Double switching from normal to anomalous dispersion via trichromatic phase manipulation of electromagnetically induced transparency

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We present a theoretical analysis of trichromatic phase manipulation of electromagnetically induced transparency and show that it is possible to obtain double switching, in which switching from normal to anomalous dispersion occurs in two separate frequency regimes. In particular, a four-level system in the N configuration is considered, in which electromagnetically induced transparency is established in the Λ subsystem and the additional transition is connected to the probe transition and is coupled by a trichromatic field. It is shown that the sum of the relative phases of the sideband components compared to that of the central component plays a crucial role in the absorption and dispersion spectra. Normal dispersion with negligible absorption or anomalous dispersion with small gain can be achieved in multiple-frequency regimes by varying the sum of the relative phases. When the sum phase is changed from 0 to π, switching from normal to anomalous dispersion occurs in the two different frequency regimes. On the other hand, so long as we fix the sum of two relative phases, the absorption and dispersion spectra keep their own features unchanged no matter how we vary the respective relative phases.

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I. INTRODUCTION

Recently great interest has been directed to the modification of dispersion and absorption properties of an absorbing medium by atomic coherence induced by coherent fields (see [1–5] for reviews). The atomic coherence leads to normal (positive) or anomalous (negative) dispersion, which corresponds to subluminal or superluminal light propagation with negligible absorption or gain. Harris *et al.* [6] first suggested that the group velocity was reduced in a three-level Λ system exhibiting electromagnetically induced transparency. Previously group velocities were reduced to $c/13.2$ [7], $c/165$ [8], and $c/3000$ [9] where c is the light velocity in vacuum. Hau *et al.* [10] used a Bose condensate and reduced the group velocity to 17 m/s. Later, the group velocities in hot gases were reduced to 90 m/s [11] and 8 m/s [12], and in a solid to 45 m/s [13]. Following an earlier proposal of Steinberg and Chiao [14], Wang *et al.* [15] demonstrated superluminal light propagation which was obtained by using a pair of Raman gain features via bichromatic excitation in the three-level Λ system. Agarwal and Dasgupta [16] predicted that the Raman gain process led to superluminal propagation in an N-shaped system. Negative dispersion without absorption was predicted [17–19] and demonstrated [20–22] for driven two-level systems.

More recently, much attention has been paid to the switching from positive to negative dispersion at the same frequency regime simply by parameter control. Several schemes have been proposed. Agarwal *et al.* [23] suggested a scheme in which a microwave field coupled the two lower states of the three-level Λ system. Dispersion switching appeared as the intensity of the microwave field was changed. Wilson-Gordon and Friedmann [24] considered a three-level Λ system in which a single pump was used. The dispersion displayed switching behavior with increasing intensity of the single driving field. Sahrai *et al.* [25] proposed to use two beams of lasers and one beam of microwave field in a double-Λ system. It was predicted that the group velocity could be controlled by changing the relative phases between different fields. Kang *et al.* [26] employed an N system and demonstrated that dispersion switching occurred as the intensity of the additional coupling field was increased. They measured a slow group velocity of $\sim 3\times 10^{-4}c$ and a negative group velocity of $\sim -4\times 10^{-4}c$.

So far, however, the switching from normal to anomalous dispersion has been limited to a single frequency regime. Turning to practical applications, one needs to achieve switching from normal to anomalous dispersion in different frequency regimes. It is well known that quantum entanglement of radiation fields lies at the heart of quantum-information processing. Entangled states of photons at different frequencies are generated and manipulated via their nonlinear interaction in an ensemble of atoms [27,28]. The efficiency of the nonlinear interaction depends crucially on the interaction time. Only for sufficiently long interaction time will a very efficient interaction take place [27–29]. It requires that group velocities of different pulses are equal whether they are subluminal or superluminal.

In this paper we show that it is possible to obtain double switching, in which switching from normal to anomalous dispersion occurs in two separate frequency regimes. The group velocities are equal whether the light propagation is subluminal or superluminal, when absorption or gain is negligibly small. The double switching is established by employing a trichromatic field to manipulate electromagnetically induced transparency. In particular, the present scheme is based on an N system, three transitions of which are coupled in sequence by the coupling field, the probe field,

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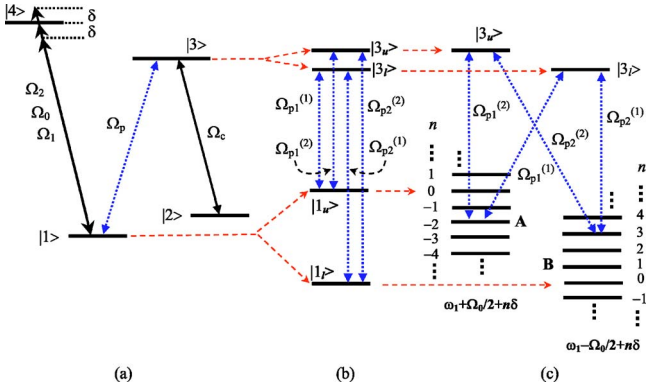

