Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement

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We present a way for symmetric multiparty-controlled teleportation of an arbitrary two-particle entangled state based on Bell-basis measurements by using two Greenberger-Horne-Zeilinger states, i.e., a sender transmits an arbitrary two-particle entangled state to a distant receiver, an arbitrary one of the $n+1$ agents, via the control of the others in a network. It will be shown that the outcomes in the cases that n is odd or is even are different in principle as the receiver has to perform a controlled-NOT operation on his particles for reconstructing the original arbitrary entangled state in addition to some local unitary operations in the former. Also we discuss the applications of this controlled teleporation for quantum secret sharing of classical and quantum information. As all the instances can be used to carry useful information, its efficiency for qubit approaches the maximal value.

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I. INTRODUCTION

The principles of quantum mechanics supplied many interesting applications in the field of information in the last decade, such as quantum computer, quantum cryptography, quantum teleportation, quantum secret sharing, and so on. The quantum-teleportation process allows the two remote parties, the sender Alice and the receiver Bob, to utilize the nonlocal correlations of the quantum channel, Einstein-Podolsky-Rosen (EPR) [1] pair shared initially, to teleport an unknown quantum state $|\chi\rangle=a|\uparrow\rangle+b|\downarrow\rangle$; Alice makes a Bell-basis measurement on her EPR particle and the unknown quantum system χ, and Bob reconstructs the state $|\chi\rangle$ with a local unitary operation on his EPR particle according to the classical information published by Alice [2]. Quantum teleportation has been demonstrated by some groups [3–6] since Bennett *et al.* [2] proposed the theoretical protocol for teleporting an unknown single qubit in 1993. Subsequently, the protocols for teleporting an entangled state are proposed with some pure entangled states or maximal multiparticle entangled states [7–13]. For example, Lu and Guo [11] introduced some ways for teleporting an entangled state $\alpha|00\rangle+\beta|11\rangle$ with entanglement swapping [14–16] by using EPR pairs or pure entangled states as the quantum channels in 2000. Lee proposed a protocol [12] for teleporting an entangled state $\alpha|10\rangle+\beta|01\rangle$ with the four-particle Greenberger-Horne-Zeilinger (GHZ) state $|\psi\rangle_L=(1/\sqrt{2})(|1010\rangle+|0101\rangle)$. Recently, Rigolin [17] showed a way to teleport an arbitrary two-qubit entangled state with a four-particle entangled state $|\psi\rangle_R=\frac{1}{2}(|0000\rangle+|0101\rangle+|1010\rangle+|1111\rangle)$ and four-particle joint measurements.

Quantum secret sharing (QSS) is an important branch of quantum communication and is used to complete the task of classical secret sharing with the principles of quantum mechanics. The basic idea of secret sharing [18] in a simple case (there are three parties of communication, say Alice, Bob, and Charlie) is that a secret is divided into two pieces which will be distributed to two parties, respectively, and they can recover the secret if and only if both act in concert. A pioneering QSS scheme was proposed by Hillery, Bužek, and Berthiaume [19] in 1999 by using the three-particle and four-particle entangled GHZ states for sharing classical information. Now, there are a lot of works focused on QSS in both the theoretical [19–30] and experimental [31,32] aspects. Different from classical secret sharing, QSS can be used to share both classical and quantum information. For instance, the QSS protocols in Refs. [19–23] are used to split a quantum secret.

Recently, controlled teleporation for a single-qubit $|\chi\rangle=a|\uparrow\rangle+b|\downarrow\rangle$ [33,34] or m-qubit message $\Pi_{i=1}^{m}\otimes(\alpha_i|0\rangle_i+\beta_i|1\rangle_i)$ [35] have been studied. In those teleportation protocols, the qubits can be regenerated by one of the receivers with the help of the others. Those principles can be used to split a quantum secret in QSS [19]. In this paper, we will present a symmetric protocol for multiparty-controlled teleportation of an arbitrary two-particle entangled state with two GHZ states and Bell-basis measurements. It can be used to share classical information and an entangled quantum secret. Different from the protocols for teleportation of a two-particle entangled state with a GHZ state in which the unknown state should be an EPR-class entangled state [7,8], i.e., $|\Phi\rangle_\chi=\alpha|uv\rangle+\beta|\bar{u}\bar{v}\rangle$ ($u,v\in\{0,1\}$, and $\bar{u}=1-u$), the unknown quantum system in this protocol is in an arbitrary

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two-particle state. Moreover, the receiver is an arbitrary one in the $n+1$ agents via the control of the others in the network. As the whole quantum source is used to carry the useful quantum information, the efficiency for the qubits approaches the maximal value and the procedure for controlled teleportation is an optimal one.

The paper is organized as follows. In Sec. II, we present a way for the symmetric controlled teleportation of an arbitrary two-particle entangled state with two three-particle GHZ states. That is, there is one controller who controls the process of quantum teleportation. We generalize it to the case with $n+1$ agents in which one is the receiver and the other n agents are the controllers in the network in Sec. III, and discuss the difference between the two cases where the number of controllers is even or odd. In Sec. IV, we apply the method for the controlled teleportation to share classical and quantum information. A brief discussion and summary are given in Sec. V.

