Radiative $np \to 1s+\gamma$ transitions induced by strong low-energy interactions in kaonic atoms

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We calculate the radiative transition rates $np \to 1s+\gamma$ in kaonic hydrogen and kaonic deuterium, induced by strong low-energy interactions and enhanced by Coulomb interactions. The obtained results should be taken into account for the theoretical analysis of the experimental data on the x-ray spectra and yields in kaonic atoms.

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I. KAONIC HYDROGEN. CONTEMPORARY EXPERIMENTAL AND THEORETICAL STATUS

The contemporary experimental and theoretical status of kaonic atoms has been recently outlined by Gasser [1]. The most recent experimental value on the energy level displacement of the ground state of kaonic hydrogen

$$-\epsilon_{1s}^{(\mathrm{expt})} + i\frac{\Gamma_{1s}^{(\mathrm{expt})}}{2} = (-194 \pm 41) + i(125 \pm 59)\ \mathrm{eV}, \quad (1)$$

obtained by the DEAR Collaboration [2], by a factor 2 smaller than the experimental value measured by the KEK Collaboration [3]

$$-\epsilon_{1s}^{(\mathrm{expt})} + i\frac{\Gamma_{1s}^{(\mathrm{expt})}}{2} = (-323 \pm 64) + i(204 \pm 115)\ \mathrm{eV}. \quad (2)$$

The theoretical analysis of the experimental data on the energy level displacement of the ground state of kaonic hydrogen, obtained by the DEAR Collaboration, Eq. (1), has been carried out in Refs. [4–6].

In Ref. [4] (see also Refs. [7,8]) we have proposed a quantum field theoretic model for strong low-energy $\bar{K}N$ interactions at threshold and computed the complex S-wave scattering lengths a_0^0 and a_0^1 of $\bar{K}N$ scattering with isospin $I=0$ and $I=1$:

$$a_0^0 = (-1.221 \pm 0.072) + i(0.537 \pm 0.064)\ \mathrm{fm}, \quad (3)$$

$$a_0^1 = (+0.258 \pm 0.024) + i(0.001 \pm 0.000)\ \mathrm{fm}. \quad (3)$$

The theoretical value for the energy level displacement of the ground state of kaonic hydrogen, calculated for the S-wave scattering lengths Eq. (3), is

$$\begin{aligned} -\epsilon_{1s}^{(0)} + i\frac{\Gamma_{1s}^{(0)}}{2} &= 2\alpha^3\mu^2 f_0^{K^-p}(0) \\ &= 412.13[\mathrm{Re}\, f_0^{K^-p}(0) + i\,\mathrm{Im}\, f_0^{K^-p}(0)] \\ &= (-203 \pm 15) + i(113 \pm 14)\ \mathrm{eV}, \end{aligned} \quad (4)$$

where $2\alpha^3\mu^2 = 412.13\ \mathrm{eV/fm}$ with $\mu = m_K m_p/(m_K + m_p) = 323.48$ MeV is the reduced mass of the K^-p pair, computed for $m_K = 493.68$ MeV and $m_p = 938.27$ MeV, and $\alpha = 1/137.036$ is the fine-structure constant [9], then $f_0^{K^-p}(0)$ equal

$$\begin{aligned} f_0^{K^-p}(0) &= \frac{a_0^0 + a_0^1}{2} \\ &= \mathrm{Re}\, f_0^{K^-p}(0) + i\,\mathrm{Im}\, f_0^{K^-p}(0) \\ &= (-0.482 \pm 0.034) + i(0.269 \pm 0.032)\ \mathrm{fm} \end{aligned} \quad (5)$$

is the S-wave amplitude of K^-p scattering at threshold [4].

The theoretical values of the energy level shift and width Eq. (4) agree well with the experimental data by the DEAR Collaboration [2] Eq. (1) and only qualitatively with the experimental data by the KEK Collaboration [3].

The quantum field theoretic model of low-energy $\bar{K}N$ interactions is formulated within effective field theory approach based on effective chiral Lagrangians and chiral perturbation theory by Gasser and Leutwyler [10–13].

In the model of strong low-energy $\bar{K}N$ interactions near threshold proposed in [4,7,8] the imaginary part of the S-wave amplitude of K^-p scattering is defined by the contributions of strange baryon resonances $\Lambda(1405)$, $\Lambda(1800)$, and $\Sigma(1750)$. According to Gell-Mann's $SU(3)$ classification of hadrons, the $\Lambda(1405)$ resonance is an $SU(3)$ singlet, whereas the resonances $\Lambda(1800)$ and $\Sigma(1750)$ are components of an $SU(3)$ octet [9]. This allows to describe the experimental data on the cross sections for inelastic reactions $K^-p \to Y\pi$, where $Y\pi = \Sigma^-\pi^+$, $\Sigma^+\pi^-$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$, at threshold of K^-p scattering [14,15]

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$$\gamma = \frac{\sigma(K^- p \to \Sigma^- \pi^+)}{\sigma(K^- p \to \Sigma^+ \pi^-)} = 2.360 \pm 0.040,$$

$$R_c = \frac{\sigma(K^- p \to \Sigma^- \pi^+) + \sigma(K^- p \to \Sigma^+ \pi^-)}{\sigma(K^- p \to \Sigma^- \pi^+) + \sigma(K^- p \to \Sigma^+ \pi^-) + \sigma(K^- p \to \Sigma^0 \pi^0) + \sigma(K^- p \to \Lambda^0 \pi^0)} = 0.664 \pm 0.011,$$

$$R_n = \frac{\sigma(K^- p \to \Lambda^0 \pi^0)}{\sigma(K^- p \to \Sigma^0 \pi^0) + \sigma(K^- p \to \Lambda^0 \pi^0)} = 0.189 \pm 0.015 \qquad (6)$$

with an accuracy of about 6% [4]. From the experimental data Eq. (6) and due to $SU(3)$ symmetry of low-energy interactions of baryon resonances with octets of baryons and pseudoscalar mesons, leading to certain relations between S-wave amplitudes of inelastic channels, we have obtained that (i) the $\Lambda(1800)$ resonance decouples from the K^-p system at low energies, (ii) one of the parameters (γ, R_c, R_n) is a function of two others and (iii) using the optical theorem the imaginary part $\mathrm{Im}\, f_0^{K^-p}(0)$ of the S-wave amplitude of K^-p scattering can be expressed in terms of the parameters γ and R_c and the S-wave amplitude $f(K^-p \to \Sigma^-\pi^+)$ of the inelastic channel $K^-p \to \Sigma^-\pi^+$ [4]:

$$\mathrm{Im}\, f_0^{K^-p}(0) = \frac{1}{R_c}\left(1 + \frac{1}{\gamma}\right)|f(K^-p \to \Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+} = (0.269 \pm 0.032)\ \mathrm{fm}, \qquad (7)$$

where $k_{\Sigma^-\pi^+} = 173.85$ MeV is a relative momentum of the $\Sigma^-\pi^+$ system at threshold of K^-p scattering. The S-wave amplitude $f(K^-p \to \Sigma^-\pi^+)$ is computed with an accuracy of about 6%. The main contribution comes from the $\Lambda(1405)$ resonance, whereas the contribution of the $\Sigma(1750)$ resonance makes up about 2.7% [4]. This agrees well with an assertion that the $\Lambda(1405)$ resonance dominates in low-energy interactions of the K^-p pair in the S-wave state [16]. For the calculation of the amplitude $f(K^-p \to \Sigma^-\pi^+)$ we need two input parameters g_1 and g_2, the coupling constants of the interactions $\Lambda(1405)BP$ and $\Sigma(1750)BP$, where $B = (N, \Sigma, \Lambda^0, \Xi)$ and $P = [\pi, K, \bar{K}, \eta(550)]$ are octets of low-lying baryons and pseudoscalar mesons. The coupling constants $g_1 = 0.907$ and $g_2 = 1.123$ have been computed in Ref. [4], using the recommended values for the masses and widths of the resonances $\Lambda(1405)$ and $\Sigma(1750)$ [9]. We would like to remind that the resonances $\Lambda(1405)$ and $\Sigma(1750)$ have status **** and ***, respectively [9].

The real part $\mathrm{Re}\, f_0^{K^-p}(0)$ of the S-wave amplitude of K^-p scattering at threshold, i.e., the S-wave scattering length $\mathrm{Re}\, f_0^{K^-p}(0) = a_0^{K^-p}$ of K^-p scattering,

$$\mathrm{Re}\, f_0^{K^-p}(0) = \mathrm{Re}\, f_0^{K^-p}(0)_R + \mathrm{Re}\, \tilde{f}_0^{K^-p}(0), \qquad (8)$$

is defined by the contribution (i) of the strange baryon resonances $\Lambda(1405)$ and $\Sigma(1750)$ in the s channel of low-energy elastic K^-p scattering, (ii) of the exotic four quark (or $K\bar{K}$ molecules) scalar states $a_0(980)$ and $f_0(980)$ in the t channel of low-energy elastic K^-p scattering, and (iii) of hadrons with nonexotic quark structures, i.e., $q\bar{q}$ for mesons and qqq for baryons [9], where $q = u$, d, or s quarks.

The contribution of the strange resonances $\Lambda(1405)$ and $\Sigma(1750)$, which we denote as $\mathrm{Re}\, f_0^{K^-p}(0)_R = (-0.154 \pm 0.009)$ fm, explains 32% of the mean value of the S-wave scattering length of K^-p scattering [4,8]: $\mathrm{Re}\, f_0^{K^-p}(0) = (-0.482 \pm 0.034)$ fm. The contribution of the $\Sigma(1750)$ resonance to $\mathrm{Re}\, f_0^{K^-p}(0)_R$ makes up only 5.5%. This corroborates again the dominance of the $\Lambda(1405)$ resonance in low-energy interactions of the K^-p pair in the S-wave state [16].

The contribution of the exotic scalar mesons $a_0(980)$ and $f_0(980)$ and nonexotic hadrons, calculated to leading order in chiral expansion, is equal to [4]

$$\mathrm{Re}\, \tilde{f}_0^{K^-p}(0) = 0.398 - 0.614\, \xi\ \mathrm{fm}, \qquad (9)$$

where the first term is defined by strong low-energy interactions of nonexotic hadrons, whereas the second one is caused by the exotic scalar mesons $a_0(980)$ and $f_0(980)$ coupled to the K^- meson and the proton. The parameter ξ is a low-energy constant (LEC), defining low-energy $a_0(980)NN$ and $f_0(980)NN$ interactions.

Using the hypothesis of the quark-hadron duality, formulated by Shifman, Veinshtein and Zakharov within nonperturbative QCD [17], and the effective quark model with chiral $U(3) \times U(3)$ symmetry [18], we have calculated the parameter ξ [4]: $\xi = 1.2 \pm 0.1$.

The validity of the application of the effective quark model with chiral $U(3) \times U(3)$ symmetry [18] to the quantitative analysis of strong low-energy $\bar{K}N$ interactions at threshold has been corroborated by example of the calculation of the S-wave scattering length of elastic K^-n scattering in connection with the calculation of the energy level displacement of the ground state of kaonic deuterium [7].

For the description of the contributions of the strange baryon resonances $\Lambda(1405)$, $\Lambda(1800)$, and $\Sigma(1750)$ and the exotic scalar mesons $a_0(980)$ and $f_0(980)$ we have used effective chiral Lagrangians with $\Lambda(1405)$, $\Lambda(1800)$, $\Sigma(1750)$, $a_0(980)$ and $f_0(980)$ treated as elementary particles, represented by local interpolating fields. Such a description of baryon resonances and exotic mesons does not contradict ChPT by Gasser and Leutwyler [10] and power counting

[11–13]. As has been pointed out by Brown and Rho with co-workers [11], the inclusion of the resonance $\Lambda(1405)$ as well as $\Lambda(1800)$ and $\Sigma(1750)$ as elementary particle fields allows to compute the amplitudes of $\bar{K}N$ scattering at threshold to leading order effective chiral Lagrangians. Such an assertion is fully confirmed in our model of strong low-energy $\bar{K}N$ interactions near threshold [4,7,8].

