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Final-state probabilities for β^--decaying light atoms

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The general theory of atomic processes stimulated by the nuclear β^- decay is considered. The final-state probabilities are determined for a number of β^--decaying light atoms (He-Ar) with the use of sudden approximation.

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I. INTRODUCTION

In the process of nuclear $\beta^{\pm}$ decay a nucleus with atomic number (or nuclear charge) Z emits an electron e^- (or positron e^+) and a neutrino ν (or antineutrino $\bar{\nu}$). The energy released in the β^- decay is shared between the outgoing electron and the neutrino (antineutrino). The nuclear recoil is negligible because of the very large nuclear mass. The incident nucleus is transformed into the daughter (or secondary) nucleus with the electric charge $Z\pm 1$. A large number of nuclear $\beta^{\pm}$ decays have been studied since the middle of the 1930s (see, e.g. [1], and references therein). Currently, the nuclear β decay is of great interest in various applications to modern technology. Let us mention only a few widely known β^--decaying nuclei, which are of paramount importance for nuclear technology: ^{3}H, ^{233}Th, ^{239}U, and ^{247}Pu. The three last isotopes are used to produce the fissionable ^{233}U, ^{239}Pu, and ^{247}Cm nuclei. Many other β^--decaying nuclei are of increasing interest for medical applications. Note also that more than 80% of all nuclei which form in nuclear reactors and nuclear explosions are $\beta^{\pm}$ decaying isotopes [2]. The ionization of inner-shell electrons leads to Auger transitions and to optical radiation that may be important for remote diagnostics in reactors.

The general equation of the β^- decay can be written in the form

$$Q \rightarrow (Q+1)^+ + e^- + \nu \qquad (1)$$

where Q is the nuclear charge of the incident nucleus, while e^- and ν are the emitted (fast) electron and neutrino, respectively. The Lorentz γ factor ($\gamma = E/m_e c^2$) of the electron emitted in the β^- decay is usually bounded between 2 and 10. In some cases, the Lorentz γ factor can be $\approx 15-18$. This means that the momentum of the emitted electron (or β^- electron) is significantly larger ($\approx 10^3-10^4$ times) than the momenta of atomic electrons. This forms a basis for application of the sudden approximation (see below) to describe a number of processes in electronic shells of incident atoms which are stimulated by the nuclear β^- decay.

In general, the nuclear β^- decay proceeds in many-electron atoms/ions, rather than in bare nuclei. This means that the emitted β^- electron interacts with the atomic electrons. The electron density distribution in the original atom/ion is also affected by a sudden change of the nuclear charge. These two effects produce excitations and even ionization of the outer and internal electrons in the considered atomic system. In other words, the ion which is formed after the nuclear β^- decay in the original atom can be found in various excited states, including unbound states. A number of experiments have been performed to measure the final-state probabilities for many $\beta^{\pm}$-decaying isotopes [3–5]. The work [5] contains an extensive review of experimental literature related to various atomic phenomena observed during the nuclear $\beta^{\pm}$ decays in atoms.

The main goal of this study is to determine the final-state probabilities for a large number of β^--decaying atoms. These probabilities are of interest in a number of applications. Below, such probabilities are determined with the use of sudden approximation. The sudden approximation for the nuclear β^- decay in atoms has been proposed by Migdal [6]. In fact, the sudden approximation can be also used to describe many other atomic processes stimulated by the nuclear β^- decays in atoms and molecules. The present work has the following structure. In the next section we discuss the sudden approximation which plays an important role for the consideration of the nuclear β^- decays in atoms. In the third section the sudden approximation is applied to compute the final-state probabilities in one-electron atoms/ions, i.e., for the hydrogenlike systems. In this case the closed analytic expressions for the final state probabilities can be produced. In this section we also discuss the approximate calculations of the ionization probabilities for the K electrons and final state probabilities for transitions which include the outermost electrons in the negatively charged ions and many-electron atoms. The fourth section contains analogous results for some many-electron atoms. The concluding remarks can be found in the last section.

