Entanglement, Bell violation, and phase decoherence of two atoms inside an optical cavity

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The simple system of two atoms couple to single mode optical cavity with the phase decoherence is studied for investigating the entanglement and Bell violation between atoms and cavity or between two atoms. We show that in the resonance case (i) atom-field entanglement rapidly decays with phase decoherence and disappears in the stationary state, (ii) atom-atom entanglement is more robust against phase decoherence and survives in the stationary state. In the nonresonance case, the pairwise atom-atom and atom-field entanglement is sensitive to the detuning parameter and is not completely destroyed during evolution. On the other hand, violation of Bell-CHSH inequality is very fragile against the phase decoherence and finally disappears in the stationary state. The phenomenon that the more Bell violation, the less entanglement, or vice versa in such a realistic physical system, is revealed. This phenomenon maybe is the consequence of the choice of concurrence as the entanglement measure and the observables to build the Bell-CHSH inequality. The genuine three-partite entanglement is also analyzed by making use of the state preparation fidelity. It is shown that the genuine three-partite entanglement can appear in the evolution of the system even in the presence of the phase decoherence.

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I. INTRODUCTION

The rapidly increasing in quantum information processing has stimulated the interest of studying quantum entanglement [1]. Entanglement can exhibit the nature of a nonlocal correlation between quantum systems that have no classical interpretation. However, real quantum systems will unavoidably be influenced by surrounding environments. The interaction between the environment and quantum systems of interest can lead to decoherence. It is therefore of great importance to prevent or minimize the influence of environmental noise in the practical realization of quantum information processing [2]. Instead of attempting to shield the system from the environmental noise, Plenio and Huelge [3] use white noise to play a constructive role and generate the controllable entanglement by incoherent sources. Similar work on this aspect has also been considered by other authors [4]. The creation of entangled atom-photon states in a Bose-Einstein condensate superradiance experiment has been discussed [5,6]. In this paper, we investigate two two-level atoms coupled to a single mode optical cavity with the phase decoherence and show the rich dynamical features of entanglement arising between atoms and cavity or between two atoms with the phase decoherence. The explicit expression of the density matrix for the system is found and used to calculate the concurrence characterizing the entanglement between two atoms or between atoms and cavity field. We find that in the resonance case (i) atom-field entanglement rapidly decays with phase decoherence and disappears in the stationary state, (ii) atom-atom entanglement is more robust against phase decoherence and survives in the stationary state. In the nonresonance case, the pairwise entanglement between various subsystems is sensitive with the detuning parameter and not completely destroyed by the phase decoherence. Furthermore, we show that even if one of the atoms is initially prepared in a maximally mixed state, it can still entangle with another atom. The Bell violation is also discussed. It is found that two atoms possibly achieve more violation, but less entanglement, or vice versa. Finally, we investigate the genuine three-partite entanglement in this system by exploring the state preparation fidelity which is related to the sufficient condition that distinguish between the genuinely three-partite entangled states and those in which only two-partite is entangled [7,8]. It is shown that the genuine three-partite entanglement can appear in the evolution of the system even in the presence of the phase decoherence.

This paper is organized as follows. In Sec. II, we obtain the explicit analytical solution of two atoms inside an optical cavity with phase decoherence. In Sec. III, the pairwise entanglement between atoms and cavity or between two atoms is investigated, and the nonlocality of two atoms is also discussed. In Sec. IV, we investigate the genuine three-partite entanglement in this system by exploring the state preparation fidelity. In Sec. V, there are some concluding remarks.

II. SOLUTION OF TWO ATOMS INSIDE OPTICAL CAVITY WITH PHASE DECOHERENCE

We consider the situation that two atoms are trapped inside a single mode optical cavity initially prepared in the vacuum state. The Hamiltonian for the system can be written as [9,10] ($\hbar=1$),

$$H=\frac{\omega_0}{2}\sigma_z^{(1)}+\frac{\omega_0}{2}\sigma_z^{(2)}+\omega a^\dagger a+g(a\sigma_+^{(1)}+a^\dagger\sigma_-^{(1)})+g(a\sigma_+^{(2)}+a^\dagger\sigma_-^{(2)}), \tag{1}$$

where $\sigma_z^{(i)},\sigma_\pm^{(i)}$ $(i=1,2)$ are atomic operators, ω_0 is atomic

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transition frequency, g is the coupling constant of the atoms to cavity field, and $a(a^\dagger)$ is the annihilation (creation) operator of cavity field with frequency ω. The generation of the entangled state in the system (1) in the laboratory has been implemented [10]. Various modifications and generalizations of the system (1) have been studied for preparing entangled states or realizing various kinds of quantum information processes [11–14]. In this paper, we investigate the entanglement between atoms and cavity or between two atoms by considering the pure phase decoherence mechanism only. In this situation, the master equation governing the time evolution for the system under the Markovian approximation is given by [15]

