Teleportation of a two-atom entangled state with a thermal cavity

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(Received 21 December 2004; published 19 August 2005)

We present a scheme to teleport an unknown atomic entangled state in driven cavity QED. In our scheme, the success probability can reach 1.0. In addition, the scheme is insensitive to the cavity decay and the thermal field.

DOI: 10.1103/PhysRevA.72.024305 PACS number(s): 03.67.Mn

Since the first scheme for quantum teleportation was proposed by Bennett *et al.* [1], much attention has been paid to it. Most theoretical schemes for teleportating an unknown quantum state have been considered with the certain quantum channels, such as composing of the Einstein-Podolsky-Rosen (EPR) pair or GHZ triplet as well as combination of the EPR pair and GHZ triplet [1–7]. In these schemes, to realize the teleportation, the sender must operate a joint Bell state measurement on the particle that carries the unknown quantum information and one of the entangled particles he possesses. Then the sender will inform the receiver of his measurement result, and the receiver can perform a unitary operation to reconstruct the initial state on his particle.

Recently, cavity QED technology has been widely applied to the quantum information field. Zheng [8] proposed a scheme for realizing two-qubit quantum phase gates with atoms in a thermal cavity. Solano *et al.* [9] proposed a method of generating multipartite entanglement by considering the interaction of a system of N two-level atoms in a cavity of high quality factor with a strong classical driving field. Many people [10–16] proposed schemes for the teleportation of unknown atomic states using cavity QED. In particular, Zheng [14] suggested a scheme of teleporting a single atomic state using a resonant atom-cavity interaction. In the scheme, the cavity decay and the thermal field affected the scheme strongly, and the success probability is 0.25. Ye and Guo [15] suggested a scheme of teleporting an atomic entangled state using a GHZ state as a quantum channel in the case of great detuning. Its success probability is 0.5. Yang *et al.* [16] made a proposal for teleporting a single atomic state with a probability 1.0 by adding a classical driving field.

In this paper, we propose a scheme about teleporting a two-atom entangled state. We make the two atoms simultaneously interact with a single-mode cavity mode and a classical field, so that the effects of thermal field and cavity decay are all eliminated. The success probability is 1.0.

Suppose the atomic entangled state that the sender Alice wants to transmit to the receiver Bob is expressed as

$$|\psi\rangle_{12} = \alpha|ge\rangle_{12} + \beta|eg\rangle_{12}, \tag{1}$$

where α and β are unknown coefficients, $|\alpha|^2+|\beta|^2=1$. $|g\rangle$ and $|e\rangle$ are the ground and the excited states of the atom, respectively. Atoms 3, 4, and 5 are prepared in the maximally three-atom entangled GHZ state as the quantum channel

$$|\psi\rangle_{345} = \frac{1}{\sqrt{2}}(|ege\rangle_{345} - i|geg\rangle_{345}), \tag{2}$$

where the atoms 1, 2, and 3 belong to the sender Alice and the other two atoms 4, 5 belong to the receiver Bob.

The initial state of the whole system composed of atoms (1, 2) and the quantum channel is given by

$$|\psi\rangle_{12345} = \frac{1}{\sqrt{2}}(\alpha|ge\rangle_{12} + \beta|eg\rangle_{12}) \otimes (|ege\rangle_{345} - i|geg\rangle_{345}). \tag{3}$$

Then Alice sends atoms 2 and 3 into a single-mode cavity. At the same time, the two atoms are driven by a classical field. The interaction between atoms and the cavity can be described as follows [17]

$$H = \omega_0\sum_{j=2}^{3} S_{z,j} + \omega_a a^\dagger a + \sum_{j=2}^{3}[g(a^\dagger S_j^- + aS_j^\dagger) + \Omega(S_j^\dagger e^{-i\omega_d t} + S_j^- e^{i\omega_d t})], \tag{4}$$

where ω_0, ω_a, and ω_d are atomic transition frequency ($e\leftrightarrow g$), cavity frequency and the frequency of driving field, respectively, $a^\dagger$ and a are creation and annihilation operators for the cavity mode, g is the coupling constant between atoms and cavity mode, $S_j^-=|g\rangle_j\langle e|$, $S_j^\dagger=|e\rangle_j\langle g|$, $S_{z,j}=\frac{1}{2}(|e\rangle_j\langle e| - |g\rangle_j\langle g|)$ are atomic operators, and Ω is the Rabi frequency of the classical field. We consider the case $\omega_0=\omega_d$. In the interaction picture, the evolution operator of the system is [17]

$$U(t) = e^{-iH_0 t}e^{-iH_{\mathrm{eff}} t}, \tag{5}$$

where $H_0=\Sigma_{j=2}^{3}\Omega(S_j^\dagger+S_j^-)$, H_{eff} is the effective Hamiltonian. In the large detuning $\delta \gg g/2$ and strong driving field $2\Omega \gg \delta, g$ limit, the effective Hamiltonian for this interaction can be described as follows [17]:

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$$H_{\text{eff}} = \frac{\lambda}{2}\left[\sum_{j=2}^{3}(|e\rangle_j\langle e| + |g\rangle_j\langle g|) + \sum_{j,k=2,j\neq k}^{3}(S_j^\dagger S_k^\dagger + S_j^\dagger S_k^- + \text{H.c.})\right], \tag{6}$$

where $\lambda = g^2/2\delta$ with δ being the detuning between atomic transition frequency ω_0 and frequency ω_a. From the form of the effective Hamiltonian, we conclude that the interaction Hamiltonian is independent of the photon number of the cavity field. In addition, there is no exchange of energy between atoms and cavity mode. So the effects of cavity decay and thermal field are all eliminated.

After interaction time t, the evolution of the total system can be expressed as

$$\begin{aligned}|\psi\rangle_{12345} \to \frac{1}{\sqrt{2}}e^{-i\lambda t}\{&\alpha|g\rangle_1[\cos\lambda t(\cos\Omega t|e\rangle_2 - i\sin\Omega t|g\rangle_2)(\cos\Omega t|e\rangle_3 - i\sin\Omega t|g\rangle_3) - i\sin\lambda t(\cos\Omega t|g\rangle_2 - i\sin\Omega t|e\rangle_2)\\
&\times(\cos\Omega t|g\rangle_3 - i\sin\Omega t|e\rangle_3)]|ge\rangle_{45} - i\alpha|g\rangle_1[\cos\lambda t(\cos\Omega t|e\rangle_2 - i\sin\Omega t|g\rangle_2)(\cos\Omega t|g\rangle_3 - i\sin\Omega t|e\rangle_3)\\
&- i\sin\lambda t(\cos\Omega t|g\rangle_2 - i\sin\Omega t|e\rangle_2)(\cos\Omega t|e\rangle_3 - i\sin\Omega t|g\rangle_3)]|eg\rangle_{45} + \beta|e\rangle_1[\cos\lambda t(\cos\Omega t|g\rangle_2 - i\sin\Omega t|e\rangle_2)\\
&\times(\cos\Omega t|e\rangle_3 - i\sin\Omega t|g\rangle_3) - i\sin\lambda t(\cos\Omega t|e\rangle_2 - i\sin\Omega t|g\rangle_2)(\cos\Omega t|g\rangle_3 - i\sin\Omega t|e\rangle_3)]|ge\rangle_{45}\\
&- i\beta|e\rangle_1[\cos\lambda t(\cos\Omega t|g\rangle_2 - i\sin\Omega t|e\rangle_2)(\cos\Omega t|g\rangle_3 - i\sin\Omega t|e\rangle_3) - i\sin\lambda t(\cos\Omega t|e\rangle_2 - i\sin\Omega t|g\rangle_2)(\cos\Omega t|e\rangle_3\\
&- i\sin\Omega t|g\rangle_3)]|eg\rangle_{45}\}.\end{aligned} \tag{7}$$

We can choose $\lambda t = \pi/4$, and realize the condition $\Omega t = \pi$ by modulating the driving field appropriately. Then Eq. (7) will become

$$\begin{aligned}|\psi\rangle_{12345} \to \frac{1}{2}[&\alpha|g\rangle_1(|e\rangle_2|e\rangle_3 - i|g\rangle_2|g\rangle_3)|ge\rangle_{45} - i\alpha|g\rangle_1(|e\rangle_2|g\rangle_3\\
&- i|g\rangle_2|e\rangle_3)|eg\rangle_{45} + \beta|e\rangle_1(|g\rangle_2|e\rangle_3 - i|e\rangle_2|g\rangle_3)|ge\rangle_{45}\\
&- i\beta|e\rangle_1(|g\rangle_2|g\rangle_3 - i|e\rangle_2|e\rangle_3)|eg\rangle_{45}].\end{aligned} \tag{8}$$

