

Transfer of amplitude and phase modulation to a different wavelength using coherently prepared sodium vapor

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We present a scheme, based on coherent Raman scattering, to transfer the complete information content of an optical field from one spectral band to another with good efficiency, high fidelity, and large bandwidth. We demonstrate the transfer of both amplitude and phase modulation to new frequencies by scattering in a coherently prepared sodium vapor. The scattering process for this system has a bandwidth of at least tens of MHz and preserves the field envelope with a fidelity of 98%.

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I. INTRODUCTION

The ability to transfer the information content of an optical field from one frequency band to another is central to many processes in modern optical technology, ranging from applications in telecommunications to quantum information processing. In this paper, we show that information impressed onto an optical field by either amplitude or phase modulation can be transferred to a different frequency band by means of coherent Raman scattering, and that this conversion can be performed effectively by making use of the special optical properties of coherently prepared media. The basic science and application of coherently prepared media, subjects prompted by the discovery of electromagnetically induced transparency (EIT) nearly 15 years ago [1], have become vigorous areas of investigation. In recent years, coherently prepared media have been applied to applications such as greatly enhancing absorptive and dispersive optical nonlinearities [2–4], lasing without inversion [5–8], highly sensitive magnetometry [9–12], making more accurate atomic clocks [13,14], slowing [15–17] and storing [18–22] light, and optical switching [23–27]. The original context of EIT, however, was the efficient generation of new frequencies of light [1,28]. EIT and related ideas have led to efficient infrared up conversion [29,30], efficient phase conjugation [31], and coherent Raman scattering [32–36]. Such studies have emphasized that power could be transferred from pump wave(s) to the desired frequency. The idea that coherently prepared media may effectively transfer *information* from one frequency band to another has received much less attention, however, while similar ideas are currently being explored in the context of traditional Raman scattering, e.g., Ref. [37]. Recently, coherent Raman scattering was used to shift the center frequency of an incoherent light field, preserving the spectral power distribution in the process [38]. But scattering stimulated by a quantum coherence in the medium may be expected to preserve the phase as well as the

amplitude of the input field. If the scattering process has a flat, dispersionless frequency response, then a signal imprinted on the input field will appear without distortion in the scattered field. For example, data expressed as phase or amplitude modulation at one wavelength could be transferred to a different wavelength with good efficiency via coherently prepared media-enhanced scattering. We report here the transfer of both amplitude and phase modulation from one wavelength to another using a coherently prepared sodium vapor. The transfer fidelity is excellent, and unlike other modulation transfer schemes (e.g., Refs. [23,24,39]), the bandwidth in our scheme is not limited by the time required to establish material coherence.

The scheme is shown in Fig. 1. A pair of strong “control” fields establishes quantum coherence ρ_{12} between a pair of metastable states $|1\rangle$ and $|2\rangle$. A weaker “input” field (i) scatters off this coherence, generating a new “output” field (o). As we show below, the output field is proportional to the product of the input field with ρ_{12} . By making the control fields monochromatic, ρ_{12} is constant and the output field becomes directly proportional to the input field; the phase and amplitude of the input are transferred to the output field, effectively shifted by the frequency difference of states $|1\rangle$ and $|2\rangle$. Since the scattering is linear in the input field, an input beam composed of many different frequencies will be

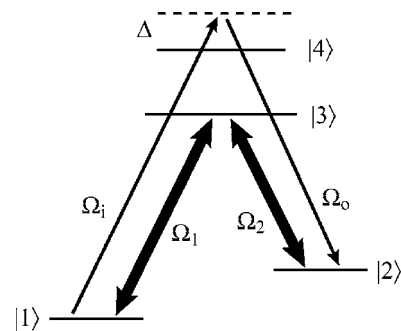


FIG. 1. The scheme for transfer of information from one frequency to another via coherent Raman scattering. The control fields Ω_1, Ω_2 create quantum coherence between the ground states $|1\rangle, |2\rangle$. A weak input field Ω_i scatters off this coherence, generating a new field Ω_o . Any information imprinted on the input field is transferred to the scattered, output field.