FIG. 1. (Color online) (a) The four-level N-type atomic system, in which the coupling field (Rabi frequency Ω_c) induces transparency in the probe field (Ω_p); a trichromatic field ($\Omega_1, \Omega_0, \Omega_2$) of frequency difference δ is used to manipulate the electromagnetically induced transparency. (b) The splitting of level $|3\rangle$ into $|3_{u,l}\rangle$ due to the coupling of the medium to the coupling field Ω_c and the splitting of level $|1\rangle$ into $|1_{u,l}\rangle$ due to the coupling of the medium to the central component Ω_0 of the trichromatic field. Correspondingly, the probe transition $|1\rangle$-$|3\rangle$ is split into four with Rabi frequencies $\Omega_{pi}^{(j)}$, $i,j=1,2$. (c) Further level splitting of the levels $|1_{u,l}\rangle$ into two infinite sets of sublevels A and B. Each of the above four transitions is split into four infinite sets of transitions.

and the trichromatic driving field [Fig. 1(a)]. By varying the sum of the relative phases of the sideband components compared to that of the central component we obtain the double switching from positive to negative dispersion. On the other hand, when we fix the sum of the relative phases, the pulse propagation retains its own features though the respective relative phases change arbitrarily.

It should be noted that there have been some schemes only for multiple positive dispersion. Lukin *et al.* [30] first showed the appearance of a pair of positive dispersions in double transparency induced by a new coupling transition. The experimental demonstration was realized by Chen *et al.* [31] and Ye *et al.* [32]. McGloin *et al.* [33] and Paspalakis *et al.* [34] showed the presence of multiple transparency and multiple positive dispersion in an n-level ladder system ($n>3$) and in an n-level system with an excited state and $n-1$ ground states, respectively. Composite transparency media were also proposed to entangle two photons at different frequencies [28,29]. Such systems contained two subsystems in the Λ configuration. Wang *et al.* recently realized an experimental demonstration for bichromatic transparency [35] in a three-level Λ system. Trichromatic excitation led to the phase dependence of the multiple transparency [36]. In contrast, to our knowledge, there have been few reports on negative dispersion in multiple frequency regimes. An exception is the work of Harshawardhan and Agarwal [37], who suggested using simultaneously a modulated field on the coupling transition and an incoherent field on the probe transition in the V system. So far there has been no proposal for multiple dispersion switching between positive and negative dispersions. Here we present a scheme for double dispersion switching.

II. MODEL AND EQUATION

The model system we consider is shown in Fig. 1(a). The atom has two metastable states $|1\rangle$ and $|2\rangle$ and two excited states $|3\rangle$ and $|4\rangle$. The four transitions between two metastable states and two excited states $|1,2\rangle$-$|3,4\rangle$ are dipole allowed, while the transitions $|1\rangle$-$|2\rangle$ and $|3\rangle$-$|4\rangle$ are dipole forbidden. A coupling pump field $\tilde{\mathbf{E}}_c(t)=\frac{1}{2}\mathbf{E}_c e^{-i\omega_c t}+\text{c.c.}$ is applied to the $|2\rangle$-$|3\rangle$ transition, and a probe field $\tilde{\mathbf{E}}_p(t)=\frac{1}{2}\mathbf{E}_p e^{-i\omega_p t}+\text{c.c.}$ is coupled to the $|1\rangle$-$|3\rangle$ transition, where $\mathbf{E}_c$ and $\mathbf{E}_p$ are the amplitudes of the pump field and the probe field, respectively, and ω_c and ω_p are the corresponding frequencies. A trichromatic field $\tilde{\mathbf{E}}_t(t)=\frac{1}{2}(\mathbf{E}_0+\mathbf{E}_1 e^{i\delta t}+\mathbf{E}_2 e^{-i\delta t})e^{-i\omega_s t}+\text{c.c.}$ is coupled to the $|1\rangle$-$|4\rangle$ transition, where $\mathbf{E}_0, \mathbf{E}_1$, and $\mathbf{E}_2$ are the amplitudes of the trichromatic pump components, and $\omega_s, \omega_s-\delta$, and $\omega_s+\delta$ are the corresponding frequencies, and δ is the frequency difference.

