Controlled hole burning in Fock space via resonant interaction

A. T. Avelar[1,2,*] and B. Baseia[2]
[1]*Instituto de Física, Universidade de Brasília, 70.919-970, Brasília (DF), Brazil*
[2]*Instituto de Física, Universidade Federal de Goiás, 74.001-970, Goiânia (GO), Brazil*
(Received 4 April 2005; published 8 August 2005)

We propose an alternative and simple method to create states of the quantized radiation field with controlled holes in their photon-number distribution. The scheme relies on resonant interaction of a cavity field with two-level Rydberg atoms plus selective atomic detection.

DOI: 10.1103/PhysRevA.72.025801 PACS number(s): 42.50.Dv

As argued previously [1], states having controlled holes in their photon-number distribution (PND) are candidates having potential application in optical data storage and optics communication. Each hole of these states can be associated with some signal (yes, 1, or +), its absence being associated with the opposite signal (no, 0, or −). Such states may be tailored either in stationary modes of electromagnetic field trapped inside a high-Q microwave cavity [1–3] or traveling modes [4,5]. In the first case, the generation could be implemented either via dispersive atom-field interaction [1,2] or via Raman interaction [3]. In the second case, one can employ a Mach-Zehnder interferometer including a nonlinear Kerr medium [4] or via a simple scheme using a single beam splitter plus a single detector [5]. The production of such states has also been considered in the context of mesoscopic Josephson junctions [6].

According to a theorem by Hillery [7] such a state is always nonclassical, since it is not a coherent state. In fact, as explicitly shown by Mandel and Wolf [8], an arbitrary field state: $\hat{\rho}=\int P(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$ has its PND given by $P_n=\int P(\alpha)|\langle n|\alpha\rangle|^2 d^2\alpha$; since $|\langle n|\alpha\rangle|^2>0$ then $P_n\neq 0$ for all n when $P(\alpha)$ is a true probability density. Hence $P_n=0$, for some value of n, corresponds to a state having no classical analog, being purely quantum mechanical. So, making holes in PND corresponds to generating nonclassical states.

In the present paper we will consider an alternative method which simplifies the generation of the mentioned states inside a cavity QED. Differently from the previous scheme [1], the present method employs a resonant interaction instead of a dispersive one. Here we will use the Jaynes-Cummings model describing a system having two-level atoms interacting resonantly with a single-mode cavity field [9]. We show that the time required to prepare the desired state is considerably reduced, which is relevant in view of decoherence processes destroying the state.

Figure 1 displays the setup preparing the mentioned states inside a high-Q cavity; S_A represents "source of atoms," "excitation" prepares highly excited Rydberg atoms, "C" represents the cavity supporting the field mode, and "MG" stands for "microwave generator" preparing a single field mode in a coherent state [10], inside the cavity.

Consider a two-level atom initially prepared in the excited state $|e\rangle$, entering the cavity and interacting resonantly with a field mode prepared in a coherent state $|\alpha\rangle$. The effective Hamiltonian for the atom-field system in the interaction picture is [11]

$$\hat{H}=\lambda(\hat{a}^{\dagger}\hat{S}^{-}+\hat{a}\hat{S}^{+}), \tag{1}$$

where $\hat{a}$ ($\hat{a}^{\dagger}$) is the annihilation (creation) operator for the cavity field, $\hat{S}^{+}$ ($\hat{S}^{-}$) stands for the raising (lowering) operator for the atomic system, and λ is the atom-field coupling constant. Since the Hamiltonian (1) is time independent, the evolution operator is given by $\hat{U}(t)=\exp(-it\hat{H}/\hbar)$ and its application to the initial state describing the whole atom-field system, $|\Psi_{AF}(0)=|e\rangle|\alpha\rangle$, results

$$|\Psi_{AF}(t)\rangle=e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\{\cos(\Omega_n\tau)|e,n\rangle - i\sin(\Omega_n\tau)|g,n+1\rangle\}, \tag{2}$$

where $\Omega_n=\sqrt{n+1}\lambda$. As a consequence, if one detects the atom in the state $|e\rangle$, after the time τ_1, then the cavity field collapses onto the state

$$|\Psi_F(\tau_1)\rangle=\mathcal{N}_1\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\cos(\Omega_n\tau_1)|n\rangle, \tag{3}$$

where $\mathcal{N}_1$ is a normalization factor. Now, by choosing the interaction time τ_1 conveniently, so that $\sqrt{n_1+1}\lambda\tau_1=\pi/2$, the component $|n_1\rangle$ in Eq. (3) vanishes.