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transferred to the output field. The bandwidth of the scattering process is just the range of input frequencies that are scattered efficiently; in a Doppler-broadened medium the bandwidth is of the order of 1 GHz. Because the control fields are not modulated, the bandwidth of the transfer process is not limited by the time required to change ρ_{12} , which is typically chosen to be long (a sizeable fraction of a second) so that the coherence may build up to its maximal value.[1]¹

II. THEORY

To justify these ideas, we consider the theory of propagation in such a medium. We suppose that the input and output fields are weak enough so as not to affect the material coherence ρ_{12} . (The case of four-wave mixing with strong fields has been analyzed in Ref. [40].) For simplicity, we suppose that the control fields are exactly resonant with the two-photon transition between $|1\rangle$ and $|2\rangle$. In the slowly varying amplitude approximation, the input and output fields evolve according to

$$\frac{d}{dz}E_i = \frac{i}{2}k_i N \mu_{41}^* \rho_{41}, \quad (1)$$

$$\frac{d}{dz}E_o = \frac{i}{2}k_o N \mu_{42}^* \rho_{42}, \quad (2)$$

where E denotes electric field, k denotes free-space wave number, N denotes atomic number density, and $\mu_{jk} \equiv \langle j | (-er) | k \rangle$ is the electric dipole matrix element. To lowest order in the weak fields we have [41]

$$\rho_{41} = \frac{i}{2(\gamma - i\Delta)}(\rho_{11}\Omega_i + \rho_{21}\Omega_o), \quad (3)$$

$$\rho_{42} = \frac{i}{2(\gamma - i\Delta)}(\rho_{12}\Omega_i + \rho_{22}\Omega_o), \quad (4)$$

where Ω is the Rabi frequency of the associated field and γ and Δ are the decay rate and detuning from the scattering state $|4\rangle$ which may be a real state or an effective state encompassing a distribution of states. The values of ρ_{11} , ρ_{22} , and ρ_{21} are determined by the control fields and can be obtained from many existing analyses of EIT. With strong fields, one finds that

$$\rho_{11} \approx \frac{|\Omega_2|^2}{|\Omega_1|^2 + |\Omega_2|^2},$$

$$\rho_{22} \approx \frac{|\Omega_1|^2}{|\Omega_1|^2 + |\Omega_2|^2},$$

¹This is the case for schemes based on EIT in a three-level system in which the transfer of modulation results from modifying the atomic coherence.

$$\rho_{21} \approx -\frac{\Omega_2^* \Omega_1}{|\Omega_1|^2 + |\Omega_2|^2}. \quad (5)$$

The propagation equations can then be written as

$$\frac{d}{dz} \begin{pmatrix} \Omega_i \\ \Omega_o \end{pmatrix} = \begin{pmatrix} -\kappa_i & \kappa_i \frac{\Omega_1}{\Omega_2} \\ \kappa_o \frac{\Omega_2}{\Omega_1} & -\kappa_o \end{pmatrix} \begin{pmatrix} \Omega_i \\ \Omega_o \end{pmatrix}, \quad (6)$$

where

$$\kappa_i = \frac{k_i N |\mu_{41}|^2}{2\hbar \epsilon_0 (\gamma - i\Delta)} \frac{|\Omega_2|^2}{|\Omega_1|^2 + |\Omega_2|^2}, \quad (7)$$

$$\kappa_o = \frac{k_o N |\mu_{42}|^2}{2\hbar \epsilon_0 (\gamma - i\Delta)} \frac{|\Omega_1|^2}{|\Omega_1|^2 + |\Omega_2|^2}.$$

The eigenvalues of this system are 0 and $-(\kappa_i + \kappa_o)$. Supposing that there is no ‘‘output’’ field incident on the medium, the fields are given by

$$\Omega_i(z) = \frac{\kappa_o + \kappa_i e^{-(\kappa_i + \kappa_o)z}}{\kappa_i + \kappa_o} \Omega_i(0), \quad (8)$$

$$\Omega_o(z) = \frac{\Omega_2 \kappa_o (1 - e^{-(\kappa_i + \kappa_o)z})}{\Omega_1 (\kappa_i + \kappa_o)} \Omega_i(0). \quad (9)$$

As claimed above, the scattered field is proportional to the input field. For resonant scattering ($\Delta=0$), κ_i and κ_o are real and energy is transferred from the input to the output in a distance $(\kappa_i + \kappa_o)^{-1}$, which is on the order of the absorption lengths $(2 \text{Re } \kappa_i)^{-1}$ and $(2 \text{Re } \kappa_o)^{-1}$. For nonresonant scattering ($|\Delta| \gg \gamma$), κ_i and κ_o are mostly imaginary; energy oscillates between the input and the output with a spatial period of $2\pi/\text{Im}(\kappa_i + \kappa_o)$, which is much shorter than either of the absorption lengths. In both cases, the fields evolve to the condition $\Omega_i(\infty)/\Omega_o(\infty) = \Omega_1/\Omega_2$ and propagate without further interaction with the medium. This behavior is illustrated in Figs. 2(a) and 2(b), which were obtained by numerical solution of Eqs. (6) for these two limiting cases.