The master equation is derived in an appropriate rotating frame and in the dipole approximation as [38]

$$\dot{\rho}=-\frac{i}{\hbar}[H_1+H_2,\rho]+\sum_{i=1,2;j=3,4}\mathcal{L}_{ij}\rho, \tag{1}$$

where the system Hamiltonian H_1+H_2 is written in the form

$$H_1=-\hbar(\Delta_p-\Delta_c)\sigma_{22}-\hbar\Delta_p\sigma_{33}-\hbar\Delta_0\sigma_{44}, \tag{2}$$

$$H_2=-\frac{\hbar}{2}\Omega_p\sigma_{31}-\frac{\hbar}{2}\Omega_c\sigma_{32}-\frac{\hbar}{2}(\Omega_0+\Omega_1 e^{i\delta t}+\Omega_2 e^{-i\delta t})\sigma_{41}+\text{H.c.} \tag{3}$$

Here we have defined the atom-field detunings $\Delta_p=\omega_p-\omega_{31}$, $\Delta_c=\omega_c-\omega_{32}$, and $\Delta_0=\omega_s-\omega_{41}$, where ω_{31}, ω_{32}, and ω_{41} are the atomic resonance frequencies of the transitions $|1\rangle$-$|3\rangle$, $|2\rangle$-$|3\rangle$, and $|1\rangle$-$|4\rangle$, respectively. $\Omega_p=\mathbf{d}_{31}\cdot\mathbf{E}_p/\hbar$ and $\Omega_c=\mathbf{d}_{32}\cdot\mathbf{E}_c/\hbar$ are Rabi frequencies involved in the probe and coupling fields, respectively, and $\Omega_i=\mathbf{d}_{41}\cdot\mathbf{E}_i/\hbar$ $(i=0,1,2)$ denote the Rabi frequencies associated with three components, respectively, of the trichromatic field. $\sigma_{ij}=|i\rangle\langle j|$ $(i,j=1$–$4)$ are projection operators for $i=j$ and spin-flip operators for $i\neq j$. $\mathcal{L}_{ij}\rho$ represents the atomic decay with rate γ_{ji} from level $|j\rangle$ to level $|i\rangle$, and takes the form

$$\mathcal{L}_{ij}\rho=\frac{\gamma_{ji}}{2}(2\sigma_{ij}\rho\sigma_{ji}-\sigma_{ji}\sigma_{ij}\rho-\rho\sigma_{ji}\sigma_{ij}). \tag{4}$$

The equations of motion for the density matrix elements are derived as

$$\dot{\rho}_{11}=\gamma_{31}\rho_{33}+\gamma_{41}\rho_{44}+\frac{i}{2}\Omega_p^*\rho_{31}-\frac{i}{2}\Omega_p\rho_{13}+\frac{i}{2}(\Omega_0^*+\Omega_1^* e^{-i\delta t}+\Omega_2^* e^{i\delta t})\rho_{41}-\frac{i}{2}(\Omega_0+\Omega_1 e^{i\delta t}+\Omega_2 e^{-i\delta t})\rho_{14}, \tag{5}$$

$$\dot{\rho}_{22}=\gamma_{32}\rho_{33}+\gamma_{42}\rho_{44}+\frac{i}{2}\Omega_c^*\rho_{32}-\frac{i}{2}\Omega_c\rho_{23}, \tag{6}$$

$$\dot{\rho}_{33} = -(\gamma_{31}+\gamma_{32})\rho_{33} + \frac{i}{2}\Omega_p\rho_{13} - \frac{i}{2}\Omega_p^*\rho_{31} + \frac{i}{2}\Omega_c\rho_{23} - \frac{i}{2}\Omega_c^*\rho_{32}, \tag{7}$$

$$\dot{\rho}_{12} = -\Gamma_{12}\rho_{12} + \frac{i}{2}\Omega_p^*\rho_{32} + \frac{i}{2}(\Omega_0^* + \Omega_1^* e^{-i\delta t} + \Omega_2^* e^{i\delta t})\rho_{42} - \frac{i}{2}\Omega_c\rho_{13}, \tag{8}$$

$$\dot{\rho}_{13} = -\Gamma_{13}\rho_{13} + \frac{i}{2}\Omega_p^*(\rho_{33}-\rho_{11}) - \frac{i}{2}\Omega_c^*\rho_{12} + \frac{i}{2}(\Omega_0^* + \Omega_1^* e^{-i\delta t} + \Omega_2^* e^{i\delta t})\rho_{43}, \tag{9}$$

$$\dot{\rho}_{14} = -\Gamma_{14}\rho_{14} + \frac{i}{2}(\Omega_0^* + \Omega_1^* e^{-i\delta t} + \Omega_2^* e^{i\delta t})(\rho_{44}-\rho_{11}) + \frac{i}{2}\Omega_p^*\rho_{34}, \tag{10}$$

$$\dot{\rho}_{23} = -\Gamma_{23}\rho_{23} + \frac{i}{2}\Omega_c^*(\rho_{33}-\rho_{22}) - \frac{i}{2}\Omega_p^*\rho_{21}, \tag{11}$$