II. SUDDEN APPROXIMATION

As mentioned above the momentum of the emitted β^- electron is significantly larger than the maximal momentum of atomic electrons. In other words, the velocity of this fast electron is much larger than the usual velocities of atomic electrons. Therefore the emitted β^- electron leaves the external shells of an atom for time which is approximately $\tau_\beta \approx a_0/c = \alpha\tau_a$, where α is the fine structure constant, a_0 is the Bohr radius, c is the speed of light in vacuum, and $\tau_a = \hbar/(e^4 m_e) \approx 2.418884 \times 10^{-17}$ s is the atomic time (or one atomic second, for short). For internal atomic shells the corresponding passing time decreases with the radii of these shells. This means that the sudden approximation [6] can be applied to determine the final-state probabilities in various atoms. The goal of this section is to discuss the sudden approximation and its applications to the β^- decay in various atomic systems.

The principal question for the β^- decaying atoms/ions is to explain the mechanism of excitation and ionization of atomic electrons. In general, the atomic electrons can be excited (and even ionized) due to the two following processes: (1) excitation due to the direct electron-electron interaction, and (2) excitation (ionization) related to the sudden change of nuclear charge Q. Following [7] and [8] we shall show below that only the second mechanism plays a noticeable role in actual applications. In other words, the electron excitation (ionization) due to the direct electron-electron interaction is very small and even negligible in most cases. We consider the single-particle excitation from a state ψ_m to a state ψ_n with the beta electron in a plane wave state $\mathbf{k}$ scattering to an outgoing state $\mathbf{k}'$. The probability for this process in first order perturbation theory is

$$W = \left| \frac{V_{nm}(\mathbf{k}-\mathbf{k}')}{E_n - E_m} \right|^2, \tag{2}$$

where

$$\begin{aligned} V_{nm}(\mathbf{k}-\mathbf{k}') &= \frac{1}{(2\pi)^3} \int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} |\mathbf{r}-\mathbf{r}'|^{-1} \psi_n(\mathbf{r}')^* \psi_m(\mathbf{r}') d\mathbf{r} d\mathbf{r}' \\ &= \frac{2}{\pi} \frac{1}{|\mathbf{k}-\mathbf{k}'|^2} \int e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} \psi_n(\mathbf{r})^* \psi_m(\mathbf{r}) d\mathbf{r}. \end{aligned} \tag{3}$$

The excitation probability is obtained by integrating over the direction of $\mathbf{k}'$. We note that

$$\begin{aligned} \int f(|\mathbf{k}-\mathbf{k}'|) d\Omega_{\mathbf{k}'} &= 2\pi \int_{-1}^{1} f([k^2 - 2kk't + k'^2]^{1/2}) dt \\ &= \frac{2\pi}{kk'} \int_{|k-k'|}^{k+k'} f(s) s\, ds \\ &= \frac{2\pi}{kk'} [F(|k-k'|) - F(k+k')] \\ &\sim \frac{2\pi}{kk'} F(|k-k'|) \end{aligned} \tag{4}$$

where $F'(s) = sf(s)$.

Since kk' is proportional to the beta decay energy, it can be concluded that the probability of excitation by this mechanism is negligible. However, this assertion must be qualified since the outgoing electron is relativistic, and the wave number should be replaced by c. The probability is therefore proportional to e^4/c^2, i.e., to α^2.

Consider now the excitation of atomic electrons due to the sudden change of nuclear charge Q. As follows from the general theory of sudden processes, in this case, the probability amplitude equals the overlap integral between the initial and final wave functions, i.e., $A_{i,f} = \langle \Psi_i(Q;\mathbf{r}) | \Psi_f(Q+1;\mathbf{r}) \rangle$. Indeed, the time for which the emitted β^- electron leaves the atom is significantly shorter than the corresponding atomic time. This means the electron density distribution cannot change during the process and the initial atom suddenly finds itself in the different nuclear field. For many-electron atoms/ion this means that the initial wave function is projected to the system of final stationary states (or final wave functions) which correspond to an atom/ion with the different nuclear charge $Q+1$. Note that the corresponding partial probabilities $P_{i,f} = |A_{i,f}|^2$ do not contain any small parameter. In general, such probabilities are the factors close to unity (more accurately: each of them varies between zero and unity).

Below, our main goal is to determine such probabilities numerically for a number of many-electron atomic systems. In the next section, however, we consider some simplified models in which the final state probabilities can be determined analytically, or numerically with the use of very simple formulas.