For the analysis of the energy level displacement of the ground state of kaonic hydrogen Meißner, Raha, and Rusetsky [5] have assumed a dominant role of the $\bar{K}^0n$ cusp, the intermediate $\bar{K}^0n$ state below threshold, for the S-wave amplitude of K^-p scattering. Such a hypothesis has a long history. For the first time it has been proposed by Dalitz and Tuan in 1960 [19] (see also Ref. [20]) within the K-matrix approach in the zero-range approximation. It has been also discussed by Deloff and Law [21]. The S-wave amplitude of K^-p scattering at threshold, obtained under the assumption of the $\bar{K}^0n$-cusp enhancement, is equal to [19–21]

$$\tilde{f}_0^{K^-p}(0) = \frac{\dfrac{a_0^0+a_0^1}{2} + q_0 a_0^0 a_0^1}{1 + \left(\dfrac{a_0^0+a_0^1}{2}\right) q_0}, \tag{10}$$

where $q_0 = \sqrt{2\mu(m_{\bar{K}^0} - m_{K^-} + m_n - m_p)} = 58.4\ \mathrm{MeV} = O(\sqrt{\alpha})$ is a relative momentum of the $\bar{K}^0n$ pair below threshold of K^-p scattering, calculated for $m_{\bar{K}^0} - m_{K^-} = 3.97$ MeV and $m_n - m_p = 1.29$ MeV [9], a_0^0 and a_0^1 are complex S-wave scattering lengths of $\bar{K}N$ scattering with isospin $I=0$ and $I=1$.

Meißner *et al.* have argued [22] that the S-wave amplitude Eq. (10) can be reproduced within a nonrelativistic effective Lagrangian approach within ChPT by Gasser and Leutwyler [10] by summing up an infinite set of bubble diagrams in Fig. 2 of Ref. [5] [see Eq. (17) of Ref. [5]]. In addition, they have given a systematic analysis of corrections to the S-wave amplitude of K^-p scattering at threshold, caused by the isospin-breaking interactions. The results are presented in terms of a_0^0 and a_0^1 [see Eq. (10)], the mass difference $(m_d - m_u)$ of current quark masses of the u and d quarks, a quantity of order $(m_d - m_u) \sim O(\alpha)$ [5], and the fine-structure constant α. As has been shown in Ref. [5] (see also Ref. [8]), these corrections are negligible in comparison with the main part of the S-wave amplitude of K^-p scattering at threshold.

For the analysis of the contribution of the $\bar{K}^0n$ cusp it is convenient to transcribe the S-wave amplitude Eq. (10) into the form

$$\tilde{f}_0^{K^-p}(0) = a_0^{K^-p} + \tilde{f}_0^{K^-p}(0)_{\mathrm{cusp}}, \tag{11}$$

where $a_0^{K^-p} = (a_0^0 + a_0^1)/2$ is the S-wave scattering length of K^-p scattering in the limit of isospin invariance, i.e., at $q_0=0$, and $\tilde{f}_0^{K^-p}(0)_{\mathrm{cusp}}$ is defined by the $\bar{K}^0n$ cusp for $q_0 \neq 0$:

$$\tilde{f}_0^{K^-p}(0)_{\mathrm{cusp}} = -\left(\frac{a_0^0 - a_0^1}{2}\right)^2 \frac{q_0}{1 + \left(\dfrac{a_0^0+a_0^1}{2}\right) q_0} = (-0.149 \pm 0.042) + i(0.151 \pm 0.037)\ \mathrm{fm}. \tag{12}$$

The S-wave amplitude $\tilde{f}_0^{K^-p}(0)_{\mathrm{cusp}}$ has a distinct form of the contribution of the inelastic channel $K^-p \to \bar{K}^0n$ with the final-state $\bar{K}^0n$ interaction, calculated in the zero-range approximation. The momentum q_0 in the numerator of Eq. (12) is caused by the phase volume of the $\bar{K}^0n$ pair in the S-wave state. The numerical value of $\tilde{f}_0^{K^-p}(0)_{\mathrm{cusp}}$ is calculated for the S-wave scattering lengths Eq. (3).

For $m_d = m_u$ and the S-wave scattering lengths Eq. (3) the energy level displacement of the ground state of kaonic hydrogen, computed for the S-wave amplitude of K^-p scattering at threshold Eq. (10) caused by the $\bar{K}^0n$-cusp enhancement, is equal to

$$-\epsilon_{1s}^{(\mathrm{th})} + i\frac{\Gamma_{1s}^{(\mathrm{th})}}{2} = (-266 \pm 23) + i(177 \pm 21)\ \mathrm{eV}. \tag{13}$$

This agrees with both our theoretical result (4) and the experimental data by the DEAR Collaboration (1) within 1.5 standard deviations for the shift and one standard deviation for the width. Within experimental error bars the numerical values for the energy level shift and width Eq. (13) do not contradict also the experimental data by the KEK Collaboration. The corrections, caused by the $\bar{K}^0n$ cusp, make up about 24% and 36% of the mean values of the energy level shift and width given by Eq. (13), respectively.

The solution of the inverse problem, i.e., using the experimental data on the energy level displacement of the ground state of kaonic hydrogen and the theoretical formula Eq. (10) to determine the complex S-wave scattering lengths a_0^0 and a_0^1, is rather ambiguous, since one needs to define four parameters ($\mathrm{Re}\,a_0^0$, $\mathrm{Im}\,a_0^0$, $\mathrm{Re}\,a_0^1$, $\mathrm{Im}\,a_0^1$) in terms of only two experimental values ($\epsilon_{1s}^{(\mathrm{expt})}$, $\Gamma_{1s}^{(\mathrm{expt})}$).

Another approach to the analysis of the S-wave amplitude of K^-p scattering near threshold and the energy level displacement of the ground state of kaonic hydrogen is related to the chiral $SU(3)$ effective field theory and relativistic coupled channels technique [6]. For the first time, the chiral $SU(3)$ effective field theory with coupled channels has been applied to the analysis of strong low-energy $\bar{K}N$ interactions near threshold by Kaiser, Siegel, and Weise in Ref. [16]. The starting point of this coupled channels approach is the effective chiral Lagrangian, which incorporates the same symmetries and symmetry breaking patterns as QCD and describes the interactions of octets of baryons $(N, \Lambda^0, \Sigma, \Xi)$ and pseudoscalar mesons $[\pi, K, \eta(550)]$ [6,16]. The pure mesonic part is defined up to second order in ChPT by Gasser and Leutwyler [10], whereas the meson-baryon interactions are taken at lowest order [6]. This means that in chiral $SU(3)$ effective field theory with coupled channels the strange baryon resonance $\Lambda(1405)$ is not included as an elementary particle field. The S-wave amplitudes of elastic and inelastic channels of K^-p scattering near threshold are defined in terms of the matrix elements of the $\mathbb{T}$ matrix, which are the solutions of the Lippmann-Schwinger equation in the momentum representation with a separable potential [16]. As has been shown in Ref. [16], such a technique allows to reproduce the dominant contribution of the $\Lambda(1405)$ resonance in reactions of low-energy K^-p interactions near threshold. The S-wave amplitude of K^-p scattering at threshold, calculated in Ref. [16], is equal to $f_0^{K^-p}(0)=-0.97+i1.10$ fm. This disagrees with the experimental data by the DEAR and the KEK Collaborations.