II. CONTROLLED TELEPORTATION VIA THE CONTROL OF ONE AGENT

An EPR pair is in one of the four Bell states shown as follows [36]:

$$|\psi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B \pm |1\rangle_A|0\rangle_B), \tag{1}$$

$$|\phi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B), \tag{2}$$

where $|0\rangle$ and $|1\rangle$ are the eigenvectors of the operator σ_z. The four unitary operations $\{U_i\}$ $(i=0,1,2,3)$ can transfer each one of the four Bell states into another,

$$U_0 = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad U_1 = |0\rangle\langle 0| - |1\rangle\langle 1|,$$

$$U_2 = |1\rangle\langle 0| + |0\rangle\langle 1|, \quad U_3 = |0\rangle\langle 1| - |1\rangle\langle 0|. \tag{3}$$

Suppose the unknown two-particle state teleported is

$$|\Phi\rangle_{xy} = a|00\rangle_{xy} + b|01\rangle_{xy} + c|10\rangle_{xy} + d|11\rangle_{xy}, \tag{4}$$

where

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \tag{5}$$

and the three-particle GHZ state prepared by Alice is

$$|\text{GHZ}\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle). \tag{6}$$

With a Hadamard (H) operation on each particle,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{7}$$

the state becomes

$$|\text{GHZ}'\rangle_{ABC} = \frac{1}{\sqrt{2}}(|+x+x+x\rangle + |-x-x-x\rangle), \tag{8}$$

where $|+x\rangle=(1/\sqrt{2})(|0\rangle+|1\rangle)$ and $|-x\rangle=(1/\sqrt{2})(|0\rangle-|1\rangle)$ are the two eigenvectors of the operator σ_x.

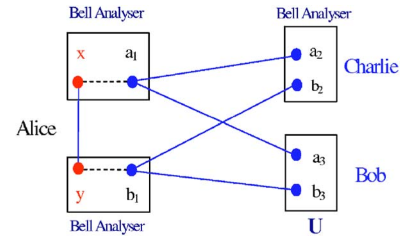

FIG. 1. (Color online) Symmetric controlled teleportation of an arbitrary two-particle entangled state with two GHZ states. Alice, Bob, and Charlie each keep one of the three particles in each GHZ state. The bold lines connect qubits in GHZ states or the two-particle arbitrary entangled state $|\Phi\rangle_{xy}$.

The basic idea of this symmetric controlled teleportation of an arbitrary two-particle entangled state is shown in Fig. 1. Suppose that Alice wants to send the state $|\Phi\rangle_{xy}$ to one of the two agents randomly and the receiver can reconstruct the state only when he/she obtains the help of the other agent, i.e., Bob reconstructs it with the control of Charlie's, or vice versa. To this end, Alice prepares two three-particle GHZ states $|\Psi\rangle_{a_1a_2a_3}$ and $|\Psi\rangle_{b_1b_2b_3}$,

$$|\Psi\rangle_{a_1a_2a_3} = |\Psi\rangle_{b_1b_2b_3} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \tag{9}$$

and she sends the particles a_2 and b_2 to Charlie, and a_3 and b_3 to Bob. The state of the composite quantum system composed of the eight particles x, y, a_1, a_2, a_3, b_1, b_2, and b_3 can be written as

$$\begin{aligned}|\Psi\rangle_s &\equiv |\Phi\rangle_{xy} \otimes |\Psi\rangle_{a_1a_2a_3} \otimes |\Psi\rangle_{b_1b_2b_3} \\ &= (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{xy} \\ &\quad \otimes \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{a_1a_2a_3} \\ &\quad \otimes \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{b_1b_2b_3}.\end{aligned} \tag{10}$$

Alice performs Bell-basis measurements on the particles x and a_1, and y and b_1, respectively, and then publishes the outcomes. If Bob wants to reconstruct the state $|\Phi\rangle_{xy}$, Charlie does the Bell-basis measurement on her particles a_2 and b_2, or vice versa. Without loss of generalization, we assume that Bob will obtain the original state with the help of Charlie, shown in Fig. 1.

In fact, Bob can only get an EPR-class entangled state, i.e., $|\Phi\rangle_u=\alpha|uv\rangle+\beta|\bar{u}\bar{v}\rangle$ $(u,v\in\{0,1\}$ and $|\alpha|^2+|\beta|^2=1)$, similar to those in Refs. [7,8,37], if Alice and Charlie perform Bell-basis measurements on the composite quantum system $|\Psi\rangle_s$ directly. For example, if the results of the Bell-basis measurements are $|\phi^+\rangle_{xa_1}$, $|\phi^+\rangle_{yb_1}$, and $|\phi^+\rangle_{a_2b_2}$, then the particles a_3 and b_3 are in the state

TABLE I. The relation between the unitary operations and the results R_{xa_1}, R_{yb_1}, and $R_{a_2b_2}$ in the case that each of Alice, Bob, and Charlie keeps one of the three particles in each GHZ state. $\Phi_{a_3b_3}$ is the state of the two particles held by Bob after all the Bell-basis measurements are done by the sender Alice and the controller Charlie.