$$\dot{\rho}_{24} = -\Gamma_{24}\rho_{24} - \frac{i}{2}(\Omega_0^* + \Omega_1^* e^{-i\delta t} + \Omega_2^* e^{i\delta t})\rho_{21} + \frac{i}{2}\Omega_c^*\rho_{34}, \tag{12}$$

$$\dot{\rho}_{34} = -\Gamma_{34}\rho_{34} + \frac{i}{2}\Omega_p\rho_{14} + \frac{i}{2}\Omega_c\rho_{24} - \frac{i}{2}(\Omega_0^* + \Omega_1^* e^{-i\delta t} + \Omega_2^* e^{i\delta t})\rho_{31}, \tag{13}$$

together with the complex conjugates for Eqs. (8)–(13) and the closure relation $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44}=1$. The parameters that appear in Eqs. (8)–(13) are

$$\Gamma_{12} = \gamma_{21} + i(\Delta_p - \Delta_c),$$

$$\Gamma_{13} = \tfrac{1}{2}(\gamma_3 + \gamma_{21}) + i\Delta_p,$$

$$\Gamma_{14} = \tfrac{1}{2}(\gamma_4 + \gamma_{21}) + i\Delta_0,$$

$$\Gamma_{23} = \tfrac{1}{2}(\gamma_3 + \gamma_{21}) + i\Delta_c,$$

$$\Gamma_{24} = \tfrac{1}{2}(\gamma_4 + \gamma_{21}) - i(\Delta_p - \Delta_c - \Delta_0),$$

$$\Gamma_{34} = \tfrac{1}{2}(\gamma_3 + \gamma_4) + i(\Delta_0 - \Delta_p), \tag{14}$$

where $\gamma_3=\gamma_{31}+\gamma_{32}$ ($\gamma_4=\gamma_{41}+\gamma_{42}$) is the atomic rate of decay from $|3\rangle$ ($|4\rangle$) to both $|1\rangle$ and $|2\rangle$, and γ_{21} is the dephasing rate between levels $|1\rangle$ and $|2\rangle$.

In order to solve the motion equations of the density-matrix elements, we can apply the harmonic expansion method and expand the density matrix elements as

$$\rho_{jk} = \sum_{l=-\infty}^{+\infty} \rho_{jk}^{(l)} e^{il\delta t} \quad (j,k = 1\text{–}4), \tag{15}$$

where the $\rho_{jk}^{(l)}$'s represent the slowly varying amplitudes and are needed to determine the absorption and dispersion properties of the optical medium. Substitution of Eq. (15) into Eqs. (5)–(13) and the complex conjugates of Eqs. (8)–(13) leads to an infinite set of equations where l is an integer that varies from $-\infty$ to ∞. The steady-state solutions can be solved by the method of matrix inversion. To do so, we construct a column vector $\mathbf{X}$ as

$$\mathbf{X} = (X_1^{(-N)},\ldots,X_{15}^{(-N)};\ldots;X_1^{(0)},\ldots,X_{15}^{(0)};\ldots;X_1^{(N)},\ldots,X_{15}^{(N)})^T, \tag{16}$$

$$X_{1-15}^{(l)} = (\rho_{21}^{(l)},\rho_{12}^{(l)},\rho_{31}^{(l)},\rho_{13}^{(l)},\rho_{41}^{(l)},\rho_{14}^{(l)},\rho_{32}^{(l)},\rho_{23}^{(l)},\rho_{42}^{(l)},\rho_{24}^{(l)},\rho_{43}^{(l)},\rho_{34}^{(l)},\rho_{11}^{(l)},\rho_{22}^{(l)},\rho_{33}^{(l)})^T, \tag{17}$$

where the value of the integer N is chosen appropriately to determine the dimension of the vector $\mathbf{X}$ and the accuracy of our matrix inversion method. For a given value of N, the dimension of the vector $\mathbf{X}$ is $15\times(2N+1)$. The set of equations for slowly varying amplitudes can be written in the form

$$\dot{\mathbf{X}} + \mathbf{Q}\mathbf{X} = \mathbf{R}, \tag{18}$$

where $\mathbf{Q}$ is a $[15\times(2N+1)]\times[15\times(2N+1)]$ matrix and $\mathbf{R}$ is a $15\times(2N+1)$ column vector, whose elements can be determined from the above equations. The steady-state solution to the vector $\mathbf{X}$ is obtained by setting the time derivatives to zero and using matrix inversion as

$$\mathbf{X} = \mathbf{Q}^{-1}\mathbf{R}. \tag{19}$$

The complex polarization of the medium is written in the present rotating frame as

