Nonadherence to the conventional group velocity for nanosecond light pulses in Rb vapor

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Propagation of nanosecond light pulses in a resonantly absorbing medium is studied. Deviation from the conventional group velocity and superluminal-to-subluminal transition were observed for the natural atomic transitions of Rb. The observed frequency and atomic density dependences of the propagation delay in the anomalous dispersion region, where the conventional concept of the group velocity breaks down, are reproduced well by the theory of the net group delay.

Recently various interesting phenomena concerned with the propagation of light pulses have been discussed. Slowing the velocity down to the sound one [1], stopping the light [2–5], and realizing negative velocity [6,7] have been reported. These phenomena are made possible in quantum controlled materials, where narrow transparent window and steep refractive index profile are produced in an absorbing region by using an artificial processing, the electromagnetically induced transparency (EIT) [8,9]. The EIT resulting from the quantum interference effect is a nonlinear phenomenon. However, the light propagation itself for the produced absorbing structure is a linear phenomenon. Even in natural resonantly absorbing media propagation of light pulses is not understood completely.

The pulse propagation in a resonant absorbing medium is a very classical problem. The problem of light propagation through dispersive media was first studied by Sommerfeld and Brillouin [10,11] early in the 20th century. They showed that main signal propagates through the medium with a velocity always less than the light velocity c in vacuum, while the front edge of the precursor can travel at c. Since then the study of the group velocity in anomalous dispersion regions has attracted a lot of interest.

Garret and McCumber [12] and Crisp [13] studied, both analytically and numerically, the propagation of a Gaussian shaped light pulse through a resonantly absorbing medium. The remarkable consequence from their discussion is that, when the incident frequency lies in the anomalous dispersion region, superluminal or even negative pulse velocity is possible, but it never violates the causality.

Experimental observation of the negative pulse velocity was reported by Chu and Wong [14] for a thin sample of GaP:N. In their experiment the pulses were detected by a second-order autocorrelation technique and, as is pointed out by Katz and Alfano [15], such a technique cannot reveal any change in pulse shape. A numerical simulation [16] of their data shows that the output pulse is in fact strongly distorted, where the concept of pulse velocity has not a clear physical significance. Recently we observed the superluminal propagation of optical pulses by a direct detection of the pulse intensity in the time domain in natural atomic transitions [17].

Talukder *et al.* [18] observed the propagation velocity in dye solutions and verified the differences from the conventional group velocity, which have been examined theoretically by Tanaka *et al.* [19] and by Peatross *et al.* [20]. In their experiment, however, the second-order autocorrelation technique was used again, and superluminal propagation was not observed. To clarify the light propagation in the anomalous dispersion region, more direct and detailed investigation is desired.

Absorption coefficient and refractive index of a Rb cell at 50 °C is shown in Fig. 1. The refractive index $n(\omega)$ is obtained from the observed absorption coefficient by applying the Kramers-Kronig relations [21]. Propagation time of pulses in the Rb vapor can be calculated from the group velocity $d\omega/dk$ in Eq. (5) of Ref. [17] and the refractive index $n(\omega)$. Here we refer to the velocity $d\omega/dk$ as conventional group velocity. Delay of propagation time is expected in the normal dispersion region, and negative delay is expected in the anomalous dispersion region. The group velocity can exceed the light velocity in vacuum and can be negative.

In the present paper we report on direct observation of deviation from the conventional group velocity $d\omega/dk$ in Rb vapor with a true shape detection of the pulse intensity. The frequency and atomic density dependences of the propagation delay are observed in the anomalous dispersion region, where the conventional concept of the group velocity breaks down, and the results are compared with those obtained from the net group delay proposed by Peatross *et al.* [20].