V_{xa_1}	V_{total}	P_{yb_1}	P_{total}	$\Phi_{a_3b_3}$	Operations
0	0	+	+	$a\|00\rangle+b\|01\rangle+d\|10\rangle+c\|11\rangle$	$U_0\otimes U_0$+CNOT
0	0	+	−	$a\|00\rangle+b\|01\rangle-d\|10\rangle-c\|11\rangle$	$U_1\otimes U_0$+CNOT
0	0	−	+	$a\|00\rangle-b\|01\rangle+d\|10\rangle-c\|11\rangle$	$U_0\otimes U_1$+CNOT
0	0	−	−	$a\|00\rangle-b\|01\rangle-d\|10\rangle+c\|11\rangle$	$U_1\otimes U_1$+CNOT
0	1	+	+	$b\|00\rangle+a\|01\rangle+c\|10\rangle+d\|11\rangle$	$U_0\otimes U_2$+CNOT
0	1	+	−	$b\|00\rangle+a\|01\rangle-c\|10\rangle-d\|11\rangle$	$U_1\otimes U_2$+CNOT
0	1	−	+	$b\|00\rangle-a\|01\rangle+c\|10\rangle-d\|11\rangle$	$U_0\otimes U_3$+CNOT
0	1	−	−	$b\|00\rangle-a\|01\rangle-c\|10\rangle+d\|11\rangle$	$U_1\otimes U_3$+CNOT
1	0	+	+	$d\|00\rangle+c\|01\rangle+a\|10\rangle+b\|11\rangle$	$U_2\otimes U_0$+CNOT
1	0	+	−	$d\|00\rangle+c\|01\rangle-a\|10\rangle-b\|11\rangle$	$U_3\otimes U_0$+CNOT
1	0	−	+	$d\|00\rangle-c\|01\rangle+a\|10\rangle-b\|11\rangle$	$U_2\otimes U_1$+CNOT
1	0	−	−	$d\|00\rangle-c\|01\rangle-a\|10\rangle+b\|11\rangle$	$U_3\otimes U_1$+CNOT
1	1	+	+	$c\|00\rangle+d\|01\rangle+b\|10\rangle+a\|11\rangle$	$U_2\otimes U_2$+CNOT
1	1	+	−	$c\|00\rangle+d\|01\rangle-b\|10\rangle-a\|11\rangle$	$U_3\otimes U_2$+CNOT
1	1	−	+	$c\|00\rangle-d\|01\rangle+b\|10\rangle-a\|11\rangle$	$U_2\otimes U_3$+CNOT
1	1	−	−	$c\|00\rangle-d\|01\rangle-b\|10\rangle+a\|11\rangle$	$U_3\otimes U_3$+CNOT

$$|\Psi\rangle_{a_3b_3} = {}_{a_2b_2}\langle\phi^+| \otimes {}_{yb_1}\langle\phi^+| \otimes {}_{xa_1}\langle\phi^+|\Phi\rangle_s = \frac{1}{4\sqrt{2}}(a|00\rangle + d|11\rangle) \Rightarrow (\alpha|00\rangle + \beta|11\rangle). \quad (11)$$

It is just a superposition of the two product states $|00\rangle$ and $|11\rangle$. Fortunately, the case will be changed with just a little of modification. Instead of sending the three particles in the state $|\Psi\rangle_{b_1b_2b_3}=(1/\sqrt{2})(|000\rangle+|111\rangle)$ directly, Alice transfers it into $|\Psi'\rangle_{b_1b_2b_3}=(1/\sqrt{2})(|+x+x+x\rangle+|-x-x-x\rangle)$ with a H operation on each particle. Then the joint state of the composite quantum system is transferred to

$$|\Psi\rangle_{joint} \equiv |\Phi\rangle_{xy} \otimes |\Psi\rangle_{a_1a_2a_3} \otimes |\Psi'\rangle_{b_1b_2b_3} = \frac{1}{2}(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{xy} \otimes (|000\rangle + |111\rangle)_{a_1a_2a_3} \otimes (|+x+x+x\rangle + |-x-x-x\rangle)_{b_1b_2b_3}. \quad (12)$$

Using the decomposition into Bell states, we can get the relation between the measurement results (i.e., R_{xa_1}, R_{yb_1}, $R_{a_2b_2}$) and the final state of the two particles a_3 and b_3, $|\Phi\rangle_{a_3b_3}$, shown in Table I.

