx))^2\right]^2} \tag{7}$$

where we normalized it to the rate of spontaneous photon scattering. A similar result, without the normalization to P_e, was obtained in Refs. [15,16]. It follows that the optimum cavity detuning is

$$\Delta_C \approx -\kappa - \Gamma_0 + U_0 \approx -\kappa + U_0, \tag{8}$$

which is exactly what we used for the plots in Fig. 1. The maximum friction coefficient, spatially averaged, is

$$\frac{\beta}{2\gamma P_e} = \frac{\hbar k^2}{2m\gamma}\left(\frac{g}{\kappa}\right)^2, \tag{9}$$

which proves the invariance of the friction coefficient per saturation as a function of the atomic detuning. Dislike with Doppler cooling in free laser fields, going away from resonance with the saturation kept constant does not make the damping processes vanish. For $g \sim \kappa$, the friction force equals the maximum of the Doppler friction force. Since the relation of g and γ is irrelevant here, the regime of strong-coupling cavity QED is not necessary. Optical lattices in resonators fulfilling $g \sim \kappa$, e.g. Ref. [17] are suitable for this type of generalized FORT scheme.

Stable trapping is achieved if the steady-state temperature is well below the trap depth. The former can be estimated by taking the large Δ_A limit of the results of Refs. [18,19]. It yields, for uniform distribution,

$$k_B T = \hbar\kappa\left(1 + \frac{\kappa\gamma}{g^2}\right), \tag{10}$$

which is in accordance with the limiting temperature in Fig. 1(a). In an intermediate range of g the simulated temperature is in fact lower, an effect due to the localization where Eq. (10) holds only approximately. As the temperature scales with κ, it can be independently controlled and chosen far below the trap depth, the latter being about $\hbar U_0|\alpha|^2 = \hbar\Delta_A P_e$.

For very large detuning the trap frequency ν starts to dominate the other frequencies of the system (resolved sideband limit). For about 5% saturated Rb atoms the limit is $\Delta_A \gtrsim 10^4\gamma$, but it can be pushed to higher detunings Δ_A by reducing the saturation. In the resolved sideband limit the validity of the analytical calculations [Eq. (7)] breaks down, since they rely on the assumption of small atomic velocity ($kv \ll \kappa$). The simulations presented above would work in this limit as well. With different tuning, $\Delta_C \approx -2\nu$, cavity sideband cooling is possible though very inefficient in the given geometry (only two-phonon transitions are allowed). In this case the use of an external probe field tuned to the red sideband is a straightforward solution. Cavity cooling in the resolved sideband limit is considered elsewhere [7,20], here we concentrate only on the cooling mechanism around $\Delta_C = -\kappa$.

The role of the cavity cooling mechanism can be seized in the variation of steady-state properties as a function of the detuning between the pump laser frequency and the cavity resonance. In Fig. 2 we plot the mean escape time of the atom from a trapping site, which is an experimentally accessible quantity. In the simulations the initial distribution represents atoms at the antinode with a small velocity spread. It is the dipole force fluctuations and, to a lesser extent, the spontaneous recoil noise which heats up the atom to escape. A significant peak can be observed at about $\Delta_C \approx -\kappa$. The variance of the escape times is also presented in the same plot. From the fact that the mean and the variance are almost equal we can deduce that the distribution of the escape times is close to an exponential one, which is expected in a steady

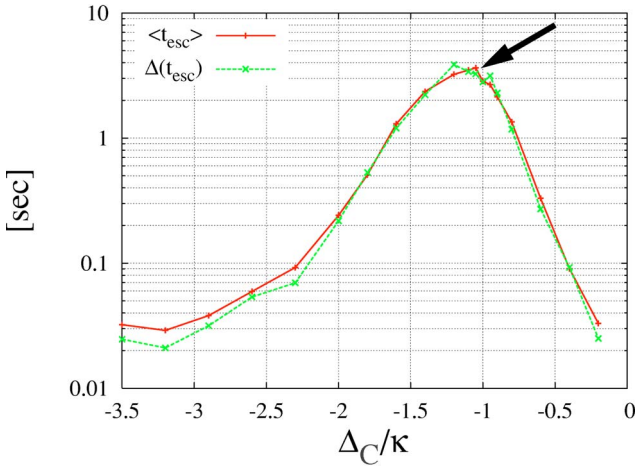

FIG. 2. (Color online) Trapping times as a function of the detuning of the field from the cavity resonance. Parameters are $\Delta_A = -1000\ \kappa$ and $g=\kappa=\gamma/2$, corresponding to the point indicated by the arrow in Fig. 1. $P_e=5\%$ on the whole range of Δ_C.

state. The parameters were set such that the point for detuning $\Delta_C=-\kappa+U_0$ in this plot (see arrow) corresponds to the point indicated by the arrow in Fig. 1(b). Note that, for reducing the numerical effort, this working point is far from the strongest localization. In fact, the magnitude of the atomic detuning Δ_A is also far below the one used in FORT. Though using larger detunings would yield much longer trapping times, it would make the simulations intractably long.

In a recent work [21], we found that atoms in a cavity being transversely pumped by a standing-wave laser field are captured for very long times in every directions, an effect that has been observed recently [22]. The effect was attributed to the peculiar spectrum of the field scattered from the side pump into the cavity, which we named as anomalous Doppler effect. Here we found that long trapping can be achieved in the simple one-dimensional geometry with the cavity being pumped. Common in the two schemes is that the spectrum of the coupled atom-field system has resonances other than the Lorentzian of a simple polarizable medium, regardless of the geometry of the scheme. We proved that the cooling force at a fixed rate of spontaneous photon scattering does not vanish in the limit of very far detuning associated with large potential depths. From another point of view, the very popular FORT scheme was generalized and its efficiency was shown to be greatly enhanced, embedding a cooling mechanism, if the field is enclosed in a cavity.

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