$$P(t) = n_a d_{13}\rho_{31} = n_a d_{13}\sum_{l=-\infty}^{+\infty}\rho_{31}^{(l)} e^{il\delta t}, \tag{20}$$

where n_a is the atomic number density. Using the complex polarization components $p(\omega_p)=n_a d_{13}\rho_{31}^{(0)}$ we have the susceptibility $\chi(\omega_p)=p(\omega_p)/\varepsilon_0 E_p$. The refraction (the deviation of the refractive index from unity) and the absorption are determined by the real and imaginary parts of the susceptibility $\chi'=\mathrm{Re}\,\chi$ and $\chi''=\mathrm{Im}\,\chi$, respectively,

$$\chi'(\omega_p) = \frac{n_a|d_{13}|^2}{\varepsilon_0\hbar}\mathrm{Re}\left(\frac{\rho_{31}^{(0)}}{\Omega_p}\right), \tag{21}$$

$$\chi''(\omega_p) = \frac{n_a|d_{13}|^2}{\varepsilon_0\hbar}\mathrm{Im}\left(\frac{\rho_{31}^{(0)}}{\Omega_p}\right). \tag{22}$$

The present method is applicable for the case of arbitrary intensity of the probe field. Here we focus only on the linear susceptibility, which is obtained simply by assuming the probe field to be weak, $|\Omega_p| \ll \gamma_3, \gamma_4, |\Omega_i|$ $(i=0,1,2,c)$.

The dispersion is the derivative of the refraction with respect to Δ_p. The group velocity v_g of the probe pulse is given by [2]

$$v_g = \frac{c}{n_R + \omega_p \partial n_R / \partial \omega_p}, \tag{23}$$

where $n_R = 1 + 2\pi\chi'(\omega_p)$ is the real part of the refractive index. For normal dispersion ($\partial\chi'/\partial\omega_p > 0$), the group velocity is less than the phase velocity ($v_g < c/n_R$) while for anomalous dispersion ($\partial\chi'/\partial\omega_p < 0$), the group velocity exceeds the phase velocity ($v_g > c/n_R$) or becomes negative ($v_g < 0$).

III. ABSORPTION AND DISPERSION SPECTRA

In what follows we present our numerical results. In general we can take $\gamma_{31} = \gamma_{32}$, $\gamma_{41} = \gamma_{42}$. The dipole matrix element is expressed via experimentally measurable quantities, the radiative decay rate γ_{31} and the wavelength λ_p, as [38]

$$\frac{|d_{13}|^2}{\varepsilon_0 \hbar} = \frac{3\lambda_p^3 \gamma_{31}}{8\pi^2}. \tag{24}$$

In these units Eqs. (21) and (22) read

$$\chi'(\omega_p) = \frac{3n_a\lambda_p^3}{16\pi^2}\mathrm{Re}\left(\frac{\rho_{31}^{(0)}}{\Omega_p/\gamma_3}\right), \tag{25}$$

$$\chi''(\omega_p) = \frac{3n_a\lambda_p^3}{16\pi^2}\mathrm{Im}\left(\frac{\rho_{31}^{(0)}}{\Omega_p/\gamma_3}\right), \tag{26}$$

where we have used the relation $\gamma_3 = \gamma_{31} + \gamma_{32} = 2\gamma_{31}$. Equations (25) and (26) show that the susceptibility can be scaled in units of $3n_a\lambda_p^3/16\pi^2$. For the realistic example we consider a four-level ^{87}Rb system in which the transition rates, the wavelength, and the density are $\gamma_3 = 2\pi \times 5.3 \times 10^6\ \mathrm{s}^{-1}$, $\gamma_4 = 2\pi \times 5.9 \times 10^6\ \mathrm{s}^{-1}$, $\lambda_p = 794$ nm, $n_a = 2 \times 10^{12}\ \mathrm{cm}^{-3}$. This system was used to demonstrate the switching from normal to anomalous dispersion in a single frequency regime [26]. In our calculation we scale frequencies, detunings, Rabi frequencies, and decay rates in units of $\gamma_3 = \gamma_{31} + \gamma_{32}$. In the scaled unit, we have $\gamma_{31} = \gamma_{32} = 0.5$, $\gamma_{41} = \gamma_{42} = 0.557$. Without loss of generality, we assume Ω_c and Ω_0 to be real. We use $\Omega_j = |\Omega_j| e^{-i\phi_j}$ ($j = 1, 2$) where $\phi_{1,2}$ are the relative phases of the sideband components $E_{1,2}$ compared to that of the central component E_0, respectively, and define the sum of the relative phases of the sideband components to that of the central component $\Phi = \phi_1 + \phi_2$. The real part (dotted line) and imaginary part (solid line) of the susceptibility are plotted in Fig. 2 as functions of the probe detuning Δ_p for three groups of different parameters: (a), (b) $\Omega_c = \Omega_0 = |\Omega_1| = |\Omega_2| = 5$, $\delta = 5$; (c), (d) $\Omega_c = \Omega_0 = |\Omega_1| = |\Omega_2| = 10$, $\delta = 10$; and (e), (f) $\Omega_0 = |\Omega_1| = |\Omega_2| = 5$, $\delta = 5$, $\Omega_c = 10$. The left column (a), (c), (e) is for $\Phi = 0$ while the right column (b), (d), (f) is for $\Phi = \pi$. The other parameters are chosen as $\Omega_p = 0.01$, $\gamma_{21} = 0.001$, $\Delta_0 = 0$, and $\Delta_c = 0$. The absorption and dispersion spectra depend not only on the Rabi frequencies ($\Omega_c, \Omega_0, |\Omega_1|, |\Omega_2|$) and the frequency difference (δ) but also on the sum phase (Φ). The characteristic features are presented as follows.