Recently, Borasoy, Nißler, and Weise [6] have applied the chiral $SU(3)$ effective field theory and relativistic coupled channels technique to the theoretical analysis of experimental data by the DEAR Collaboration Eq. (1). For the details of the calculations we relegate readers to Ref. [6] (see also Ref. [23,24]).

The theoretical prediction for the S-wave amplitude of K^-p scattering at threshold $f_0^{K^-p}(0)=(-0.57+i0.46)$ fm has been obtained as *a result of an "optimal" compromise between the various existing data sets* [6]. This means that input parameters of the approach have been defined from the fit of the experimental data on the cross sections $\sigma(K^-p\to K^-p)$, $\sigma(K^-p\to \bar{K}^0 n)$, and $\sigma(K^-p\to\Sigma^{\pm}\pi^{\mp})$ for the laboratory momenta of the incident K^- meson from domain $50\ \mathrm{MeV}/c<p_{\mathrm{lab}}\leqslant 250\ \mathrm{MeV}/c$ (or relative momenta of the K^-p pair from domain $40\ \mathrm{MeV}/c<p_{cm}\leqslant 200\ \mathrm{MeV}/c$) and the $\pi\Sigma$ mass spectrum in the isospin $I=0$ channel [6]. The theoretical analysis of the experimental data on the ratios of the inelastic cross sections at threshold Eq. (6), carried out in Ref. [6], has led also to the results: $\gamma=2.380$, $R_c=0.631$, and $R_n=0.176$, which agree well with the experimental data Eq. (6).

For the S-wave amplitude of K^-p scattering at threshold $f_0^{K^-p}=(-0.57+i0.46)$ fm the energy level displacement of the ground state of kaonic hydrogen is equal to

$$\begin{aligned} -\epsilon_{1s}+i\frac{\Gamma_{1s}}{2} &= 412.13\, f_0^{K^-p}(0) \\ &= 412.13(-0.56+i0.46) \\ &= -235+i195\ \mathrm{eV}. \end{aligned} \tag{14}$$

This does not contradict the experimental data by both the DEAR Collaboration within two standard deviations and the KEK Collaboration within the experimental error bars.

Recently [8] we have shown that the hypothesis on the dominant role of the $\bar{K}^0 n$ cusp can be realized in our approach in complete agreement with Meißner *et al.* [5].

A preliminary analysis of the cross sections for the reactions $K^-p\to K^-p$, $K^-p\to\bar{K}^0 n$, and $K^-p\to\Sigma^{\mp}\pi^{\pm}$, carried out in our model of low-energy $\bar{K}N$ interactions for the laboratory momenta of the K^- meson from domain $70\ \mathrm{MeV}/c<p_{\mathrm{lab}}\leqslant 150\ \mathrm{MeV}/c$, shows an agreement with the results obtained by Borasoy *et al.* and the experimental data (practically, within two standard deviations) used in Ref. [6] for the fit of input parameters.

Thus, the theoretical analysis of the experimental data on the energy level displacement of the ground state of kaonic hydrogen confirms Gasser's conclusion that "*The theory of $\bar{K}p$ scattering leaves many questions open. More precise data will reveal whether present techniques are able to describe the complicated situation properly*." [1].

Hence, the improvement of the precision of the experimental data on the energy level shift and width of the ground state of kaonic atoms is a meaningful current problem in physics of kaonic atoms.

II. INTRODUCTION

The x-ray spectra and yields [25–31], produced by the atomic transitions $np\to 1s$ in kaonic hydrogen, where n is the *principal quantum number* of the energy levels, are the main experimental tool for the measurement of the energy level displacement of the ground state of kaonic hydrogen, caused by strong low-energy $\bar{K}N$ interactions [2]. It is known [28–31] that the x-ray yields related to the K_α, K_β, and K_γ lines of kaonic hydrogen are very sensitive to the value of Γ_{2p}, the rate of hadronic decays of kaonic hydrogen from the $2p$ state. Usually Γ_{2p} is used as an input parameter in the theories of the atomic cascades [28–31].

Recently [32] we have calculated the rate Γ_{np} of hadronic decays of kaonic hydrogen from the np state. For the $2p$ state we have obtained: $\Gamma_{2p}=2\ \mathrm{meV}=3.0\times 10^{12}\ \mathrm{sec}^{-1}$. This agrees well with the assumption by Koike, Harada, and Akaishi [28]. In order to reconcile the experimental data on the K_α, K_β, and K_γ lines with the theoretical analysis they assumed that $\Gamma_{2p}>1\ \mathrm{meV}=1.5\times 10^{12}\ \mathrm{sec}^{-1}$.

Recently Faifman and Men'shikov have presented the calculated yields for the K series of x rays for kaonic hydrogen in dependence of the hydrogen density [33]. They have shown that the use of the theoretical value $\Gamma_{2p}=2$ meV of the width of the $2p$ state of kaonic hydrogen, computed in our work, leads to good agreement with the experimental data, measured for the K_α line by the KEK Collaboration [3]. They have also shown that the results of cascade calculations with other values of the width of the $2p$ excited state of kaonic hydrogen, used as an input parameter, disagree with the available experimental data. The results obtained by Faifman and Men'shikov contradict those by Jensen and Markushin [31]. Therefore, as has been accentuated by Faifman and Men'shikov [33], the further analysis of the experimental data by the DEAR Collaboration should allow to perform a more detailed comparison of the theoretical value $\Gamma_{2p}=2$ meV with other phenomenological values of the width of the $2p$ state of kaonic hydrogen Γ_{2p}, used as input parameters.

In this paper we continue the analysis of the influence of strong low-energy interactions on the transitions from the np excited states in kaonic hydrogen, which we have started in Ref. [32]. We investigate the radiative transitions $np\to 1s+\gamma$, induced by strong low-energy interactions and enhanced by the Coulomb interaction of the K^-p pair, in kaonic hydrogen and apply the obtained results to kaonic deuterium.

The paper is organized as follows. In Sec. III we calculate the radiative transition rates $np\to 1s+\gamma$ in kaonic hydrogen, induced by strong low-energy interactions and enhanced by the Coulomb interaction. The Coulomb interaction is taken into account in the form of the explicit nonrelativistic Coulomb wave functions of the relative motion of the K^-p pairs and the explicit nonrelativistic Coulomb Green functions for the calculation of the amplitude of the kaon-proton *bremsstrahlung* $K^-+p\to K^-+p+\gamma$, defining the radiative atomic transition rate $(K^-p)_{np}\to(K^-p)_{1s}+\gamma$ in our approach. In Sec. IV we calculate the radiative transition rates $np\to 1s+\gamma$ in kaonic deuterium. In the Conclusion we discuss the obtained results.