Next, assume that a second atom in the state $|e\rangle$ enters immediately after the detection of the first one; for the second atom the initial field state inside the cavity is given by the Eq. (3), corresponding to the final state projected by the

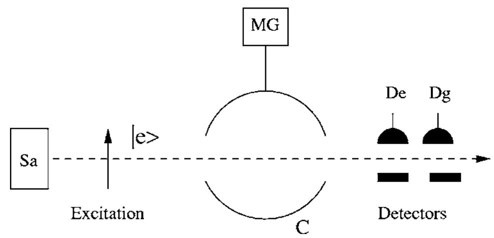

FIG. 1. Scheme of the setup creating a field inside the cavity with controlled holes in its PND.

*Electronic address: ardiley@unb.br

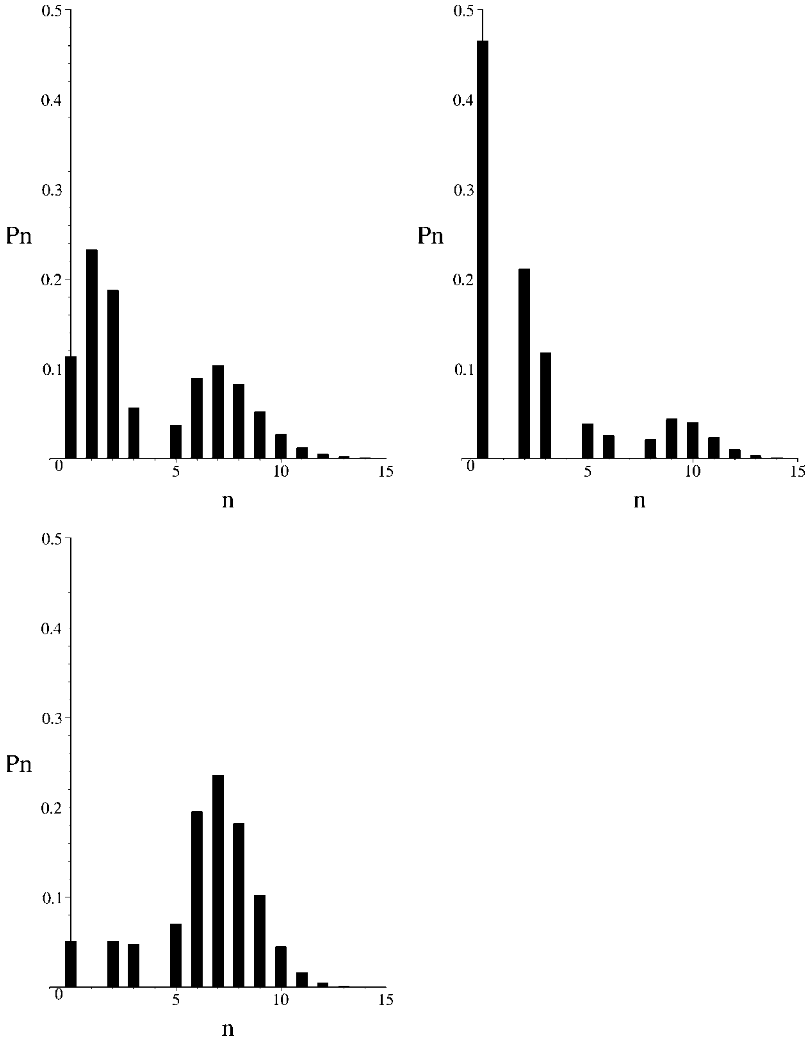

FIG. 2. Holes in the PND, for α=2.0, (a) at n_1=4, for the first atom; (b) at n_1=4 and n_2=1, for the second atom; (c) at n_1=4, n_2=1, and n_3=7, for the third atom.

detection of the first atom. As a result, the atom-field system now evolves to the state (up to normalization),

$$|\Psi_{AF}(\tau_2)\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\{\cos(\Omega_n\tau_2)\cos(\Omega_n\tau_1)|e,n\rangle - \cos(\Omega_n\tau_1)\sin(\Omega_n\tau_2)|g,n+1\rangle\}. \tag{4}$$

Hence, if one detects the second atom in the state $|e\rangle$, then the cavity field collapses onto the state

$$|\Psi_F(\tau_2)\rangle = \mathcal{N}_2\sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\{\cos(\Omega_n\tau_2)\cos(\Omega_n\tau_1)|n\rangle\}, \tag{5}$$

where $\mathcal{N}_2$ is a normalization factor. Here the choice $\sqrt{n_2+1}\lambda\tau_2=\pi/2$ produces another hole, now at the component $|n_2\rangle$.