Now, let us describe the notations in Table I. Here we define V as the bit value of the Bell state, i.e., $V_{|\phi^\pm\rangle}\equiv 0$, $V_{|\psi^\pm\rangle}\equiv 1$. That is, the bit value $V=0$ if the states of the two particles in a Bell state are parallel, otherwise $V=1$. $V_{total} \equiv V_{xa_1}\oplus V_{yb_1}\oplus V_{a_2b_2}$. P denotes the parity of the result of the Bell-basis measurement on the two-particle quantum system $R_i\in\{|\psi^+\rangle,|\psi^-\rangle,|\phi^+\rangle,|\phi^-\rangle\}$, i.e., $P_{|\psi^\pm\rangle}\equiv\pm$, $P_{|\phi^\pm\rangle}\equiv\pm$ and $P_{total}\equiv\Pi_{i=1}\otimes P_{R_i}=P_{R_{xa_1}}\otimes P_{R_{yb_1}}\otimes P_{R_{a_2b_2}}$; $\Phi_{a_3b_3}$ is the state of the two particles a_3 and b_3 after all the Bell-basis measurements are taken by Alice and Charlie; the unitary operations $U_i\otimes U_j$+CNOT $(i,j\in\{0,1,2,3\})$ means performing the unitary operation U_i on the particle a_3 and the operation U_j on the particle b_3, respectively, and then taking a controlled-NOT (CNOT) gate on those two particles for reconstructing the state $|\Phi\rangle_{xy}$, shown in Fig. 2. For example, if the results of R_{xa_1}, R_{yb_1}, and $R_{a_2b_2}$ are $|\psi^-\rangle_{xa_1}$, $|\phi^-\rangle_{yb_1}$, and $|\psi^-\rangle_{a_2b_2}$, respectively, then $V_{R_{xa_1}}=1$, $V_{total}=V_{xa_1}\oplus V_{yb_1}\oplus V_{a_2b_2}=1\oplus 0\oplus 1=0$, $P_{yb_1}=-$, $P_{total}=(-)\otimes(-)\otimes(-)=-$, and Bob first performs the unitary operations U_3 and U_1 on the particles a_3 and b_3, respectively, and then does the CNOT operation on those two particles for reconstructing the state $|\Phi\rangle_{xy}$.

Unlike those in Refs. [7,8,37], the original entangled state is an arbitrary one, i.e., $|\Phi\rangle_{xy}=a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$ is an arbitrary state in the Hilbert space $\mathcal{H}^2\otimes\mathcal{H}^2$ for two particles. Another feature in this controlled teleportation is that the receiver should perform a CNOT gate on the two particles for recovering the state $|\Phi\rangle_{xy}$. Moreover, the whole quantum source is used to carry useful information and the efficiency for the qubits $\eta_q\equiv q_u/q_t$ approaches the maximal value $\frac{1}{3}$ as

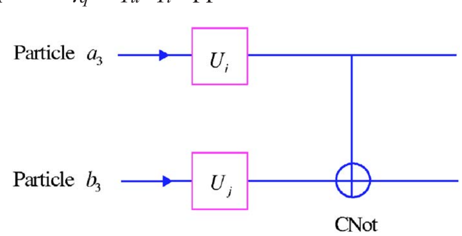

FIG. 2. (Color online) The operations that Bob needs to perform on the two particles for reconstructing the original entangled state. $U_i, U_j\in\{U_0,U_1,U_2,U_3\}$.

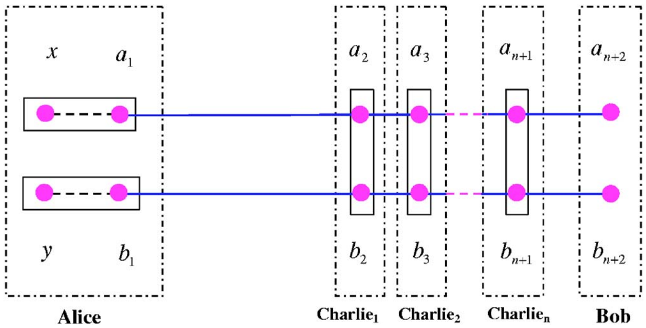

FIG. 3. (Color online) The principle of the controlled teleportation of an arbitrary two-particle entangled state in the case that there are n controllers. The rectangles represent the Bell-basis measurements done by Alice or the controllers; Charlie i are the n controllers in the $n+1$ agents; Bob is just the agent who will obtain the original entangled state with unitary operations.

the receiver can recover the two-qubit entangled state with a six-qubit quantum source, where q_u is the number of useful qubits and q_t is the number of qubits used for teleportation.

III. CONTROLLED TELEPORTATION VIA THE CONTROL OF *n* AGENTS

In this section, we will generalize the method discussed above to the case where there are n controllers who control the teleportation of an arbitrary two-particle entangled state $|\Phi\rangle_{xy}=a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$, say Charlie$_i$ $\{i=1,2,\ldots,n\}$, shown in Fig. 3.

For the controlled teleportation, Alice prepares two $(n+2)$-particle GHZ states. The state of the composite quantum system can be written as

$$\begin{aligned}|\Psi\rangle_s &\equiv |\Phi\rangle_{xy}\otimes|\Psi\rangle_{s_1}\otimes|\Psi\rangle_{s_2}\\ &=(a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle)_{xy}\\ &\quad\otimes\frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n+2}|0\rangle_{a_i}+\prod_{i=1}^{n+2}|1\rangle_{a_i}\right)\\ &\quad\otimes\frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n+2}|+x\rangle_{b_i}+\prod_{i=1}^{n+2}|-x\rangle_{b_i}\right).\end{aligned} \quad (13)$$

After the Bell-basis measurements on the particles x and a_1, and y and b_1, respectively, are done by Alice, the state of the subsystem (without being normalized) becomes

$$\begin{aligned}\Psi_{sub}=&\left(\prod_{i=2}^{n+2}|0\rangle_{a_i}\right)\otimes\left[\alpha\left(\prod_{i=2}^{n+2}|+x\rangle_{b_i}\right)+\beta\left(\prod_{i=2}^{n+2}|-x\rangle_{b_i}\right)\right]\\ &+\left(\prod_{i=2}^{n+2}|1\rangle_{a_i}\right)\otimes\left[\gamma\left(\prod_{i=2}^{n+2}|+x\rangle_{b_i}\right)+\delta\left(\prod_{i=2}^{n+2}|-x\rangle_{b_i}\right)\right].\end{aligned} \quad (14)$$

The relation between the numbers $\alpha,\beta,\gamma,\delta$ and the results R_{xa_1},R_{yb_1} is shown in Table II.