III. $np\to 1s+\gamma$ TRANSITIONS IN KAONIC HYDROGEN

The radiative transition rate $(K^-p)_{np}\to(K^-p)_{1s}+\gamma$ can be defined by [4]

$$\Gamma((K^-p)_{np}\to(K^-p)_{1s}\gamma) = \frac{1}{8\pi}\frac{\omega}{(m_K+m_p)^2}\overline{|M((K^-p)_{np}\to(K^-p)_{1s}\gamma)|^2}, \quad (15)$$

where $\omega=E_{np}-E_{1s}=\alpha^2\mu(n^2-1)/2n^2$ is the photon energy, $\alpha=1/137.036$ is the fine-structure constant, $\mu=m_Km_p/(m_K+m_p)=323.48$ MeV is the reduced mass of the K^-p pair calculated for $m_K=493.68$ MeV and $m_p=938.27$ MeV. Then the notation $\overline{|M((K^-p)_{np}\to(K^-p)_{1s}\gamma)|^2}$ means

$$\overline{|M((K^-p)_{np}\to(K^-p)_{1s}\gamma)|^2} = \frac{1}{3}\sum_{m=0,\pm1}\frac{1}{2}\sum_{\sigma=\pm1/2}\sum_{\sigma'=\pm1/2}\sum_{\lambda=\pm1}|M((K^-p)_{np}\to(K^-p)_{1s}\gamma)|^2, \quad (16)$$

where $m=0,\pm1$ is a magnetic quantum number of the $(K^-p)_{np}$ pair in the bound np excited state of kaonic hydrogen, $\sigma=\pm1/2$ and $\sigma'=\pm1/2$ are polarizations of the initial and final protons, and $\lambda=\pm1$ are polarizations of the photon. We have averaged over the magnetic quantum number and the polarization of the initial proton and summed up over polarizations of the final proton and the photon.

The amplitude $M((K^-p)_{np}\to(K^-p)_{1s}\gamma)$ is given by [4]

$$M((K^-p)_{np}\to(K^-p)_{1s}\gamma) = \frac{1}{2\mu}\int\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\Phi^*_{100}(\vec{k})\Phi_{n1m}(\vec{q})\times M(K^-p\to K^-p\gamma), \quad (17)$$

where $\Phi_{100}(\vec{k})=\Phi_{1s}(k)$ and $\Phi_{n1m}(\vec{q})=-i\sqrt{4\pi}\Phi_{np}(q)Y_{1m}(\vartheta_{\vec{q}},\varphi_{\vec{q}})$ are the wave functions of kaonic hydrogen in the ground $1s$ and the np state in the momentum representation [34]; $Y_{1m}(\vartheta_{\vec{q}},\varphi_{\vec{q}})$ is a spherical harmonic and $\Phi_{1s}(k)$ and $\Phi_{np}(q)$ are radial wave functions of kaonic hydrogen in the momentum representation [34] (see also Refs. [32,35]). Then, $M(K^-p\to K^-p\gamma)$ is the amplitude of the kaon-proton bremsstrahlung $K^-+p\to K^-+p+\gamma$:

$$M(K^-p\to K^-p\gamma) = M\left(K^-\left(\vec{q}+\frac{m_K}{m_K+m_p}\vec{Q}\right)p\left(-\vec{q}+\frac{m_p}{m_K+m_p}\vec{Q},\sigma\right)\to K^-(\vec{k})p(-\vec{k},\sigma')\gamma(\vec{Q},\lambda)\right) + M\left(K^-(\vec{q})p(-\vec{q},\sigma)\to K^-\left(\vec{k}-\frac{m_K}{m_K+m_p}\vec{Q}\right)p\left(-\vec{k}-\frac{m_p}{m_K+m_p}\vec{Q},\sigma'\right)\gamma(\vec{Q},\lambda)\right). \quad (18)$$

The amplitude $M(K^-p\to K^-p\gamma)$ is defined by the Feynman diagrams depicted in Fig. 1. For the calculation of this amplitude we use the following effective Lagrangian, describing strong low-energy and electromagnetic interactions of the K^-p pairs:

$$\mathcal{L}_{\rm eff}(x) = \partial_\mu K^{-\dagger}(x)\partial^\mu K^-(x) - m_K^2K^{-\dagger}(x)K^-(x) + \bar{p}(x)(i\gamma^\mu\partial_\mu - m_p)p(x) + 4\pi\left(1+\frac{m_K}{m_p}\right)a_0^{K^-p}\bar{p}(x)p(x)K^{-\dagger}(x)K^-(x) - e\bar{p}(x)\gamma_\mu p(x)A^\mu(x) + ie(K^{-\dagger}(x)\partial_\mu K^-(x) - \partial_\mu K^{-\dagger}(x)K^-(x))A^\mu(x) + e^2K^{-\dagger}(x)K^-(x)A_\mu(x)A^\mu(x), \quad (19)$$

where e is the electric charge of the proton $e=\sqrt{4\pi\alpha}$, $a_0^{K^-p}$ is the S-wave scattering length of K^-p scattering [4,7], $A^\mu(x)$ is the vector potential of the quantized electromagnetic field. In the effective Lagrangian Eq. (19) strong low-energy interactions of the K^-p pair are described by $\mathcal{L}_{KKpp}(x)=4\pi(1+m_K/m_p)a_0^{K^-p}\bar{p}(x)p(x)K^{-\dagger}(x)K^-(x)$.

In the nonrelativistic limit the amplitude of the kaon-proton bremsstrahlung $K^-+p\to K^-+p+\gamma$ in Fig. 1 reads

$$M(K^-p\to K^-p\gamma) = 8\pi(m_K+m_p)a_0^{K^-p}\frac{ie}{2\mu\omega}\vec{e}^{\,*}(\vec{p},\lambda)\cdot(\vec{k}+\vec{q}). \quad (20)$$

For the calculation of the amplitude of the atomic transition $(K^-p)_{np}\to(K^-p)_{1s}+\gamma$ the amplitude of the kaon-proton bremsstrahlung Eq. (20) should be weighted with the wave functions of kaonic hydrogen in the ground and excited np states Eq. (17). Since the wave function of the ground state is spherical symmetric, the integration over $\vec{k}$ leads to the vanishing of the term proportional to $\vec{k}\cdot\vec{e}^{\,*}(\vec{p},\lambda)$. Therefore, we omit it in the following.