III. RESULTS FOR SINGLE-ELECTRON SYSTEMS

In some cases the final-state probabilities during the nuclear β^- decay in atoms/ions can be evaluated numerically and even analytically. Such cases include the one-electron atomic systems, e.g., the tritium atom and various one-electron ions. In other cases the electron density distribution of the internal electrons (e.g., the K and L electrons) can be approximated by some simple analytic functions (e.g., hydrogenic functions) and this simplifies significantly all calculations at the following steps.

First, consider the one-electron, hydrogenlike atom/ion. For one-electron systems the corresponding results can be obtained in closed analytic form. First, it can be shown that the angular momentum ℓ and its z projection m are conserved during the nuclear β^- decay in the one-electron atom/ion [7]. This follows from the central symmetry of electric field (apart from a negligible possible change in a hyperfine interaction) which is created by the atomic nucleus. Moreover, the final-state probabilities $P^{\ell}_{n_1 n_2}$ for one-electron systems can be obtained analytically. In these cases, the initial and final systems are the one-electron or H-like ions/atoms. The corresponding analytic formulas for $P^{\ell}_{in,fi}$ can be easily obtained by applying the radial parts of the reduced hydrogenlike wave functions in the form [8]:

$$\begin{aligned} \Psi_{n,\ell}(Q,r) &= \frac{1}{n} \left[\frac{Q(n-l-1)!}{(n+l)!} \right]^{1/2} \left[\frac{2Qr}{n} \right]^{l+1} \\ &\quad \times \sum_{k=0}^{n-l-1} \frac{(-1)^k}{k!} \binom{n+l}{2l+k+1} \left[\frac{2Qr}{n} \right]^k \exp\left(-\frac{Qr}{n} \right), \end{aligned} \tag{5}$$

where Q is the nuclear (i.e., positive) charge. By applying this formula one finds an analytic finite sum expression for the transition probability $P^{\ell}_{n_1,n_2}$ which corresponds to the $\beta^{\pm}$ decay from the initial (Q_1, n_1, l) to the final $(Q_2 = Q_1 \pm 1, n_2, l)$ states. The final analytic formula for $A^{\ell}_{n_1,n_2}$ takes the form

$$A^{\ell}_{n_1,n_2} = \frac{1}{n_1 n_2}\left[\frac{Q_1 Q_2 (n_1-l-1)\,!\,(n_2-l-1)!}{(n_1+l)\,!\,(n_2+l)!}\right]^{1/2} \times \left[\frac{4Q_1Q_2}{n_1n_2}\right]^{l+1} \sum_{k_1=0}^{n_1-l-1}\sum_{k_2=0}^{n_2-l-1} \frac{(-1)^{k_1+k_2}}{k_1\,!\,k_2!}\binom{n_1+l}{2l+k_1+1} \times \binom{n_2+l}{2l+k_2+1}(2l+k_1+k_2+2)\,! \times \left[\frac{n_1n_2}{Q_1n_2+Q_2n_1}\right]^{2l+k_1+k_2+3}. \qquad (6)$$

The numerical calculation of different transition probabilities with the use of Eq. (6) is straightforward and obtained results [9,10] agree very well with the known probabilities for the β^{-} decay in the tritium atom. The overall probability to detect the resulting He^{+} ion in one of its bound states is more than 98%.

The second example is related to the ionization of internal K shell in the β^{-}-decaying many-electron atom. The ionization of an electron from the K shell produces a vacancy which is later filled by an electron from outer shells with the emission of soft X rays. This explains why the ionization of the electron from the K shell is so important. From a theoretical point of view the involvement of K electron in the ionization process allows one to obtain the closed analytic formulas for its probability. Indeed, the wave function of a K electron can be approximated to a good accuracy by the wave function of the ground state of a hydrogen atom with the nuclear charge Q, i.e., $R_i^Q(r)=Q\sqrt{Q/\pi}\exp(-Qr)$ in atomic units ($\hbar=1, e=1, m_e=1$). The final wave function of the continuous spectrum is [8]

$$R_f^{Q+1}(r) = \sqrt{\frac{8\pi}{\nu[1-\exp(-2\pi\nu)]}}\exp(-ipr)\ {}_1F_1(i\nu + 1,2;2ipr), \qquad (7)$$