In the case of maximal coherence ($\rho_{12}=1/2$), a steady state is reached in which the input and output each have 25% of the total incident power. This result holds because the input and output fields are resonant with their respective transitions, losing power to absorption until the system enters a double-dark state. However, if the ground states are not completely coherent as assumed in the derivation above, in other words $|\rho_{12}|^2 < \rho_{11}\rho_{22}$, a steady state does not exist; the Raman scattering is eventually dominated by absorption. In the case of resonant scattering with 50% coherence ($\rho_{11} = \rho_{22} = 1/2, \rho_{12} = 1/4$), the scattered power peaks after about two absorption lengths at about 4% of the input power, as illustrated in the numerical simulation shown in Fig. 2(c). In this case, in order to obtain higher conversion efficiency one must detune the input and increase the optical thickness of the medium.

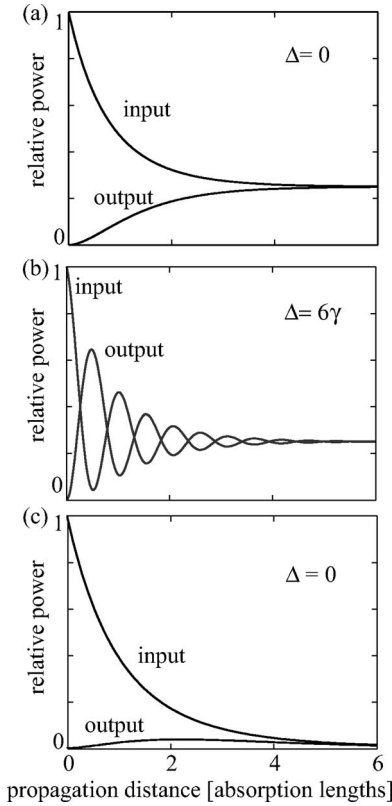


FIG. 2. Spatial evolution of the fields in coherent Raman scattering. (a) Resonant scattering. (b) Nonresonant scattering. (c) Resonant scattering with 50% of maximal coherence. The horizontal axis is scaled in each case by $(2 \operatorname{Re} \kappa_i)^{-1} = (2 \operatorname{Re} \kappa_o)^{-1}$, the absorption length in the absence of quantum coherent effects. Note that because the fields are nonresonant in (b), the absorption length is larger than in (a) and (c).

In principle, it is possible to obtain almost perfect conversion efficiency if the input field is tuned far away from resonance ($|\Delta| \gg \gamma$). In this case energy oscillates back and forth between the input and output fields, as shown in Fig. 2(b), making it possible to select the propagation distance to maximize the conversion efficiency. However, this would require a medium with a large optical thickness. For a system prepared in an imperfect dark state, the residual absorption might become a problem in this case.

Our primary interest in this paper is in the situation in which the input field is time varying, or equivalently, contains multiple components with different detunings Δ . The model just developed can be used to predict the maximum modulation bandwidth that can be efficiently transferred to the new frequency band. In the limit of a thin medium, such that $(\kappa_i + \kappa_o)z \ll 1$, we find from the above that

$$\Omega_o = \kappa_o z \frac{\Omega_2}{\Omega_1} \Omega_i. \quad (10)$$

The maximum bandwidth is thus determined by the frequency width of the coupling constant κ_o , which according to Eq. (7) is of the order of γ , that is, the bandwidth of the

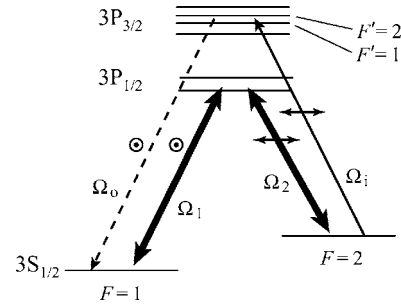


FIG. 3. The scheme for modulation transfer by coherent Raman scattering in sodium. A modulated input field Ω_i scatters off the ground hyperfine coherence created by the control fields, creating a modulated field Ω_o .