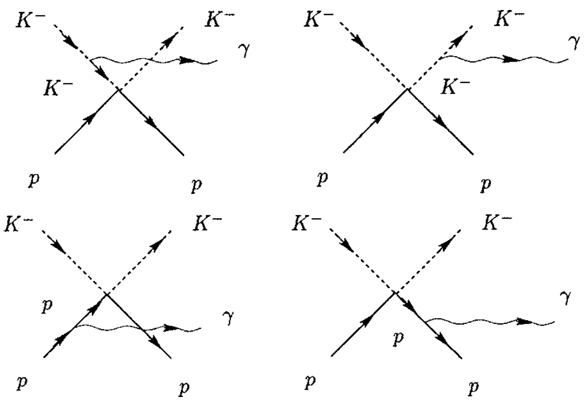

FIG. 1. Feynman diagrams for the radiative transitions $(K^-p)_{np}\to(K^-p)_{1s}+\gamma$ caused by strong low-energy interactions.

The appearance of the photon energy ω in the denominator of the amplitude of the kaon-proton bremsstrahlung $K^-+p\to K^-+p+\gamma$ is due to the fact that the virtual K^- meson and the proton are practically on-mass shell.

Our approach to the calculation of the amplitude of the kaon-proton bremsstrahlung $K^-+p\to K^-+p+\gamma$ is similar to that which has been used in Ref. [7] for the derivation of the Ericson-Weise formula for the S-wave scattering amplitude of K^-d scattering [36] and runs parallel the nonrelativistic effective field theory (EFT) approach, based on chiral perturbation theory (ChPT) by Gasser and Leutwyler [10], which has been applied by Meißner *et al.* [5] to the calculation of isospin-breaking corrections to the S-wave amplitude of K^-p scattering at threshold.

Due to the Coulomb interaction of the K^-p pair in initial and final states [37–39] the amplitude of the kaon-proton bremsstrahlung Eq. (20) changes as follows:

$$M(K^-p\to K^-p\gamma)=8\pi(m_K+m_p)a_0^{K^-p}\frac{ie}{2\mu\omega}\vec{e}^{\,*}(\vec{p},\lambda)\cdot\vec{q}\times e^{\pi/2ka_B}\Gamma(1-i/ka_B)e^{\pi/2qa_B}\Gamma(2-i/qa_B), \quad (21)$$

where $a_B=1/\alpha\mu$ is the Bohr radius and the Γ functions are defined by [40]

$$e^{\pi z/2}\Gamma(1-iz)=\sqrt{\frac{2\pi z}{1-e^{-2\pi z}}}\times\exp\left\{i\left[\gamma z+\sum_{k=1}^{\infty}\left(\frac{z}{k}-\arctan\left(\frac{z}{k}\right)\right)\right]\right\},$$

$$e^{\pi z/2}\Gamma(2-iz)=(1-iz)e^{\pi z/2}\Gamma(1-iz), \quad (22)$$

with Euler's constant $\gamma=0.57721\ldots$ [40].

For the calculation of the amplitude Eq. (21), taking into account the Coulomb interaction of the K^-p pair in the initial and final state, we have used the potential model approach. In this approach we describe strong low-energy interactions by the effective zero-range potential

$$V(\vec{r})=-\frac{2\pi}{\mu}a_0^{K^-p}\delta^{(3)}(\vec{r}), \quad (23)$$

which is equivalent to the effective local strong low-energy $KKpp$ interaction $\mathcal{L}_{KKpp}(x)=4\pi(1+m_K/m_p)a_0^{K^-p}\bar{p}(x)p(x)K^{-\dagger}(x)K^-(x)$ in Eq. (19).

By virtue of the Coulomb interaction the term $i\vec{e}^{\,*}(\vec{p},\lambda)\cdot\vec{q}$ in Eq. (20) is replaced by

$$i\vec{e}^{\,*}(\vec{p},\lambda)\cdot\vec{q}\to\vec{e}^{\,*}(\vec{p},\lambda)\int d^3x\;\delta^{(3)}(\vec{r})\,\nabla\,\psi^C_{K^-p}(\vec{q},\vec{r})=i\vec{e}^{\,*}(\vec{p},\lambda)\cdot\vec{q}e^{\pi/2qa_B}\Gamma(2-i/qa_B), \quad (24)$$

where $\psi^C_{K^-p}(\vec{q},\vec{r})$ is the exact nonrelativistic Coulomb wave function of the relative motion of the K^-p pair in the incoming scattering state with relative momentum $\vec{q}$. It is given by [39]

$$\psi^C_{K^-p}(\vec{q},\vec{r})=e^{\pi/2qa_B}\Gamma(1-i/qa_B)e^{i\vec{q}\cdot\vec{r}}F(i/qa_B,1,iqr-i\vec{q}\cdot\vec{r}). \quad (25)$$

Here $F(i/qa_B,1,iqr-i\vec{q}\cdot\vec{r})$ is the confluent hypergeometric function [39,40].

The factor $e^{\pi/2ka_B}\Gamma(1-i/ka_B)$ in Eq. (21) is the rest of the asymptotic of the non-relativistic Coulomb Green function $G^C_{K^-p}(\vec{r},0;k^2)$ [38]

$$G^C_{K^-p}(\vec{r},0;k^2)=-\frac{1}{4\pi r}\Gamma(1-i/ka_B)W_{i/ka_B,1/2}(-2ikr) \quad (26)$$

of the relative motion of the K^-p pair at $r\to\infty$, where $W_{i/ka_B,1/2}(-2ikr)$ is the Whittaker function [40], describing the outgoing spherical wave distorted by the Coulomb interaction [38].

For the amplitude of the transition $(K^-p)_{np}\to(K^-p)_{1s}+\gamma$ we get

$$M((K^-p)_{np}\to(K^-p)_{1s}\gamma)=8\pi(m_K+m_p)a_0^{K^-p}\frac{ie}{4\mu^2\omega}\vec{e}^{\,*}(\vec{p},\lambda)\int\frac{d^3k}{(2\pi)^3}\times\Phi^*_{100}(\vec{k})e^{\pi/2ka_B}\Gamma(1-i/ka_B)\int\frac{d^3q}{(2\pi)^3}\times e^{\pi/2qa_B}\Gamma(2-i/qa_B)\vec{q}\Phi_{n1m}(\vec{q}). \quad (27)$$

The main contributions to the integrals over $\vec{k}$ and $\vec{q}$ come from the regions $k\geqslant 1/a_B$ and $q>1/na_B$ [36]. These momenta are of order of $O(\alpha)$. Therefore, the contribution of the photon momentum can be dropped, since it is of order of $O(\alpha^2)$.