where $\nu=(Q+1)/p$, p is the momentum of outgoing electron and ${}_1F_1(a,b;x)$ is the confluent hypergeometric function. The overlap integral between $R_i^Q(r)$ and $R_f^{Q+1}(r)$, i.e., the probability amplitude $A_{i;f}$, can be computed analytically

$$\frac{2Q\sqrt{2Q}}{\sqrt{\nu[1-\exp(-2\pi\nu)]}}\int_0^{\infty} R_i^Q(r)R_f^{Q+1}(r)r^2dr = \frac{4Q\sqrt{2Q}}{\sqrt{\nu[1-\exp(-2\pi\nu)]}}\frac{\exp(-2\nu\cot^{-1}\nu)}{(Q^2+p^2)(Q+ip)}. \qquad (8)$$

The expression for the probability takes the form

$$P_{i;f} = |A_{i;f}|^2 = \frac{32Q^3p}{(Q+1)[1-\exp(-2\pi\nu)]}\frac{\exp(-4\nu\cot^{-1}\nu)}{(Q^2+p^2)^3}. \qquad (9)$$

The analytic methods can be also successfully applied to determine the final state probabilities for the transitions which include the outermost electron(s). In many cases in β^{-}-decaying many-electron atoms/ions the overall changes of internal electron density are relatively small. In contrast with this, the distribution of the outermost electron(s) changes drastically, if the nuclear charge Q increases by +1. In particular, this is the case for the negatively charged ions, e.g., for the T^{-} and O^{-} ions, and also for the neutral atoms of alkali metals, e.g., for the Na and K atoms. As is well-known (see, e.g., [11]) the radial wave function of an arbitrary atomic system at large r has the following asymptotic form:

$$R_i^Q(r) \sim r^{b_1-1}\exp(-t_1r) = r^{Q^*/t_1-1}\exp(-t_1r), \qquad (10)$$

where $t_1=\sqrt{2I_1}$, $b_1=Q^*/t_1$, and $Q^*=Q-N+1$. An analogous expression can be written for the wave function (at large r) of the final atomic system

$$R_f^Q(r) \sim r^{b_2-1}\exp(-t_2r) = r^{Q^*/t_2-1}\exp(-t_2r), \qquad (11)$$

where $t_2=\sqrt{2I_2}$, $b_2=Q^*/t_2$, and $Q^*=Q-N+2$.

Note that the last two equations represent the asymptotic forms of the actual wave functions at large r. To compute the final-state probabilities for the β^{-}-decaying atom/ion one needs to know the wave function in the whole space, rather than its long-range asymptotic. Nevertheless, to perform approximate evaluations we can assume that Eqs. (10) and (11) represent the actual wave functions everywhere. The corresponding unit-norm wave functions are

$$R_{N;i}^Q(r) = \frac{(2t)^b\sqrt{2t}}{\sqrt{\Gamma(2b+1)}}r^{Q^*/t-1}\exp(-tr), \qquad (12)$$

where $b=b_1$, b_2 and $t=t_1$, t_2. By using these expressions one finds the following formula for the final ground-to-ground state probability:

$$P_{\rm gg} = \frac{(2t_1)^{2b_1+1}(2t_2)^{2b_2+1}}{\Gamma(2b_1+1)\Gamma(2b_2+1)}\frac{\Gamma^2(b_1+b_2+1)}{(t_1^2+t_2^2)^{b_1+b_2+1}}, \qquad (13)$$

where b_1 and t_1 are defined above, while $b_2=(Q-N+2)/t_2$ and $t_2=\sqrt{2I_2}$, where I_2 is the corresponding ionization potential of the secondary atomic system, i.e., the ion/atom which has been created during the β^{-} decay. This formula also has a good accuracy for the negatively charged ions and relatively good accuracy for the neutral atoms.