Then Alice performs a rotation operation R on the state of atom 1, the rotation matrix takes the following form:

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}. \tag{9}$$

After performing the rotation operation on the state of atom 1, we have

$$R|g\rangle_1 = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1),$$

$$R|e\rangle_1 = \frac{1}{\sqrt{2}}(|e\rangle_1 - |g\rangle_1). \tag{10}$$

So Eq. (8) will become

$$\begin{aligned}|\psi\rangle_{12345} \to \frac{1}{2\sqrt{2}}[&|eee\rangle_{123}(\alpha|ge\rangle_{45} - \beta|eg\rangle_{45})\\
&- i|egg\rangle_{123}(\alpha|ge\rangle_{45} + \beta|eg\rangle_{45}) + |gee\rangle_{123}(\alpha|ge\rangle_{45}\\
&+ \beta|eg\rangle_{45}) - i|ggg\rangle_{123}(\alpha|ge\rangle_{45} - \beta|eg\rangle_{45})\\
&- i|eeg\rangle_{123}(\alpha|eg\rangle_{45} + \beta|ge\rangle_{45}) - |ege\rangle_{123}(\alpha|eg\rangle_{45}\\
&- \beta|ge\rangle_{45}) - i|geg\rangle_{123}(\alpha|eg\rangle_{45} - \beta|ge\rangle_{45})\\
&- |gge\rangle_{123}(\alpha|eg\rangle_{45} + \beta|ge\rangle_{45})].\end{aligned} \tag{11}$$

Then Alice will detect the atoms 1, 2, and 3. If the result is $|gee\rangle_{123}$, the state of the atoms 4, 5 will collapse into the following state

$$|\psi'\rangle_{45} = \frac{1}{2\sqrt{2}}(\alpha|ge\rangle_{45} + \beta|eg\rangle_{45}). \tag{12}$$

From Eq. (12), we can know that the probability of successful teleportation is $\frac{1}{8}$. Similarly, as depicted in Table I, it can be easily proven if the measurement results are $|eee\rangle_{123}$, $|eeg\rangle_{123}$, $|egg\rangle_{123}$, $|gge\rangle_{123}$, $|ege\rangle_{123}$, $|geg\rangle_{123}$, and $|ggg\rangle_{123}$, the teleportation also can succeed with the same probabilities. That is to say, the total success probability is 1.0, which is higher than Ref. [16]. Table I lists the measurement results

TABLE I. The results of the teleportation scheme. R denotes the measurement result on atoms 1, 2, and 3. Operation denotes the operation needed for the receiver. σ_{ij} is a Pauli operation σ_j on particle i.

R	$\|\psi\rangle_{45}$	Operation
$\|eee\rangle_{123}, \|ggg\rangle_{123}$	$\frac{1}{2\sqrt{2}}(\alpha\|ge\rangle_{45} - \beta\|eg\rangle_{45})$	$I_4 \otimes \sigma_{5z}$
$\|gee\rangle_{123}, \|egg\rangle_{123}$	$\frac{1}{2\sqrt{2}}(\alpha\|ge\rangle_{45} + \beta\|eg\rangle_{45})$	$I_4 \otimes I_5$
$\|eeg\rangle_{123}, \|gge\rangle_{123}$	$\frac{1}{2\sqrt{2}}(\alpha\|eg\rangle_{45} + \beta\|ge\rangle_{45})$	$\sigma_{4x} \otimes \sigma_{5x}$
$\|ege\rangle_{123}, \|geg\rangle_{123}$	$\frac{1}{2\sqrt{2}}(\alpha\|eg\rangle_{45} - \beta\|ge\rangle_{45})$	$\sigma_{4x} \otimes \sigma_{5y}$

on atoms 1, 2, and 3, the result state of atoms 4 and 5, and the operation needed for the receiver to convert the state of atoms 4 and 5 into the initial state of atoms 1, 2.

In conclusion, we have proposed a protocol for teleporting an unknown atomic entangled state from a sender to a receiver by using a three-atom entangled GHZ state. Compared to the previous scheme [15], our scheme has a main advantage. That is, our success probability can be improved because of adding a classical driving field. The effects of cavity decay and thermal field in this scheme are eliminated.

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