coupling constant to the scattering state or manifold $|4\rangle$. Note that this bandwidth is unrelated to, and generally much larger than, the bandwidth of the coherent population trapping resonance. In the limit of a thick medium, we find that

$$\Omega_o = \frac{\Omega_2}{\Omega_1} \frac{k_o |\mu_{42}|^2}{k_i |\mu_{41}|^2 + k_o |\mu_{42}|^2} \Omega_i. \quad (11)$$

This result shows no dependence on Δ , indicating that the transfer bandwidth is unlimited. The bandwidth is large because, within the assumptions of this model, even far-detuned components are eventually scattered given enough distance.

III. EXPERIMENT

The scheme for coherent modulation transfer was implemented in sodium vapor (Fig. 3). Strong, cross-polarized fields (Ω_1, Ω_2) tuned to the $D1$ transition create coherence between the $F=1$ and $F=2$ hyperfine ground levels. A weak field tuned to the $D2$ transition scatters off the hyperfine coherence, producing a field polarized orthogonally to the input and shifted in frequency by 1772 MHz. Although our simulations [42] predict that a relatively large hyperfine coherence can be produced in this configuration, the conversion efficiency is nevertheless significantly reduced by residual absorption of the input and scattered fields. Because absorption can occur via transitions to any of the four excited hyperfine levels of the $D2$ line, whereas selection rules allow Raman scattering through the $F'=1$ and $F'=2$ levels only, the susceptibility for absorption is always larger than that for Raman scattering. As in the case of incomplete coherent preparation [Fig. 2(c)], the conversion efficiency is limited to a few percent [42].

A tunable dye laser tuned to the sodium $D1$ line (589.6 nm) and an acousto-optic modulator were used to produce the control fields (Fig. 4). By passing the laser light through the modulator and rotating the polarization of the first-order diffracted beam, two orthogonally polarized beams 1772 MHz apart were obtained. A second dye laser was tuned to the $D2$ transition (589.0 nm) to provide an information-bearing input beam as well as a local-oscillator beam for heterodyne detection of the scattered field. The input compo-

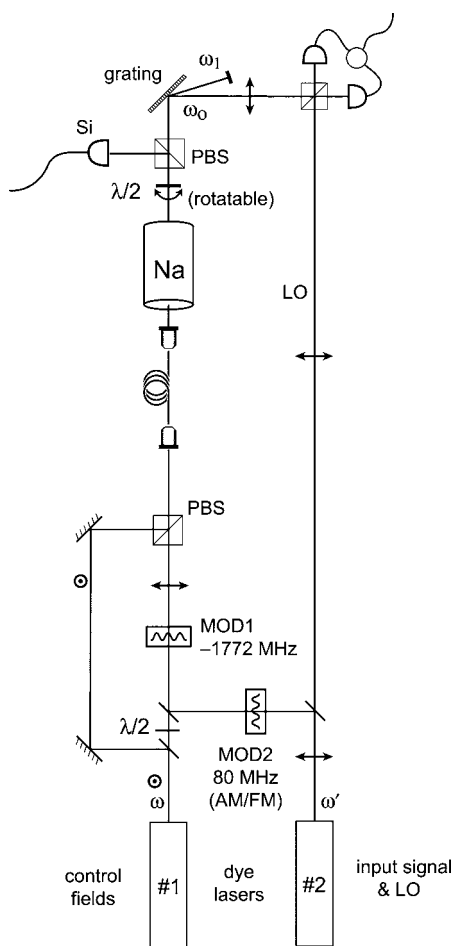


FIG. 4. Schematic of the experimental setup. The input and output field amplitudes are measured using balanced heterodyne detection. *Notation:* PBS = polarizing beam splitter; $\lambda/2$ = half wave plate; Si = silicon photodetector; MOD = acousto-optic modulator; LO = local oscillator. MOD2 has a carrier frequency of 80 MHz and is used to impress either frequency modulation or amplitude modulation onto the input probe beam. The controls fields of frequencies ω_1 and ω_2 are derived from dye laser 1, and the input signal field of frequency ω_i is derived from dye laser 2.