The formula, Eq. (10), can be used also to determine the ionization probability in the case of β^{-}-decaying negatively charged ions. In general, the nuclear β^{-} decay in the negatively charged ions is of interest in some astrophysical problems. The probability amplitude takes the form

$$\mathcal{C}\sqrt{\frac{2}{\pi k}}\int_0^{\infty}\exp(-t_1r)\sin(kr)dr = \mathcal{C}\sqrt{\frac{2}{\pi}}\frac{1}{(2I+2E)}, \qquad (14)$$

where $k=\sqrt{2E}$ and $\mathcal{C}$ is the normalization constant for the original wave function of the outermost electron. For the ionization probability, one finds

$$P_i = \frac{\mathcal{C}^2}{2\pi(I+E)^2}. \qquad (15)$$

IV. RESULTS FOR MANY-ELECTRON ATOMS

In general, the analytic methods described above can be applied to a very restricted number of actual problems. To determine the final-state probabilities in β^--decaying many-electron atoms/ions one needs to use the numerical methods based on the self-consistent field model.

As shown above the final-state probabilities for the β^--decaying atoms are expressed as the overlap integrals between the initial- and final-(bound) state wave functions. In this study, for many-electron atoms/ions we shall use the relativistic atomic structure program GRASP [12], although for the systems considered here, relativity cannot have a significant effect. Since the change in the nuclear potential is a monopole, only transitions conserving the total J and parity occur. The GRASP program has been modified by one of us [13] so that the relativistic orbitals are obtained from a local central potential constructed so that the energy, or a weighted average of energies, is stationary with respect to variations in the potential. This is a generalization to the relativistic case of the optimized potential model [14] and the multiconfiguration optimized potential model [15]. Although this effective potential approach is somewhat less general than the variation of all the orbitals independently, it eliminates the problem of maintaining orthogonality of the orbitals, and it appears to have more satisfactory convergence properties. Results in the single-configuration, nonrelativistic case are essentially the same as Hartree-Fock results [16].

The relativistic wave functions are, in the notation of [12],

$$|\Gamma PJM\rangle = \sum_r c_{r\Gamma}|rPJM\rangle, \tag{16}$$

where the $c_{r\Gamma}$ are the mixing coefficients obtained by diagonalizing the Hamiltonian. The $|rPJM\rangle$ are configuration state functions obtained from a particular configuration defined by a choice of N orbitals from the finite basis:

$$|rPJM\rangle = \sum_{m_1\cdots m_N} \langle rPJM|m_1\cdots m_N\rangle \times \sqrt{\frac{1}{N!}}\det[\psi_{n_1\kappa_1 m_1}(\mathbf{r}_1)\cdots\psi_{n_N\kappa_N m_N}(\mathbf{r}_N)]. \tag{17}$$

The index r labels the various PJ states obtained from a particular configuration. If the complete Hamiltonian is diagonalized, the results are independent of how the PJ projection is performed.

The matrix elements of the overlap are given by

$$\langle\Gamma' P'J'M'|\Gamma PJM\rangle = \sum_{rr'}\langle r'P'J'M'|rPJM\rangle = \sum_{rr'}\langle r'PJM|rPJM\rangle\delta_{PP'}\delta_{JJ'}\delta_{MM'}. \tag{18}$$

The configurations are defined by N pairs $(n_1\kappa_1)\cdots(n_N\kappa_N)$ and are orthogonal unless $(n_1\kappa_1)\cdots(n_N\kappa_N)$ is a permutation of $(n_1'\kappa_1')\cdots(n_N'\kappa_N')$. To simplify the calculation, the same configuration set is chosen for the parent and daughter atoms; the angular momentum projections are the same, and the overlaps depend only on the radial function overlaps. The overlaps are then zero unless the same N orbitals are occupied in configuration state functions r and r'. Then

$$\langle r'PJM|rPJM\rangle = \sum_{m_1\cdots m_N}\langle PJM|m_1\cdots m_N\rangle^2 \times \frac{1}{N!}\int \det[\psi'_{n_1'\kappa_1'm_1'}(\mathbf{r}_1)\cdots\psi'_{n_N'\kappa_N'm_N'}(\mathbf{r}_N)] \times \det[\psi_{n_1\kappa_1 m_1}(\mathbf{r}_1)\cdots\psi_{n_N\kappa_N m_N}(\mathbf{r}_N)] \times d\mathbf{r}_1\cdots d\mathbf{r}_N = \det[\langle\psi'_{n'\kappa'}|\psi_{n\kappa}\rangle]. \tag{19}$$

The overlaps of the initial- and final-state wave functions are therefore given by

$$\langle\Gamma' PJM|\Gamma PJM\rangle = \sum_{rr'} c_{r'\Gamma'}c_{r\Gamma}\langle r'PJM|rPJM\rangle. \tag{20}$$

TABLE I. Transition amplitudes and probabilities for the ground-state to ground-state transition for β^- decay, together with their ionization potentials (I.P.) of the initial state.