TABLE II. The relation between the values of $\alpha,\beta,\gamma,\delta$ and the results of the Bell-basis measurements on the particles x and a_1, y and b_1.

V_{xa_1}	V_{yb_1}	P_{xa_1}	P_{yb_1}	α	β	γ	δ
0	0	+	+	$+(a+b)$	$+(a-b)$	$+(c+d)$	$+(c-d)$
0	0	+	−	$+(a-b)$	$+(a+b)$	$+(c-d)$	$+(c+d)$
0	0	−	+	$+(a+b)$	$+(a-b)$	$-(c+d)$	$-(c-d)$
0	0	−	−	$+(a-b)$	$+(a+b)$	$-(c-d)$	$-(c+d)$
0	1	+	+	$+(a+b)$	$-(a-b)$	$+(c+d)$	$-(c-d)$
0	1	+	−	$+(a-b)$	$-(a+b)$	$+(c-d)$	$-(c+d)$
0	1	−	+	$+(a+b)$	$-(a-b)$	$-(c+d)$	$+(c-d)$
0	1	−	−	$+(a-b)$	$-(a+b)$	$-(c-d)$	$+(c+d)$
1	0	+	+	$+(c+d)$	$+(c-d)$	$+(a+b)$	$+(a-b)$
1	0	+	−	$+(c-d)$	$+(c+d)$	$+(a-b)$	$+(a+b)$
1	0	−	+	$-(c+d)$	$-(c-d)$	$+(a+b)$	$+(a-b)$
1	0	−	−	$-(c-d)$	$-(c+d)$	$+(a-b)$	$+(a+b)$
1	1	+	+	$+(c+d)$	$-(c-d)$	$+(a+b)$	$-(a-b)$
1	1	+	−	$+(c-d)$	$-(c+d)$	$+(a-b)$	$-(a+b)$
1	1	−	+	$-(c+d)$	$+(c-d)$	$+(a+b)$	$-(a-b)$
1	1	−	−	$-(c-d)$	$+(c+d)$	$+(a-b)$	$-(a+b)$

We can use a common formula to represent the Bell-basis measurements done by the n controllers, Charlie$_s$, i.e.,

$$M\equiv(\langle\psi^-|)^{n-m-l-k}\otimes(\langle\psi^+|)^m\otimes(\langle\phi^-|)^l\otimes(\langle\phi^+|)^k. \quad (15)$$

It means that the numbers of the controllers who obtain the results of Bell-basis measurements $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$, and $|\psi^-\rangle$

TABLE III. The relation between the results of the Bell-basis measurements and the state Ψ_f when the number of the controllers is even.

V_{xa_1}	V_{total}	P_{yb_2}	P_{total}	Ψ_f	Operations
0	0	+	+	$a\|00\rangle+b\|01\rangle+c\|10\rangle+d\|11\rangle$	$U_0\otimes U_0$
0	0	+	−	$a\|00\rangle+b\|01\rangle-c\|10\rangle-d\|11\rangle$	$U_1\otimes U_0$
0	0	−	+	$a\|00\rangle-b\|01\rangle-c\|10\rangle+d\|11\rangle$	$U_1\otimes U_1$
0	0	−	−	$a\|00\rangle-b\|01\rangle+c\|10\rangle-d\|11\rangle$	$U_0\otimes U_1$
0	1	+	+	$b\|00\rangle+a\|01\rangle+d\|10\rangle+c\|11\rangle$	$U_0\otimes U_2$
0	1	+	−	$b\|00\rangle+a\|01\rangle-d\|10\rangle-c\|11\rangle$	$U_1\otimes U_2$
0	1	−	+	$b\|00\rangle-a\|01\rangle-d\|10\rangle+c\|11\rangle$	$U_1\otimes U_3$
0	1	−	−	$b\|00\rangle-a\|01\rangle+d\|10\rangle-c\|11\rangle$	$U_0\otimes U_3$
1	0	+	+	$d\|00\rangle+c\|01\rangle+b\|10\rangle+a\|11\rangle$	$U_2\otimes U_2$
1	0	+	−	$d\|00\rangle+c\|01\rangle-b\|10\rangle-a\|11\rangle$	$U_3\otimes U_2$
1	0	−	+	$d\|00\rangle-c\|01\rangle-b\|10\rangle+a\|11\rangle$	$U_3\otimes U_3$
1	0	−	−	$d\|00\rangle-c\|01\rangle+b\|10\rangle-a\|11\rangle$	$U_2\otimes U_3$
1	1	+	+	$c\|00\rangle+d\|01\rangle+a\|10\rangle+b\|11\rangle$	$U_2\otimes U_0$
1	1	+	−	$c\|00\rangle+d\|01\rangle-a\|10\rangle-b\|11\rangle$	$U_3\otimes U_0$
1	1	−	+	$c\|00\rangle-d\|01\rangle-a\|10\rangle+b\|11\rangle$	$U_3\otimes U_1$
1	1	−	−	$c\|00\rangle-d\|01\rangle+a\|10\rangle-b\|11\rangle$	$U_2\otimes U_1$