ment was passed through an additional acousto-optic modulator (MOD2) which imparted either amplitude or frequency modulation to the beam, depending on the experiment. All three beams were coupled into a 3-m segment of single-mode, polarization maintaining fiber. The light exiting the fiber was collimated at a diameter of 2 mm and directed into a sodium cell. At the entrance to the cell, the input beam contained 1.3 mW of power while the control fields contained 13 mW each. The cell, which was 8 cm long, was heated to produce a number density of approximately $1.5 \times 10^{11} \text{ cm}^{-3}$. Helium buffer gas at 50 mbar pressure was introduced into the cell to reduce time-of-flight broadening of the ground-state coherence to a few hundred kHz. The fields emerging from the cell were separated by a polarizing beam splitter and a grating. By rotating the polarizer, we were able to isolate either the input or scattered field for heterodyne detection. The beat note between the reference field and either input or output field was measured by a fast

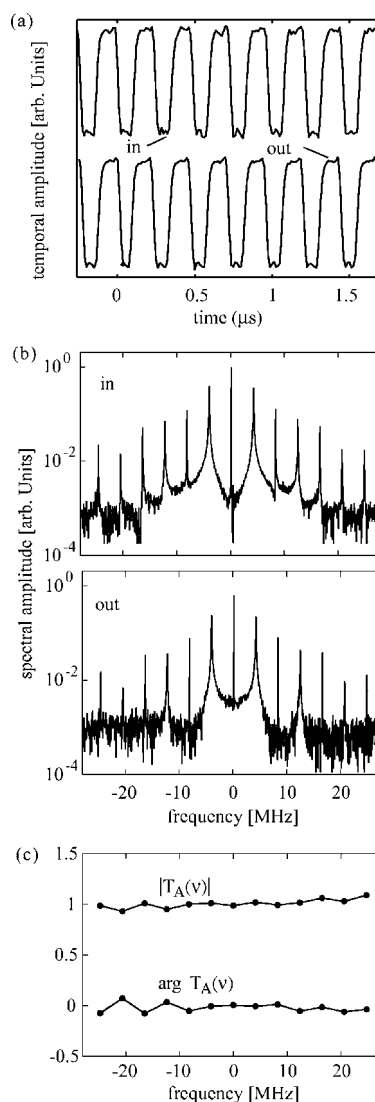


FIG. 5. Frequency conversion of an amplitude-modulated optical signal. (a) The temporal amplitudes of the input and output fields. (b) The spectral amplitudes of the input and output fields. (c) The magnitude and phase of the normalized transfer function (evaluated at locations of spectral peaks); a value of unity at all frequencies means perfect reproduction of the input field. For this particular signal, the conversion fidelity was 98%.

detector and recorded on a digital oscilloscope. The positive-frequency part of the spectrum was numerically isolated and downshifted to move the carrier frequency to zero frequency. In this way, the complex envelope of the input or output field was obtained.

For the first experiment, MOD2 was driven with an amplitude-modulating driver. A square wave was applied to the driver, resulting in an input field having nearly 100% amplitude modulation at 4 MHz. A portion of the normalized input and output field envelopes are shown in Fig. 5, along with the fields' spectra. Clearly, the scattered field is essentially identical to the input field (apart from the carrier wavelength and overall scale). In the second experiment, MOD2 was driven with a frequency-modulating driver. A sine wave was applied to the driver, resulting in a input field having

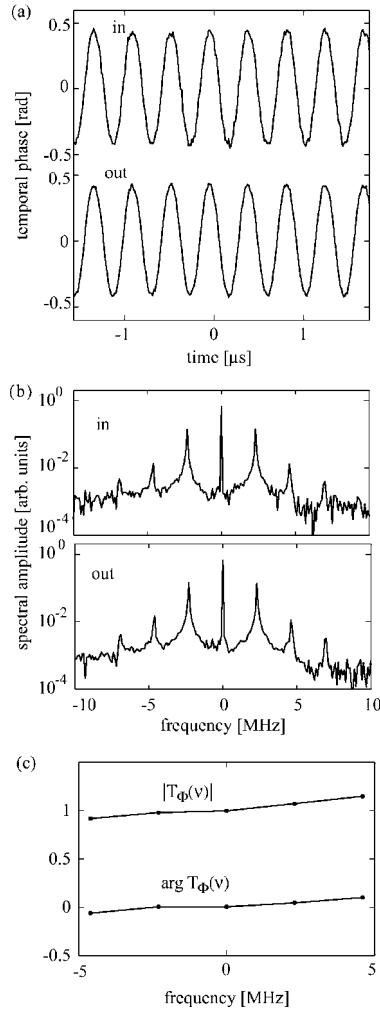


FIG. 6. Frequency conversion of a phase (frequency) modulated optical signal. (a) The temporal phases of the input and output fields. (b) The spectral amplitudes of the input and output fields. The conversion fidelity for this signal was 98%.