Atom	Ion	Amplitude	Probability	I.P.(eV)
He	Li$^+$	0.855	0.731	24.59
Li	Be$^+$	0.736	0.541	5.39
Be	B$^+$	0.762	0.581	9.32
B	C$^+$	0.794	0.630	8.30
C	N$^+$	0.820	0.673	11.26
N	O$^+$	0.842	0.709	14.53
O	F$^+$	0.858	0.736	13.62
F	Ne$^+$	0.871	0.759	17.42
Ne	Na$^+$	0.873	0.762	21.56
Na	Mg$^+$	0.791	0.625	5.14
Mg	Al$^+$	0.797	0.635	7.65
Al	Si$^+$	0.802	0.643	5.99
Si	P$^+$	0.814	0.662	8.15
P	S$^+$	0.828	0.685	10.49
S	Cl$^+$	0.839	0.704	10.36
Cl	Ar$^+$	0.849	0.722	12.97
Ar	K$^+$	0.845	0.715	15.76

Wave functions for the final state ion are obtained by minimizing the average energy, as weighted by the statistical weights $2J+1$, over the configurations of interest. For example, in the case of O, the configurations included are $2s^22p^4$, $2s^2p^33p$, and $2s2p^43s$. Thus there may be more than one final state from a given configuration state. The initial state is, as noted above, obtained using the same configuration set. However, the states are computed in the potential obtained from the single configuration ground state calculation in order to obtain the optimum ground state description.

The results shown are for transitions from the ground state of the initial atom to the ground state and lowest excited states of the positive ion. The former are shown in Table I. Transition amplitudes for the first row elements are given in Table II, and for the second row elements in Table III. It appears that, in general, the amplitudes for excitation to low-lying excited states are very small. The ionization potentials,

TABLE II. Transition amplitudes for the lowest single excitations following β^- decay.

Atom	Ion	Excitation	Amplitude	Atom	Ion	Excitation	Amplitude
He	Li^+	$1s \rightarrow 2s$	0.074	N	O^+	$2p \rightarrow 3p$	0.024
He	Li^+	$1s \rightarrow 2s$	0.042	N	O^+	$2s \rightarrow 3s$	−0.013
Li	Be^+	$2s \rightarrow 3s$	−0.014	O	F^+	$2p \rightarrow 3p$	0.014
Li	Be^+	$2s \rightarrow 4s$	−0.006	O	F^+	$2p \rightarrow 3p'$	0.030
Be	B^+	$2s \rightarrow 3s$	0.060	O	F^+	$2s \rightarrow 3s$	0.020
Be	B^+	$2s \rightarrow 4s$	0.031	F	Ne^+	$2p \rightarrow 3p$	−0.026
B	C^+	$2p \rightarrow 3p$	0.025	F	Ne^+	$2p \rightarrow 3p'$	−0.043
B	C^+	$2s \rightarrow 3s$	−0.033	F	Ne^+	$2p \rightarrow 3p''$	0.015
B	C^+	$2s \rightarrow 3s'$	−0.011	F	Ne^+	$2s \rightarrow 3s'$	−0.007
C	N^+	$2p \rightarrow 3p$	0.027	F	Ne^+	$2s \rightarrow 3s'$	−0.007
C	N^+	$2s \rightarrow 3s$	0.023	Ne	Na^+	$2p \rightarrow 3p$	0.121
C	N^+	$2s \rightarrow 3s'$	0.009	Ne	Na^+	$2s \rightarrow 3s$	0.035

which are also an indication of the atomic stability, are also shown in Table I.

V. DISCUSSION AND CONCLUSION

The general theory of atomic excitations during the nuclear $\beta^{\pm}$ decay is considered. The final-state probabilities are determined numerically for a number of many-electron atoms. In fact, our present results include the ground-ground transition probabilities P_{gg} for a number of light atoms He-Ar. All these probabilities are determined with the use of sudden approximation [6]. The sudden approximation provides a sufficient accuracy to describe atomic excitations in β^--decaying many-electron atoms. Note that the sudden approximation can be also used to study the β^- decay in molecules [17].