are k,l,m, and $n-k-l-m$, respectively. Because of the symmetry, who obtains the result $|\phi^{+}\rangle$ is not important for the final state $|\Psi\rangle_{a_{n+2}b_{n+2}}$ of the two particles a_{n+2} and b_{n+2}, but only the number. The operation M represents all the possible results of the Bell-basis measurements of the n controllers with the parameters k, l, and m. The final state $|\Psi\rangle_{a_{n+2}b_{n+2}}$ can be obtained by means of performing the operation M on the state of the subsystem Ψ_{sub}:

$$\begin{aligned}\Psi_{a_{n+2}b_{n+2}} &= M\left\{\left(\prod_{i=2}^{n+2}|0\rangle_{a_i}\right)\otimes\left[\alpha\left(\prod_{i=2}^{n+2}|+x\rangle_{b_i}\right)+\beta\left(\prod_{i=2}^{n+2}|-x\rangle_{b_i}\right)\right]+\left(\prod_{i=2}^{n+2}|1\rangle_{a_i}\right)\otimes\left[\gamma\left(\prod_{i=2}^{n+2}|+x\rangle_{b_i}\right)+\delta\left(\prod_{i=2}^{n+2}|-x\rangle_{b_i}\right)\right]\right\}\\ &= \frac{1}{2^n}\{|0\rangle_{a_{n+2}}[\alpha|+x\rangle+(-1)^{n-l-k}\beta|-x\rangle]_{b_{n+2}}+(-1)^{n-m-k}|1\rangle_{a_{n+2}}[\gamma|+x\rangle+(-1)^{k+l}\delta|-x\rangle]_{b_{n+2}}\}\\ &= \frac{\sqrt{2}}{2^{n+1}}\{[\alpha+(-1)^{n-l-k}\beta]|00\rangle+[\alpha-(-1)^{n-l-k}\beta]|01\rangle+(-1)^{n-m-k}[\gamma+(-1)^{k+l}\delta]|10\rangle+(-1)^{n-m-k}[\gamma-(-1)^{k+l}\delta]|11\rangle\}.\end{aligned} \tag{16}$$

This is just the final state Ψ_f without being normalized:

$$\begin{aligned}\Psi_f &= [\alpha+(-1)^{n-l-k}\beta]|00\rangle+[\alpha-(-1)^{n-l-k}\beta]|01\rangle\\ &+(-1)^{n-m-k}[\gamma+(-1)^{k+l}\delta]|10\rangle\\ &+(-1)^{n-m-k}[\gamma-(-1)^{k+l}\delta]|11\rangle.\end{aligned} \tag{17}$$

As in the case with one controller, let us define

$$V_{total}\equiv\sum_i \oplus V_{R_i},\quad P_{total}\equiv\prod_i \otimes P_{R_i}, \tag{18}$$

where V_{R_i}, P_{R_i} are the bit values and the parities of the results of the Bell-basis measurements done by Alice or the controllers, respectively (see them in Sec. II).

The relation between the state Ψ_f and the results V_{xa_1}, V_{total}, P_{yb_2}, and P_{total} is shown in Table III when the number of controllers n is even. When n is odd, the result is the same as that in Table I with just the modification of replacing the state $\Phi_{a_3b_3}$ with Ψ_f. The results in Tables I and III show that the unitary operations performed on Bob's particles for reconstructing the state $|\Phi\rangle_{xy}$ are different in principle when n is even or odd. In Table III, it is enough for Bob to reconstruct the state $|\Phi\rangle_{xy}$ with the two local unitary operations U_i and U_j $(i,j\in\{0,1,2,3\})$ on the particles a_{n+2} and b_{n+2}, respectively, but he has to do an additional CNOT operation on the two particles when the number of the controller is odd, which is different from the other methods for a controlled teleportation [33–35].

For a secure controlled teleportation of the state $|\Phi\rangle_{xy}$, the controllers need to keep the receiver from eavesdropping the quantum communication when they set up the quantum channel, similar to the case in quantum secret sharing. Surely, the task of the teleportation of an arbitrary two-particle entangled state can be completed with the combination of the method for teleporting an arbitrary two-qubit state [17] and quantum secure direct communication protocols [38–43], similar to the way that quantum secret sharing for classical information [30] can be finished with quantum-key-distribution protocols [44–50]. This time, the receiver is only the person who is deterministic in advance, not an arbitrary man in the $n+1$ agents. Moreover, the total efficiency η_t is not more than that in this symmetric controlled teleportation protocol, as the classical information exchanged and the quantum source will increase since the efficiency of QKD is no more than 1. Here η_t is defined as [48,49]

$$\eta_t=\frac{q_u}{q_t+b_t}, \tag{19}$$

where b_t is the number of classical bits exchanged between the parties. On the other hand, the multiparticle-entangled states must be produced in this protocol, which is not easy at present [51–53]. With the improvement of technology, it may be feasible in the future.