The experimental setup is similar to that in a previous paper [17]. We used Rb vapor as a resonantly absorbing me-
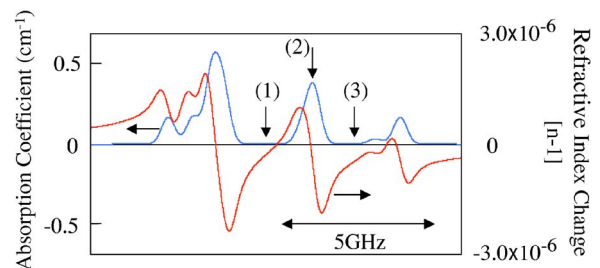

FIG. 1. (Color online) Absorption coefficient and refractive index change $n-1$ of the Rb D_1 transition for the atomic density 1.0×10^{11} cm^{-3} (50 °C). Arrows (1), (2), and (3) indicate the frequency of the probe pulse.

dium. Difference from the previous work is the light source; a Ti:sapphire laser is used instead of a laser diode. Nanosecond optical pulses are generated by an electro-optic modulator from the continuous output of the Ti:sapphire laser tuned to the Rb D_1 transition at 794.8 nm. An acousto-optic modulator, which produces 200 ns pulses, is used to improve the extinction ratio of the laser light and to prevent the hyperfine pumping. The pulses are divided into probe and reference pulses by a beam splitter. The arrival time of the reference pulse is detected by an avalanche photodiode. The delay of the arrival time of the probe pulse after passing through a Rb cell whose thickness is 1 cm is detected by another avalanche photodiode. The density of the Rb vapor is varied by changing the temperature of the cell. An optical fiber is used to give a time delay ($\sim$500 ns) and to avoid the effect of electronic noise from the electro-optic modulator at the arrival time of the pulses to the Rb cell.

Positive delay at the center of two absorption lines and negative delay at the peak of an absorption line were observed [17], where zero delay is defined for the propagation with the light velocity c in vacuum. The largest delay time observed at the center shown by arrow (1) in Fig. 1 is 40 ns at 189 °C for 3.4 ns probe pulse, where the propagation velocity of the probe pulse is 2.5×10^5 m/s and is reduced to about 8×10^{-4} of c. In other words, the probe pulse whose length is 1 m in air is shortened to less than 1 mm in the Rb cell, and appears from the cell as a 1 m pulse again. The largest advance time observed at the peak shown by arrow (2) in Fig. 1 is 2.3 ns at 95 °C, which corresponds to 68% of the pulse width. Observed frequency dependence of the delay for the 3.4 ns probe pulse is explained well by the conventional group velocity $d\omega/dk$ for all temperature range below 189 °C and for the frequency range where the transmitted intensity is detectable.

To study the effect of the spectral width of the probe pulse, propagation delay for shorter pulses was measured. Observed frequency dependences of the delay of the probe pulse is shown in Fig. 2(a), where the pulse width is 2.4 ns. The frequency range corresponds to the right half of Fig. 1. Experimental results for the three values of the atomic density ρ are shown; (1) 1.2×10^{12} cm^{-3} (82 °C), (2) 2.3×10^{12} cm^{-3} (91 °C), (3) 1.4×10^{13} cm^{-3} (120 °C). Theoretical curve calculated by applying $d\omega/dk$ is also shown. The vertical axis is the delay normalized by the atomic density ρ. Since the delay calculated from $d\omega/dk$ is proportional to the atomic density, the normalized delay curves calculated from $d\omega/dk$ for different atomic densities are shown as a single curve in Fig. 2. As is seen in Fig. 2(a), the experimental results deviate from the theoretical curve expected from $d\omega/dk$. As the temperature is increased and the atomic density is increased, the deviation becomes larger in the resonantly absorption regions. In these conditions the propagation of optical pulses cannot be explained by $d\omega/dk$, and the conventional concept of the group velocity breaks down for such an optically thick medium.