(i) For $\Phi = 0$ (left column) the system exhibits electromagnetically induced transparency with normal dispersion in a series of frequency regimes. Intervals between adjacent peaks (or between adjacent transparency windows) are determined by the frequency difference δ. For the cases in Figs. 2(a) and 2(e), the interval is equal to $\delta = 5$, while the gap is $\delta = 10$ for the case in Fig. 2(c). Whether the central absorption spectrum displays an absorption peak or a transparency window depends on Rabi frequencies. For example, when we vary the Rabi frequency Ω_c from $\Omega_c = 5$ to 10 and keep unchanged other parameters, the central absorption peak [Fig. 2(a)] is replaced by the central transparency [Fig. 2(e)].

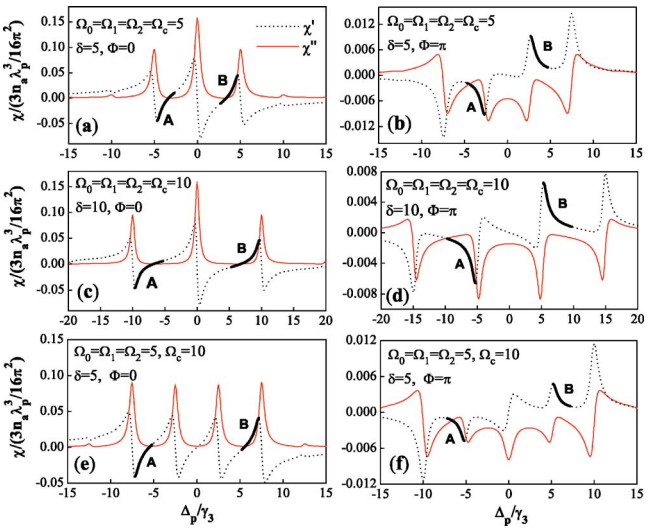

FIG. 2. (Color online) The real part (dotted line) and the imaginary part (solid line) of the susceptibility as functions of the probe detuning Δ_p for (a), (b) $\Omega_0 = |\Omega_1| = |\Omega_2| = \Omega_c = 5$, $\delta = 5$; (c), (d) $\Omega_0 = |\Omega_1| = |\Omega_2| = \Omega_c = 10$, $\delta = 10$; and (e), (f) $\Omega_0 = |\Omega_1| = |\Omega_2| = 5$, $\Omega_c = 10$, $\delta = 5$. $\Phi = 0$ is for the left column (a), (c), (e) while $\Phi = \pi$ is for the right column (b), (d), (f). The other parameters are chosen as $\Omega_p = 0.01$, $\gamma_{31} = \gamma_{32} = 0.5$, $\gamma_{41} = \gamma_{42} = 0.557$, $\gamma_{21} = 0.001$, $\Delta_0 = 0$, and $\Delta_c = 0$. Double switching from normal to anomalous dispersion occurs in the regimes indicated by arrows with the letters *A* and *B*.

(ii) When the sum phase is changed from $\Phi = 0$ (left column) to $\Phi = \pi$ (right column), the absorption features are turned into gain features in a very wide frequency range and multiple positive dispersion is transformed into multiple negative dispersion. The frequencies at which gain peaks appear for $\Phi = \pi$ correspond to those at which the transparency windows are seated for $\Phi = 0$. However, the interval between adjacent gain peaks is kept unchanged. Regardless of the change in the sum phase, the interval is determined simply by the frequency difference δ. By comparison it is easily understood that the appearance of the gain peaks reflects the role of the sum phase in the absorption spectra. First, varying the sum phase from $\Phi = 0$ to π leads to the splitting of the absorption peak into two, which are shifted from the original position by $\pm\delta/2$, respectively. The split components from the adjacent peaks superpose and turn upside down, and lead to gain features. Correspondingly, the normal dispersion between two adjacent absorption peaks is transformed into the anomalous dispersion between two adjacent gain peaks as the sum phase changes from $\Phi = 0$ to π.