IV. QUANTUM SECRET SHARING BASED ON CONTROLLED TELEPORTATION

A. Setting up the quantum channel with GHZ states

It is important for the parties of the communication to set up a quantum channel with GHZ states securely in both the symmetric controlled teleporation and quantum secret sharing. The process for constructing a quantum channel discussed in this paper is similar to that in Ref. [38] for quantum secure direct communication (QSDC) in which the classical secret is transmitted directly without creating a private key and then encrypting it. Another property, as in QSDC [38–43], is that the information about the unknown state $|\Phi\rangle_{xy}$ should not be leaked to an unauthorized user, such as a vicious eavesdropper Eve. It means that the controllers and Eve can get nothing about the final entangled state even though they eavesdrop on the quantum communication. If the quantum channel is secure, no-one can obtain the original state except for the legal receiver Bob.

If the process for constructing the entangled quantum channel is secure, then the whole process for communication is secure as no-one can read out the information about a maximal entangled quantum system from a part of it [36]. The results in Secs. II and III show that Bob's particle is randomly in one of the 16 entangled states with the same probability. The randomness of the outcomes ensures the security of the communication [36], as for the classical one-time-pad cryptosystem [54].

The method for setting up a quantum channel with a sequence of EPR pairs is discussed in Refs. [38,55]. The approach can also be used for sharing GHZ states [36]. The way is just that the legal users determine whether there is an eavesdropper in the line when they transmit the particles in the GHZ state and then purify the quantum channel if there is no-one monitoring the line or the probability for being eavesdropped is lower than a suitable threshold. The latter can be considered as quantum privacy amplification with quantum purification [56,57]. Let us use a three-particle GHZ state as an example to demonstrate the principle, as in Ref. [37]. Alice prepares a sequence of GHZ states $|\mathrm{GHZ}\rangle_{ABC}$. For each GHZ state, Alice sends the particles B and C to Bob and Charlie, respectively, and retains the particle A:

$$\begin{aligned}|\mathrm{GHZ}\rangle_{ABC} &= \frac{1}{\sqrt{2}}(|000\rangle_{ABC}+|111\rangle_{ABC})\\ &= \frac{1}{2\sqrt{2}}[(|+x\rangle_A|+x\rangle_B+|-x\rangle_A|-x\rangle_B)|+x\rangle_C\\ &\quad+(|+x\rangle_A|-x\rangle_B+|-x\rangle_A|+x\rangle_B)|-x\rangle_C]. \quad (20)\end{aligned}$$

For determining whether there is an eavesdropper in the line when the particles are transmitted, Alice picks up some of the GHZ states from the GHZ sequence randomly, and requires Bob and Charlie to choose the measuring basis σ_z or σ_x to measure their particles according to the information published by Alice. If there is an eavesdropper monitoring the quantum channel, the error rate of the samples will increase, as in the Bennett-Brassard-Mermin QKD protocol [46]. If the error rate is low, Alice, Bob, and Charlie can obtain some private GHZ states with multiparticle entanglement purification [56,57].

B. Quantum secret sharing of a classical secret and quantum information with controlled teleportation

Now, let us introduce the method for quantum secret sharing with controlled teleportation. There are two main goals in quantum secret sharing. One is to share classical information, a sequence of binary numbers, and the other is to share quantum information, an unknown quantum state. In the former, the quantum state of each two particles in all the parties of the communication is coded as a two-bit binary number (the parties store the results of the measurements on the particles as classical information). For instance, they can code the four Bell states $\{|\phi^+\rangle,|\phi^-\rangle,|\psi^+\rangle,|\psi^-\rangle\}$ as $\{0+,1-,0-,1+\}$, respectively. Here the codes $\{+,-\}$ can be used to represent the binary numbers $\{0,1\}$, respectively. For sharing an unknown quantum state, the case is similar to that for the controlled teleportation of an arbitrary two-particle state, and the agents will recover the unknown state when they collaborate.

TABLE IV. The relation between the results of the Bell-basis measurements taken by Alice and Charlie$_i$ and the state Ψ_f of the particles a_{n+2} and σ_z when the number of the controllers is even.

V_{total}	P_{total}	Ψ_f
0	+	$(U_0\otimes U_0)\Psi_c$
0	−	$(U_0\otimes U_1)\Psi_c$
1	+	$(U_0\otimes U_2)\Psi_c$
1	−	$(U_0\otimes U_3)\Psi_c$