Here we discuss the propagation of optical pulses which do not adhere to the conventional group velocity $d\omega/dk$. In our experiment, propagation of 2.4 ns optical pulses cannot be explained by the conventional group velocity, while that of 3.4 ns optical pulses is explained well. This is considered
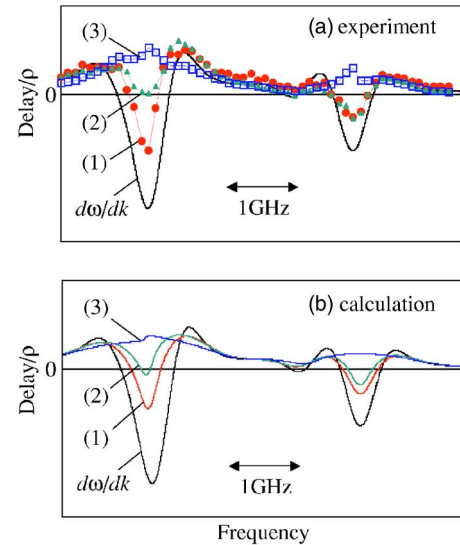

FIG. 2. (Color online) (a) Observed frequency dependences of the delay of the probe pulse, where the pulse width is 2.4 ns and the atomic density ρ is (1) 1.2×10^{12} cm^{-3} (82 °C), (2) 2.3×10^{12} cm^{-3} (91° C), and (3) 1.4×10^{13} cm^{-3} (120° C). Theoretical curve calculated by applying $d\omega/dk$ is also shown. The vertical axis is the delay normalized by the atomic density ρ. (b) Theoretical frequency dependences of the delay of the probe pulse calculated by applying the theory of Peatross *et al.*, where the spectral width of the probe pulse is taken as 400 MHz. Curves (1), (2), and (3) correspond to the three cases in (a).

to be caused by the difference in the spectral width of the pulses. The observed deviation from $d\omega/dk$ is understood by the effects of resonant absorption and violation of linear approximation in Eq. (2) of Ref. [17] in resonantly absorbing media. In 1986, in the field of plasma physics, a new concept of group velocity, which can be applied even in resonantly absorbing regions, has been proposed by Tanaka *et al.* [19,22]. In this theory the average velocity, which is defined by the traveling distance x of the pulse peak divided by its traveling time t, is given by means of the saddle point method to be

$$\frac{x}{t} = \left[\frac{d\omega}{dk}\right]_{\omega_s} = \frac{c}{\left[\dfrac{d(\mathrm{Re}[n(\omega)]\omega)}{d\omega}\right]_{\omega_s}}, \qquad (1)$$

where ω_s is the center frequency of the transmitted pulse and is given by

$$\omega_s - \omega_c = -\frac{x\delta^2}{c}\left[\frac{d(\mathrm{Im}[n(\omega)]\omega)}{d\omega}\right]_{\omega_s}, \qquad (2)$$

where ω_c is the center frequency of the incident pulse, and δ is the spectral width of the incident pulse. The difference from the conventional group velocity is that the derivative is taken not at the center frequency of the initial pulse but at that of the transmitted one. The result of the theory by Tanaka *et al.* says that the group velocity depends on the spectral width or the pulse width of the incident pulse and is determined by the center frequency of the transmitted pulse

which may be shifted from the initial one by the resonant absorption, and that the concept of the conventional group velocity does not break down if applied to the surviving spectrum instead of the initial spectrum. In this theory the propagation velocity and the frequency shift are derived analytically, and those results are instructive and are easy to be understood intuitively. However, the frequency dependence of the propagation delay calculated from the theory by Tanaka *et al.* shows discontinuous point at the absorption peak for strong absorption, which does not appear in our experiment.

To analyze our experimental results, we introduce the net group delay proposed by Peatross *et al.* [20]. They defined arrival time of pulse as that of the center-of-mass of pulse. The arrival time $\langle t \rangle$ of pulse propagating along the x axis can be expressed as follows:

$$\langle t \rangle = \frac{\int_{-\infty}^{\infty} tS(x,t)dt}{\int_{-\infty}^{\infty} S(x,t)dt}, \tag{3}$$

where $S(x,t)$ is the Poynting vector of the transmitted pulse. Fourier transformation and some calculations lead to a sum of two terms, net group delay, and reshaping delay. The reshaping delay has some contribution for frequency-chirped pulses and can be neglected for the pulses of Fourier-transform limit. Here we take only the net group delay into account. The net group delay is given by