(iii) Comparing the left column ($\Phi = 0$) with the right

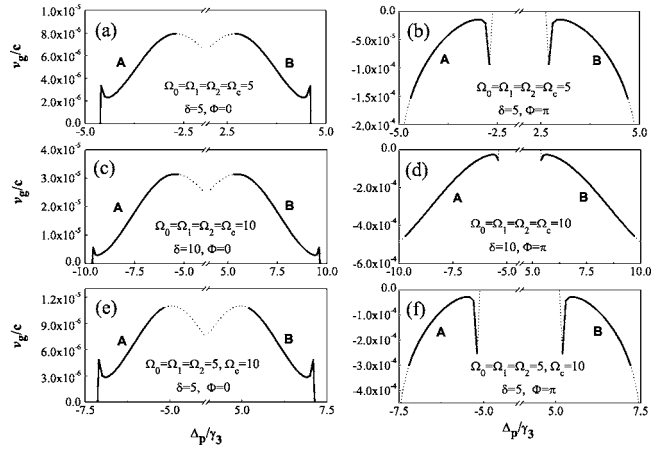

FIG. 3. The group velocity versus the probe detuning Δ_p for the same parameters as in Fig. 2. Double switching from slow group velocity to negative group velocity occurs in the regimes indicated by the letters A and B.

column ($\Phi=\pi$) it is easy to find the two separate frequency regimes (indicated by thick lines with the letters A and B), in which positive dispersion with vanishing absorption occurs for $\Phi=0$ while negative dispersion with negligible gain happens for $\Phi=\pi$. That means that when we vary the sum phase from $\Phi=0$ to π, the switching from normal to anomalous dispersion occurs in two different frequency regimes. In Fig. 3 we plot the group velocity v_g for the same parameters as in Fig. 2. It is seen that the switching from slow group velocity to negative group velocity occurs in two regimes, which are also indicated by thick lines with the letters A and B. Since $\omega_p|\partial n_R/\partial\omega_p|=\omega_p|\partial n_R/\partial\Delta_p|\gg n_R$ except near the frequency at which the $|\chi'|$'s take their maximum values, the group velocity spectra are symmetrical with respect to the atomic resonant transition frequency ω_{31} ($\Delta_p=0$). For the atomic density considered here, the positive group velocities are reduced to $v_g\approx(10^{-6}$–$10^{-5})c$ while the negative group velocities are $v_g\approx-10^{-4}c$. Such group velocities are in agreement with those measured in cold Rb atoms, where dispersion switching occurs in a single frequency regime [26]. The A and B regimes cover the detuning range $\Delta_p\approx-4.7$ to -2.7 and $\Delta_p\approx2.7$–4.7 for Figs. 3(a) and 3(b); $\Delta_p\approx-9.7$ to -5.3 and $\Delta_p\approx5.3$–9.7 for Figs. 3(c) and 3(d); $\Delta_p\approx-7.2$ to -5.2 and $\Delta_p\approx5.2$–7.2 for Figs. 3(e) and 3(f). The frequency ranges in Figs. 3(a), 3(b), 3(e), and 3(f) are about twice as large as the level width of the excited states γ_3, while the frequency range in Figs. 3(c) and 3(d) is about four times as large as the level width. These frequency ranges are almost the same as for single dispersion switching [26]. Since the interval between the adjacent absorption peaks is determined by the difference frequency δ, we can increase the frequency range for the double switching by increasing the difference frequency δ. The price is relatively less reduction of group velocity, as shown in Figs. 2(c) and 2(d). However, the reduction of the group velocity can be greatly enhanced in a Bose-Einstein condensate because of the benefit of a much greater atomic density.

(iv) On the other hand, the response of the medium can be independent of the change in the respective relative phases when the sum of the two relative phases is fixed, i.e., $\phi_1+\phi_2=C$ (constant). When we vary the respective phases $\phi_1=(C/2)-\phi$ and $\phi_2=(C/2)+\phi$ (ϕ is any real number) all properties of absorption and dispersion are not longer changed, so long as C stays unchanged. For the cases in Fig. 2, the absorption and dispersion display remarkably different features for different values $C=0$ (a), (c), (e), $C=\pi$ (b), (d), (f). But for a given value of C, the spectra exactly keep their features regardless of the respective values of ϕ_1 and ϕ_2.