For sharing classical information, Alice encodes her message (a random key or a classical secret) on her two-particle quantum state. For the convenience of the measurements, it requires that the final state of Bob's particles can be measured deterministically if all the controllers, say Charlie$_i$, perform the Bell-basis measurements on their particles, as in QSDC [38–42]. In other words, all the input states should be orthogonal. Then the quantum system composed of two particles prepared by Alice for coding the classical information is in one of the four Bell states or in EPR-class states, i.e.,

$$|\Phi'^{\pm}\rangle_{xy}=\alpha|uv\rangle\pm\beta|\overline{u}\overline{v}\rangle, \quad (21)$$

where $u,v\in\{|0\rangle,|1\rangle,|+x\rangle,|-x\rangle\}$. Without loss of generality, we suppose that the state $|\Psi\rangle_c$ prepared for carrying the classical secret is one of the four Bell states $\{|\phi^{\pm}\rangle,|\psi^{\pm}\rangle\}$. Alice prepares two GHZ states $|\Psi\rangle_{s_1}$ and $|\Psi\rangle_{s_2}$ as the quantum channel. And the state of the composite quantum system is

$$|\Psi\rangle_s\equiv|\Psi\rangle_c\otimes|\Psi\rangle_{s_1}\otimes|\Psi\rangle_{S_2}, \quad (22)$$

where

$$|\Psi\rangle_{s_1}=\frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n+2}|0\rangle_{a_i}+\prod_{i=1}^{n+2}|1\rangle_{a_i}\right), \quad (23)$$

$$|\Psi\rangle_{s_2}=\frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n+2}|+x\rangle_{b_i}+\prod_{i=1}^{n+2}|-x\rangle_{b_i}\right). \quad (24)$$

Alice and Charlie$_i$ all perform Bell-basis measurements on their particles. Bob will perform Bell-basis measurement on his two particles a_{n+2} and b_{n+2} when the number of the controllers, Charlie$_i$, is even, otherwise Bob will take a joint measurement $\sigma_x\otimes\sigma_z$ on his particles (that is, he take a σ_x measurement on the particle a_{n+2} and σ_z on the particle b_{n+2}) as the final states Ψ_f of Bob's particles in these two cases are different.

The relation between the final state Ψ_f of Bob's particles and the original entangled state Ψ_c is shown in Tables IV and V for the cases that the number of controllers is even and odd, respectively. When the n controllers and Bob want to reconstruct the classical secret, they collaborate to decode the message with the information published by Alice, according to Tables IV and V.

The difference between this QSS protocol for a classical secret and the symmetric multiparty-controlled teleportation discussed above is that the input states are orthogonal and all the agents take the measurements on their particles in the former.

A piece of quantum information can be an arbitrary state of a quantum system. For a two-particle quantum state, an arbitrary state can be an entangled state in the general form shown in Eq. (4). With the symmetric multiparty-controlled teleportation discussed above, quantum secret sharing for an entangled state is easily implemented in principle in the same way. Moreover, each of the $n+1$ agents can act as the person who will reconstruct the quantum information with the help of the others in the network.

V. DISCUSSION AND SUMMARY

In the symmetric multiparty-controlled teleportation, the second GHZ state is prepared along the x direction. It can also be prepared along the z direction, like the first GHZ state, i.e.,

$$
\begin{aligned}
|\Psi\rangle_s &\equiv |\Phi\rangle_{xy} \otimes |\Psi\rangle_{s_1} \otimes |\Psi\rangle_{s_2} \\
&= \frac{1}{\sqrt{2}}(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{xy} \\
&\quad \otimes \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n+2}|0\rangle_{a_i} + \prod_{i=1}^{n+2}|1\rangle_{a_i}\right) \\
&\quad \otimes \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{n+2}|0\rangle_{b_i} + \prod_{i=1}^{n+2}|1\rangle_{b_i}\right). \qquad (25)
\end{aligned}
$$

At this time, Alice and $n-1$ controllers do the Bell-basis measurements directly on their particles, and the last controller, say Charlie$_n$ first takes a H operation on her second particle b_{n+1} and then performs the Bell-basis measurement on her two particles. Similar to the case above, Bob can also recover the original entangled state with the unitary operations. As for sharing of a classical secret, Alice can also prepare the two GHZ states along the z direction, and all the persons in the communication perform Bell-basis measurements on their particles without the H operation.

TABLE V. The relation between the results of the Bell-basis measurements and the state Ψ_f when the number of the controllers is odd.

V_{total}	P_{total}	Ψ_f
0	+	$(\text{CNOT}+U_0\otimes U_0)\Psi_c$
0	−	$(\text{CNOT}+U_0\otimes U_1)\Psi_c$
1	+	$(\text{CNOT}+U_0\otimes U_2)\Psi_c$
1	−	$(\text{CNOT}+U_0\otimes U_3)\Psi_c$

In summary, we present a method for symmetric multiparty-controlled teleportation of an arbitrary two-particle entangled state with two GHZ states and Bell-basis measurements. Any one in the $n+1$ agents can reconstruct the original entangled state with the help of the other n agents in the network and the information published by the sender Alice. To this end, Alice prepares two $(n+2)$-particle GHZ states along the z direction and the x direction, respectively. When the number of the controllers is even, the receiver, say Bob, need only perform two local unitary operations on his particles to obtain the original entangled state with the help of the n controllers in the network; otherwise, he has to do a CNOT operation on his particles in addition to the local unitary operations. This method for a symmetric multiparty-controlled teleportation can also be used to share classical information and an arbitrary two-particle state with just a little modification. As the whole quantum source is used to carry the useful quantum information, the efficiency for qubits approaches the maximal value and the procedure for controlled teleportation is an optimal one.

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