$$\Delta t = \frac{\int_{-\infty}^{\infty} \left[\frac{d(\mathrm{Re}[k(\omega)])}{d\omega} \Delta x \right] S(x,\omega)d\omega}{\int_{-\infty}^{\infty} S(x,\omega)d\omega}, \tag{4}$$

where $S(x,\omega)$ is the Fourier transform of $S(x,t)$. The arrival time is derived to be the weighted sum of the propagation time for each frequency component of the transmitted pulse. The result of this theory gives a new context for the group velocity, where the group velocity is always meaningful even for broad band pulses and when the group velocity is superluminal or negative, if we consider the function $d\omega/dk$ as group velocity which is permitted to vary arbitrarily within the bandwidth of the pulse.

Frequency dependences of the delay of the probe pulse calculated from Eq. (4) for the observed three cases in Fig. 2(a) are shown in Fig. 2(b), where a Gaussian spectrum of 400 MHz width (full width at half maxima) for the probe pulse is assumed. This value is two times larger than that expected from the Fourier transform limit of the 2.4 ns pulse due to the broadening by the electro-optic modulator. The value 400 MHz of the spectral width was estimated from a pulse absorption experiment, where the frequency dependences of the absorption coefficient around an absorption peak were measured both for the 2.4 ns pulses and monochromatic cw light, and simulated absorption curve for the pulses was compared with the observed one. Agreement between the theoretical curves and the experimental results is
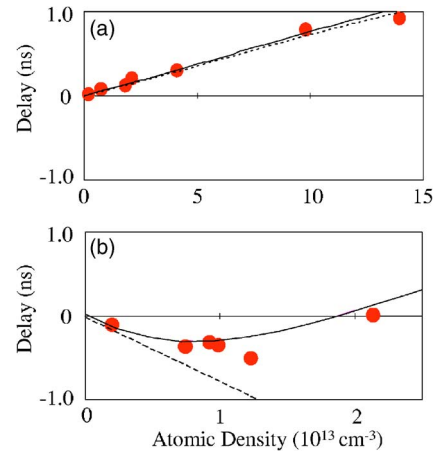

FIG. 3. (Color online) (a) Atomic density dependences of the delay of the probe pulse observed at the frequencies indicated by (a) arrow (3) and (b) arrow (2) in Fig. 1(a). Broken and solid lines are theoretical curves calculated by applying $d\omega/dk$ and the theory of Peatross *et al.*, respectively.

good. The observed pulse propagation deviated from the conventional group velocity $d\omega/dk$ can be reproduced well by the theory of Peatross *et al.*

Atomic density dependences of the delay of the probe pulse observed for the frequencies indicated by arrows (3) and (2) in Fig. 1 are shown in Fig. 3. Figure 3(a) shows a linear increase in the positive delay in the normal dispersion region. Figure 3(b) shows that the arrival time of the probe pulse in the anomalous dispersion region moves toward the negative delay for the low density indicating superluminal pulse propagation, and then turns back and moves toward the positive delay with increasing density indicating subluminal pulse propagation. The broken and solid lines in Fig. 3 are theoretical curves calculated by applying $d\omega/dk$ and the theory of Peatross *et al.*, respectively. The observed superluminal-to-subluminal transition in the anomalous dispersion region can be reproduced by the theory of Peatross *et al.*, where the conventional treatment of the group velocity $d\omega/dk$ breaks down. In the pulse propagation through absorbing medium, the effective optical propagation distance is a product of the refractive index and propagation distance, while the refractive index is linearly proportional to the atomic density. The change of the atomic density is equivalent to the change of the effective optical propagation distance. From the viewpoint of propagation distance, optical pulses in an anomalous dispersion medium can propagate superluminally for a short distance but turns back to a subluminal propagation after a long distance travel.

In conclusion, we observed propagation of nanosecond pulses in natural atomic vapor. The pulse propagation which do not adhere to the conventional group velocity $d\omega/dk$ was observed. The observed frequency dependence and the superluminal-to-subluminal transition in the propagation delay in the anomalous dispersion region were explained well by the theory of Peatross *et al.*

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