As follows from our present results for the He, Li, …, Ar atoms the ground-ground transition probabilities depend upon the nuclear charge Q and total number of electrons N_e in the considered system. In general, the ground-ground transition probabilities are maximal for the inert gases He, Ne, Ar ($P_{gg} \approx 0.715 - 0.762$) and halogens F, Cl ($P_{gg} \approx 0.722 - 0.759$). On the other hand such probabilities are minimal for alkali metals Li, Na, K$\approx 0.541 - 0.625$. In general, the maximal difference in the P_{gg} probabilities decreases when the nuclear charge increases, e.g., for the He and Li atoms $\Delta \approx 0.19$, while for the Ne and Na $\Delta \approx 0.137$.

The probabilities for transitions to low-lying excited states are relatively small. It appears that most of the strength not contained in the ground state transitions is to ionization states, probably from the inner shells. This question will be studied further in the future.

In conclusion, we want to mention that this work opens an avenue in theoretical atomic spectroscopy—atomic/optical excitations during the nuclear reactions. In the case of nuclear $\beta^{\pm}$ decays such excitations are produced by the sud-

TABLE III. Transition amplitudes for the lowest single excitations following β^- decay for the second row elements.

Atom	Ion	Excitation	Amplitude	Atom	Ion	Excitation	Amplitude
Na	Mg^+	$3s \rightarrow 4s$	0.086	P	S^+	$3s \rightarrow 4s$	0.020
Na	Mg^+	$2p \rightarrow 3p$	−0.032	P	S^+	$3s \rightarrow 4s'$	0.013
Na	Mg^+	$2p \rightarrow 3p'$	−0.044	S	Cl^+	$3p \rightarrow 4p$	−0.013
Mg	Al^+	$3s \rightarrow 4s$	0.104	S	Cl^+	$3p \rightarrow 4p'$	0.004
Mg	Al^+	$2p \rightarrow 3p$	0.048	S	Cl^+	$3s \rightarrow 4s$	−0.012
Al	Si^+	$3p \rightarrow 4p$	0.016	S	Cl^+	$3s \rightarrow 4s'$	−0.009
Al	Si^+	$3s \rightarrow 4s$	−0.007	Cl	Ar^+	$3p \rightarrow 4p$	−0.002
Al	Si^+	$3s \rightarrow 4s'$	0.007	Cl	Ar^+	$3p \rightarrow 4p'$	0.004
Si	P^+	$3s \rightarrow 4s$	−0.010	Cl	Ar^+	$3s \rightarrow 4p''$	−0.003
Si	P^+	$3s \rightarrow 4s'$	−0.011	Cl	Ar^+	$3s \rightarrow 4s$	0.010
Si	P^+	$3p \rightarrow 4p$	0.002	Cl	Ar^+	$3s \rightarrow 4s'$	0.007
P	S^+	$3p \rightarrow 4p$	−0.004	Ar	K^+	$3p \rightarrow 4p$	−0.016
P	S^+	$3p \rightarrow 4p'$	−0.004	Ar	K^+	$3p \rightarrow 4p'$	0.105
P	S^+	$3p \rightarrow 4p''$	0.009	Ar	K^+	$3s \rightarrow 4s$	0.021

den nuclear processes/reactions. The observed optical spectra of the secondary atoms (or daughter atoms) are unique and can be used to identify the $\beta^{\pm}$ decaying isotopes from large distances. In this study we have restricted ourselves to the consideration only of nuclear β^{-} decays in atomic systems. The analysis of analogous β^{+} decays and sudden neutron shake can also be performed with the use of the sudden approximation described in this study. However, the analysis of nuclear recoil in reactions with neutrons requires the consideration of atomic states with higher values of angular momenta. The analysis of atomic excitations during α-decay, nuclear fission, $(n;t)$-, $(n;\alpha)$- and $(n;p)$-reactions is significantly more complicated and requires application of the new methods. The theory of atomic excitations during such processes will be developed in our future studies.

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