By repeating the procedure N times, we obtain the generalized result for the Nth atom,

$$|\Psi_F(\tau_N)\rangle = \mathcal{N}_N\sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\prod_{j=1}^{N} \cos(\Omega_n\tau_j)|n\rangle, \tag{6}$$

where τ_j stands for the interaction time for the jth atom crossing the cavity. Equation (6) shows that the number of atoms coincides with the number of holes.

From Eq. (6) we find the expression of the PND, $P_n=|\langle n|\Psi_F(\tau_N)\rangle|^2$. In this way, a little algebra furnishes

$$P_n = \frac{(|\alpha|^{2n}/n!)\prod_{j=1}^{N} \cos^2(\Omega_n\tau_j)}{\sum_{m=0}^{\infty}(|\alpha|^{2m}/m!)\prod_{j=1}^{N} \cos^2(\Omega_m\tau_j)}. \tag{7}$$

We note that Eq. (7) defines the presence of holes in the PND through the control of the interaction times spent by the atoms to cross the cavity. When creating many holes in such a way, the creation of each of them neither affects nor is affected by the presence of others, as shown below and also discussed in [1,3,5].

To illustrate results we have plotted Fig. 2 showing the controlled production of holes in the PND.

To exemplify the procedure we take attention to the typical experimental values of the parameters involved for the Rydberg atoms with principal quantum numbers 50 and 51. This implies the coupling constant $\lambda \simeq 2\pi \times 47$ kHz [12] and leads to the interaction times $\tau_1 = \pi/(2\sqrt{n_1+1}\lambda) \simeq 3.8$ μs for $n_1 = 1$, $\tau_2 = 2.4$ μs for $n_2 = 4$, and $\tau_3 = 1.9$ μs for $n_3 = 7$. On the other hand, the cavity-field damping time $t_{cav} \simeq 10$ ms [13] and the initial coherent state $|\alpha\rangle$ with $|\alpha| = 2.0$ leads to the decoherence time $t_d = t_{cav}/2|\alpha|^2 \simeq 1.3$ ms, greater than 8.1 μs (which is the sum of total interaction time required to complete the production of the three holes specified above). Therefore, the scheme is experimentally feasible within the microwave domain where the radiative time of Rydberg is about $T_r = 30$ ms.

In summary, we have employed the (resonant) Jaynes-Cummings model describing the interaction of two-level (Rydberg) atoms with a single-mode field in a coherent state, to create controlled holes in photon-number distribution of the field state. The scheme allows us to simplify a previous one, by economizing the two Ramsey zones used in [1] and the single Ramsey zone employed in [3]. This simplification is relevant, since the time intervals spent by atoms to cross the cavity and to cross the Ramsey zones are of same order, about 1 μs [14]. In both schemes the cavity might have a high quality to sustain the prepared field state for sufficiently "large" times, in view of its degradation caused by decoherence processes [15].

Partial support from the CNPq, Brazilian agency, is gratefully acknowledged.

[1] J. M. C. Malbouisson and B. Baseia, Phys. Lett. A **290**, 234 (2001).
[2] B. Baseia, M. H. Y. Moussa, and V. S. Bagnato, Phys. Lett. A **240**, 277 (1998).
[3] A. T. Avelar, L. A. de Souza, T. M. da Rocha Filho, and B. Baseia, J. Opt. B: Quantum Semiclassical Opt. **6**, 383 (2004).
[4] C. C. Gerry and A. Benmoussa, Phys. Lett. A **303**, 30 (2002).
[5] B. M. Escher, A. T. Avelar, T. M. da Rocha Filho, and B. Baseia, Phys. Rev. A **70**, 025801 (2004).
[6] J. Zou, B. Shao, and X. Xing, Int. J. Mod. Phys. B **13**, 2727 (1999).
[7] M. Hillery, Phys. Lett. **111A**, 409 (1985).
[8] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).
[9] K. Vogel, V. M. Akulin, and W. P. Schleich, Phys. Rev. Lett. **71**, 1816 (1993).
[10] J. M. Raimond, M. Brune, and S. Haroche, Phys. Rev. Lett. **79**, 1964 (1997).
[11] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).
[12] A. Rauschenbeutel, P. Bertet, S. Osnaghi, G. Nogues, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **64**, 050301(R) (2001).
[13] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45**, 5193 (1992).
[14] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001).
[15] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).