For the integration over $\vec{q}$ we define the vector $\vec{q}$ in the spherical basis [7]

$$\vec{q} = q\sqrt{\frac{4\pi}{3}}\sum_{M=0,\pm 1} Y^*_{1M}(\vartheta_{\vec{q}},\varphi_{\vec{q}})\vec{e}_M, \quad (28)$$

where $\vec{e}_{\pm 1} = \mp(\vec{e}_x \pm i\vec{e}_y)/\sqrt{2}$ and $\vec{e}_0 = \vec{e}_z$ are spherical unit vectors expended into Cartesian unit vectors $\vec{e}_x$, $\vec{e}_y$, and $\vec{e}_z$. Using Eq. (28) for the integral over $\vec{q}$ we get [35]

$$\int \frac{d^3q}{(2\pi)^3} e^{\pi/2qa_B}\Gamma(2 - i/qa_B)\vec{q}\Phi_{n1m}(\vec{q}) = -i\frac{\vec{e}_m}{\sqrt{3}}\int \frac{d^3q}{(2\pi)^3} e^{\pi/2qa_B}\Gamma(2 - i/qa_B)q\Phi_{np}(q). \quad (29)$$

For the integration over k and q we use the wave functions $\Phi_{100}(\vec{k}) = \Phi_{1s}(k)$ and $\Phi_{np}(q)$ given by [34]

$$\Phi_{1s}(k) = \frac{8\sqrt{\pi a_B^3}}{(1 + k^2a_B^2)^2}, \quad (30)$$

$$\Phi_{np}(q) = \sqrt{\frac{\pi n^3 a_B^3}{n^2 - 1}}\frac{32\, nqa_B}{(1 + n^2q^2a_B^2)^3}C^2_{n-2}\left(\frac{n^2q^2a_B^2 - 1}{n^2q^2a_B^2 + 1}\right), \quad (30)$$

where $C^2_{n-2}(z)$ is the Gegenbauer polynomial [34,40].

Using Ref. [41] and taking into account the results obtained in Ref. [35] we make the integration over momenta k and q:

$$\int \frac{d^3k}{(2\pi)^3}\Phi^*_{1s}(k)e^{\pi/2ka_B}\Gamma(1 - i/ka_B) \simeq \xi_{1s}\sqrt{\frac{1}{\pi a_B^3}}e^{i\varphi_{1s}}, \quad (31)$$

$$\int \frac{d^3q}{(2\pi)^3}e^{\pi/2qa_B}\Gamma(2 - i/qa_B)q\Phi_{np}(q) \simeq \xi_{np}\sqrt{\frac{1}{\pi a_B^5}\frac{n^2 - 1}{n^5}}e^{i\varphi_{np}},$$

where $\xi_{1s}=1.91$ and $\xi_{2p}=3.52$, $\xi_{3p}=2.22$, $\xi_{4p}=2.85,\ldots$ and φ_{1s} and φ_{np} are real phases, which do not contribute to the transition rate $(K^-p)_{np} \to (K^-p)_{1s}+\gamma$.

The amplitude of the transition $(K^-p)_{np} \to (K^-p)_{1s}+\gamma$ is defined by

$$M((K^-p)_{np} \to (K^-p)_{1s}\gamma) = 8\pi(m_K + m_p)ia_0^{K^-p}\frac{\mu^2}{\omega}\sqrt{\frac{\alpha^9}{4\pi}}\vec{e}^{\,*}(\vec{p},\lambda)\cdot\frac{\vec{e}_m}{\sqrt{3}}\sqrt{\frac{n^2 - 1}{n^5}} \times \xi_{1s}\xi_{np}e^{i(\varphi_{1s}+\varphi_{np})}. \quad (32)$$

The transition rate $(K^-p)_{np} \to (K^-p)_{1s}+\gamma$ is equal to

$$\Gamma((K^-p)_{np} \to (K^-p)_{1s}\gamma) = \frac{8}{n^3}\frac{\xi^2_{np}}{\xi^2_{2p}}\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma), \quad (33)$$

where the rate $\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma)$ is given by

$$\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma) = \frac{\xi^2_{1s}\xi^2_{2p}}{9}\alpha^7\mu^3|a_0^{K^-p}|^2. \quad (34)$$

For the subsequent analysis of the transition rate $(K^-p)_{np} \to (K^-p)_{1s}+\gamma$ it is convenient to represent $|a_0^{K^-p}|^2$ in terms of the energy level displacement of the ground state of kaonic hydrogen

$$|a_0^{K^-p}|^2 = \frac{1}{4\alpha^6\mu^4}\left(\epsilon^2_{1s} + \frac{1}{4}\Gamma^2_{1s}\right). \quad (35)$$

This is the model-independent DGBTT (Deser, Goldberger, Baumann, Thirring [42], and Trueman [43]) formula. Substituting Eq. (35) into Eq. (34) we get

$$\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma) = \frac{\xi^2_{1s}\xi^2_{2p}}{36}\frac{\alpha}{\mu}\left(\epsilon^2_{1s} + \frac{1}{4}\Gamma^2_{1s}\right). \quad (36)$$

Using the numerical values of the parameters ξ_{1s} and ξ_{2p} we obtain

$$\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma) = 4.3 \times 10^4\left(\epsilon^2_{1s} + \frac{1}{4}\Gamma^2_{1s}\right)\ \text{sec}^{-1}, \quad (37)$$

where ϵ_{1s} and Γ_{1s} are measured in eV.

Using the theoretical value for the energy level displacement of the ground state of kaonic hydrogen Eq. (4), for the rate of the transition $\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma)$ we get

$$\Gamma((K^-p)_{2p} \to (K^-p)_{1s}\gamma) = (2.3 \pm 0.3) \times 10^9\ \text{sec}^{-1}. \quad (38)$$

According to Eq. (33), the transition rate $(K^-p)_{3p} \to (K^-p)_{1s}+\gamma$ is equal to

$$\Gamma((K^-p)_{3p} \to (K^-p)_{1s}\gamma) = (2.7 \pm 0.4) \times 10^8\ \text{sec}^{-1}. \quad (39)$$

These rates should be compared with the pure electric dipole transition rates $2p \to 1s+\gamma$ and $3p \to 1s+\gamma$ at the neglect of strong interactions.