The above results can be understood by employing dressed atomic states [39]. As above we consider $\Delta_0=\Delta_c=0$ and $\delta>0$. The dressing transform can be performed through three steps. In the first step, the coupling field Ω_c splits the level $|3\rangle$ into two sublevels $(|3_u\rangle,|3_l\rangle)$ with level shifts $\pm\hbar\Omega_c/2$, as shown in Fig. 1(b). As a consequence, the probe transition $|1\rangle$-$|3\rangle$ is split into two subtransitions $|1\rangle$-$|3_{u,l}\rangle$, which have Rabi frequencies Ω_{p1} and Ω_{p2}, respectively, and resonance frequencies $\omega_{31}\pm(\Omega_c/2)$, respectively. In the second step, the coupling of the atom to the central driving component Ω_0 gives rise to the splitting of the level $|1\rangle$ into two sublevels $(|1_u\rangle,|1_l\rangle)$, which move from the level $|1\rangle$ by $\pm\hbar\Omega_0/2$. Thus, the above two subtransitions $|1\rangle$-$|3_{u,l}\rangle$ are split into four $|1_{u,l}\rangle$-$|3_{u,l}\rangle$, which have Rabi frequencies $\Omega_{pi}^{(j)}$, $i,j=1,2$, and resonance frequencies $\omega_{31}\pm(\Omega_c\pm\Omega_0)/2$. In the third step, the coupling of the atom to the sideband components Ω_1 and Ω_2 leads to further splitting [Fig. 1(c)]. Two sublevels $|1_u\rangle$ and $|1_l\rangle$ split, respectively, into two infinite series of sublevels (indicated by A and B, respectively), which have energy levels $\hbar(\omega_1\pm\Omega_0/2+n\delta)$, $n=0,\pm1,\pm2,\ldots$. The sublevels in the same infinite series are separated by the modulation frequency δ. As a result, each of the above four probe transitions $|1_{u,l}\rangle$-$|3_{u,l}\rangle$ becomes an infinite set of transitions. These four infinite sets of transitions have the central resonance frequencies $\omega_p^{(c)}=\omega_{31}\pm(\Omega_c\pm\Omega_0)/2$ and the sideband resonance frequencies $\omega_p^{(s)}=\omega_{31}\pm(\Omega_c\pm\Omega_0)/2+n\delta$, $n=\pm1,\pm2,\ldots$.

The absorption and dispersion spectra are the result of the coherent superposition of all probe transitions. The gain is due to the optical pumping that is obtained by the spontaneous decay $|4\rangle\rightsquigarrow|2\rangle$. In the dressed picture, the spontaneous emission $|4\rangle\rightsquigarrow|2\rangle$ gives rise to the incoherent population transfer from two infinite sets of sublevels A and B to the two dressed states $(|3_u\rangle,|3_l\rangle)$. It is clear that the gain depends on atomic coherence at the same time. The dressed coherence occurs not only between the transitions in each group but also between any two different groups. The probe transitions from different groups superpose in a complicated way because the coherence depends not only strongly on all Rabi frequencies $(\Omega_0,|\Omega_1|,|\Omega_2|)$ and modulation frequency δ, but also crucially on the phases (ϕ_1,ϕ_2). Note that no phase dependence is existent for bichromatic case. Jakob and Kryuchkyan [40] showed that phase-dependent squeezed-reservoir effects occur for the lower-level coupling case but are absent for the upper-level coupling case. The phase dependence vanishes for the lower-level coupling case when the electromagnetic reservoir is reduced to the usual vacuum as considered here. Phase dependence does not appear in the bichromatically driven three-level system in Λ configuration [35]. It is true for the present system when one of the trichro-

matic components is absent ($\Omega_i=0$, $i=0,1,2$). The present system can be viewed as the combination of two bichromatically driven subsystems, each of which involves one infinite set of sublevels A or B. If the two subsystems are independent of each other, there is no phase dependence, as in the bichromatic case. It is due to the introduction of the central field component Ω_0 that two subsystems are correlated to each other. It is such a correlation that leads to phase dependence. It is obvious that the relative phases of the central component E_0 to those of the sideband components $E_{1,2}$ play their role through the phase sum $[-(\phi_1+\phi_2)]$. Compared with the bichromatic case, however, phase independence is kept under an additional condition. The condition is found that the phase sum $\phi_1+\phi_2$ is kept constant. That is to say, the final spectrum is independent of the phase difference $(\phi_1-\phi_2)$ when the sum phase Φ is fixed.

IV. CONCLUSION

In summary, we have shown that when a trichromatic field is used to manipulate the electromagnetically induced transparency in a four-level N-type system, the absorption and dispersion spectra depend crucially on the sum of the relative phases of the sideband components compared to that of the central component. When we vary the sum phase, we can have switching from normal dispersion with negligible absorption to anomalous dispersion with negligible gain in two different frequency regimes. We have also shown that when the sum phase is fixed, the dispersion and absorption spectra are independent of the respective relative phases.

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