phase modulation at 2 MHz. The time-dependent phases and spectra of the input and scattered, output fields are shown in Fig. 6. Again, the two fields have essentially identical envelopes. The similarity of the input and scattered fields can be quantified by the fidelity,

$$\mathcal{F} = \frac{\int \tilde{E}_i^*(t) \tilde{E}_o(t) dt}{\sqrt{\int |\tilde{E}_i(t)|^2 dt} \sqrt{\int |\tilde{E}_o(t)|^2 dt}}, \quad (12)$$

where $\tilde{E}_{i,o}(t) = \int E_{i,o}(\nu) e^{2\pi i \nu t} d\nu$ and ν is the frequency relative to the field's carrier frequency. For both amplitude- and phase-modulated signals, the conversion fidelity was in excess of 0.99.

A more informative measure of the faithfulness of the process is the transfer function

$$T(\nu) = \frac{E_o(\nu)}{E_i(\nu)}. \quad (13)$$

If $T(\nu)$ is constant over some frequency band, then any signal falling in that band will be perfectly reproduced. Figures 5(c) and 6(c) show the magnitude and phase of $T(\nu)/|T(0)|$ evaluated at the frequencies present in the input and output signals. In the experiment involving amplitude modulation, the transfer function $T_A(\nu)$ has a flat, uniform response across the entire range of measurable frequencies, a range of tens of MHz. The transfer function $T_\Phi(\nu)$ in the phase-modulation experiment [Fig. 6(c)] shows a slight asymmetry, but is still fairly uniform over a bandwidth of several MHz. These data were taken on a different day than the amplitude-modulation data; on this day the input laser may have been detuned slightly off the peak of the Raman gain, resulting in a small variation in amplitude and phase of the scattered field with frequency.

While the quality of the signal transfer was excellent, the conversion efficiency was smaller than one would hope. Typically, a 1.3-mW input would result in only 5–10 μ W at the new frequency. As indicated above, this is in part due to the structure of the states involved in the Raman scattering, which have a larger susceptibility for absorption than Raman scattering. In principle, one could tune the input away from resonance and use a longer cell (or increase the number density) to reduce the absorption susceptibility relative to the scattering susceptibility. However, this approach presumes that the control fields will continue to prepare the atoms with large ground-state coherence over the large propagation distances. In practice, the degree of ground-state coherence is never maximal, and decreases with distance as the control fields are attenuated by residual absorption. In a sodium vapor buffered by an inert gas, the residual absorption is significant [42]. In a wax-coated rubidium cell, however, the ground-state coherence and EIT should be greater, yielding higher conversion efficiency.

Although the frequency shifts demonstrated here are not large from an optical perspective (1.8 GHz), the principal physics remains the same for two-photon (Raman) scattering between levels with much larger energy separation. Furthermore, it is not necessary that the input and control fields reside in the same region of the spectrum; near infrared control fields could be used to produce substantial shifts in ultraviolet signals, for instance.

Even though we have demonstrated the transfer of modulation only for an amplitude- or a phase-modulated signal, it is in principle possible to transfer the complete information from the input field to the output field, as shown by Eq. (9).

IV. CONCLUSIONS

To summarize, we have presented a scheme for coherent transfer of optical information from one frequency band to another by use of coherent Raman scattering. This scheme has the advantages of excellent fidelity, large bandwidth, and

potentially good efficiency. We implemented the scheme in sodium vapor by creating quantum coherence between the ground hyperfine levels, and scattering a signal field off this coherence. Both amplitude- and phase-modulated signals were transferred to different wavelengths with extremely high fidelity, thereby demonstrating the coherent nature of the conversion. The transfer function was found to be essentially flat over at least 40 MHz, showing that the scheme is useful for arbitrary signals of large bandwidth. While the efficiency was lower than desired, and the wavelength shift was very small, we believe that the principles demonstrated

here may be extended readily to other media that will permit substantial wavelength shifts and high conversion efficiencies.

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