Using the results obtained by Bethe and Salpeter [34] and adjusting them to kaonic hydrogen we get $\Gamma_{2p\to 1s} = 4.0\times 10^{11}$ sec^{-1} and $\Gamma_{3p\to 1s} = 1.0\times 10^{11}$ sec^{-1}, respectively. Hence the transition rates $(K^-p)_{2p} \to (K^-p)_{1s}+\gamma$ and $(K^-p)_{3p} \to (K^-p)_{1s}+\gamma$, given by Eqs. (38) and (39) and induced by strong low-energy interactions, make up about 0.6% and 0.3% of the pure electric dipole transition rates $2p \to 1s+\gamma$ and $3p \to 1s+\gamma$, respectively.

For the experimental data of the energy level displacement q of the ground state of kaonic hydrogen, measured by Iwasaki *et al.* (the KEK Collaboration) [3] Eq. (2), the transition rates $(K^-p)_{2p} \to (K^-p)_{1s}+\gamma$ and $(K^-p)_{3p} \to (K^-p)_{1s}+\gamma$, given by Eqs. (38) and (39), are increased by a factor of 3.

IV. $np \to 1s+\gamma$ TRANSITIONS IN KAONIC DEUTERIUM

The formula (36) can be applied to the description of the radiative transition rates $(K^-d)_{np} \to (K^-d)_{1s}+\gamma$, induced by strong low-energy interactions and enhanced by the Coulomb interaction, in kaonic deuterium $(K^-d)_{np} \to (K^-d)_{1s}+\gamma$. One gets

$$\Gamma((K^-d)_{2p} \to (K^-d)_{1s}\gamma) = \frac{\xi_{1s}^2 \xi_{2p}^2}{36}\frac{\alpha}{\mu}\left(\epsilon_{1s}^2 + \frac{1}{4}\Gamma_{1s}^2\right) = 3.6 \times 10^4\left(\epsilon_{1s}^2 + \frac{1}{4}\Gamma_{1s}^2\right)\ \mathrm{sec}^{-1}, \quad (40)$$

where $\mu = m_K m_d/(m_K + m_d) = 390.81$ MeV is the reduced mass of the K^-d pair and $m_d = 1875.61$ MeV is the deuteron mass [9].

Recently the energy level displacement of the ground state of kaonic deuterium has been estimated in [7]:

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = (-325 \pm 60) + i(315 \pm 50)\ \mathrm{eV}. \quad (41)$$

According to Eq. (41), the radiative transition rates $(K^-d)_{2p} \to (K^-d)_{1s} + \gamma$ and $(K^-d)_{3p} \to (K^-d)_{1s} + \gamma$ are equal to

$$\Gamma((K^-d)_{2p} \to (K^-d)_{1s}\gamma) = (7.4 \pm 1.8) \times 10^9\ \mathrm{sec}^{-1},$$

$$\Gamma((K^-d)_{3p} \to (K^-d)_{1s}\gamma) = (8.7 \pm 2.1) \times 10^8\ \mathrm{sec}^{-1}. \quad (42)$$

This should be compared with the pure electric dipole transition rates $2p \to 1s + \gamma$ and $3p \to 1s + \gamma$, where are equal to $\Gamma_{2p\to 1s} = 4.8 \times 10^{11}\ \mathrm{sec}^{-1}$ and $\Gamma_{3p\to 1s} = 1.2 \times 10^{11}\ \mathrm{sec}^{-1}$, respectively [34].

Thus the radiative transition rates $(K^-d)_{2p} \to (K^-d)_{1s} + \gamma$ and $(K^-d)_{3p} \to (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions and enhanced by the Coulomb interaction, make up about 1.5% and 0.7% of the pure electric dipole transition rates $2p \to 1s + \gamma$ and $3p \to 1s + \gamma$, respectively.

For the energy level displacement of the ground state of kaonic deuterium, predicted by Barrett and Deloff [44]: $(\epsilon_{1s}^{(d)}, \Gamma_{1s}^{(d)}) = (-693, 880)$ eV, the rates of the transitions $(K^-d)_{2p} \to (K^-d)_{1s} + \gamma$ and $(K^-d)_{3p} \to (K^-d)_{1s} + \gamma$, given by Eq. (42), are increased by more than three times.

V. CONCLUSION

We have calculated the radiative transition rates $np \to 1s + \gamma$ for kaonic hydrogen and kaonic deuterium, induced by strong low-energy interactions and enhanced by the attractive Coulomb interaction of the K^-p and K^-d pairs in the kaon-proton and kaon-deuteron bremsstrahlung, $K^- + p \to K^- + p + \gamma$ and $K^- + d \to K^- + d + \gamma$. The neglect of the Coulomb interaction of the K^-p and K^-d pair s in these reactions corresponds to $\xi_{1s} = \xi_{np} = 1$ in Eq. (32). For the calculation of the amplitudes of the kaon-proton and kaon-deuteron bremsstrahlung we have used an approach analogous to that of the EFT based on ChPT by Gasser and Leutwyler [10] and used in Ref. [7] for the derivation of the Ericson-Weise formula for the S-wave scattering length of K^-d scattering [36]. It agrees also well with the potential model approach.

We have found that for the $2p$ states of kaonic atoms the contributions of the transition rates $(K^-p)_{2p} \to (K^-p)_{1s} + \gamma$ and $(K^-d)_{2p} \to (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions, relative to the pure electric dipole transition rates $2p \to 1s + \gamma$, are of the order of one percent.

Precisions of this order ±0.2% and ±1.0% have been reached in the experiments of the PSI Collaboration [45] for the measurements of the energy level shift and width of the ground state of pionic hydrogen, respectively. These precisions are defined by the accuracy of the measurements of the x-ray spectra and yields in pionic hydrogen.

Measurements of the energy level displacements of the ground states for kaonic atoms at the percent level of precision, would demand to take into account the transition rates $(K^-p)_{np} \to (K^-p)_{1s} + \gamma$ and $(K^-d)_{np} \to (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions, for the theoretical description of the experimental data on the x-ray spectra and yields in kaonic atoms.

The program of the measurements of the energy level displacement of the ground state of kaonic hydrogen with a precision of a percent level will be realized very likely in 2006 year by the DEAR/SIDDHARTA Collaborations [46].

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