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{\gamma}{2}[H,[H,\rho]], \tag{2}$$

where γ is the phase decoherence rate. Note that the equation with the similar form has been proposed to describe the intrinsic decoherence [16]. The formal solution of the master equation (2) can be expressed as follows [17]:

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\gamma t)^k}{k!} M^k(t)\rho(0)M^{\dagger k}(t), \tag{3}$$

where $\rho(0)$ is the density opera-tor of the initial atom-field system and $M^k(t)$ is defined by

$$M^k(t) = H^k \exp(-iHt)\exp\left(-\frac{\gamma t}{2}H^2\right). \tag{4}$$

As adopted in Ref. [10], we assume that the cavity field is prepared initially in the vacuum state $|0\rangle$, and atom 1 is prepared in the excited state $|e\rangle$ and atom 2 is in the ground state $|g\rangle$, i.e.,

$$\rho(0) = |0\rangle\langle 0| \otimes |eg\rangle\langle eg|. \tag{5}$$

This kind of initial condition can make two atoms achieve the nearly maximally entangled state in the evolution in the large detuning limit. Substituting $\rho(0)$ into Eq. (3), the time evolution of $\rho(t)$ can be obtained as follows:

$$\rho(t) = \rho_{0,+}|0\rangle\langle 0| \otimes |B_+\rangle\langle B_+| + \rho_{1,gg}|1\rangle\langle 1| \otimes |gg\rangle\langle gg| + \frac{1}{4}|0\rangle\langle 0| \otimes |B_-\rangle\langle B_-| + \rho_{+gg}|0\rangle\langle 1| \otimes |B_+\rangle\langle gg| + \rho_{-gg}|0\rangle\langle 1| \otimes |B_-\rangle \times\langle gg| + \rho_{+-}|0\rangle\langle 0| \otimes |B_+\rangle\langle B_-| + \mathrm{H.c.},$$

$$\rho_{0,+} = \frac{1}{8}\left[1 + \frac{\Delta^2}{\Omega^2} + \left(1 - \frac{\Delta^2}{\Omega^2}\right)\cos\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right],$$

$$\rho_{1,gg} = \frac{g^2}{\Omega^2}\left[1 - \cos\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right],$$

$$\rho_{+gg} = \frac{\sqrt{2}g}{2\Omega}\left\{\frac{\Delta}{\Omega}\left[1 - \cos\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right] + i\sin\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right\},$$

$$\rho_{-gg} = \frac{\sqrt{2}g}{2\Omega}\left[\exp\left(\frac{i\Omega t - i\Delta t}{2}\right)\exp\left(-\frac{\gamma t}{8}(\Omega-\Delta)^2\right) - \exp\left(\frac{-i\Omega t - i\Delta t}{2}\right)\exp\left(-\frac{\gamma t}{8}(\Omega+\Delta)^2\right)\right],$$

$$\rho_{+-} = \frac{1}{4}\left[\left(1 - \frac{\Delta}{\Omega}\right)e^{i(\Omega+\Delta)t/2}\exp\left(-\frac{\gamma t}{8}(\Omega+\Delta)^2\right) + \left(1 + \frac{\Delta}{\Omega}\right)e^{-i(\Omega-\Delta)t/2}\exp\left(-\frac{\gamma t}{8}(\Omega-\Delta)^2\right)\right], \tag{6}$$

where $\Delta = \omega_0 - \omega$ is the detuning between the atoms and cavity field, $\Omega = (\Delta^2 + 8g^2)^{1/2}$, and $|B_\pm\rangle = (\sqrt{2}/2)(|eg\rangle \pm |ge\rangle)$ are the Bell states. By tracing out the degree of freedom of the cavity field, we obtain the reduced density matrix $\rho_s(t)$ describing the subsystem containing only two atoms,

$$\rho_s(t) = \rho_{s++}|B_+\rangle\langle B_+| + \frac{1}{4}|B_-\rangle\langle B_-| + \rho_{sgg}|gg\rangle\langle gg| + \rho_{s+-}|B_+\rangle\langle B_-| + \mathrm{H.c.},$$

$$\rho_{s++} = \frac{1}{8}\left[1 + \frac{\Delta^2}{\Omega^2} + \left(1 - \frac{\Delta^2}{\Omega^2}\right)\cos\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right],$$

$$\rho_{sgg} = \frac{g^2}{\Omega^2}\left[1 - \cos\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right],$$

$$\rho_{s+-} = \frac{1}{4}\left[\left(1 - \frac{\Delta}{\Omega}\right)e^{i(\Omega+\Delta)t/2}\exp\left(-\frac{\gamma t}{8}(\Omega+\Delta)^2\right) + \left(1 + \frac{\Delta}{\Omega}\right)e^{-i(\Omega-\Delta)t/2}\exp\left(-\frac{\gamma t}{8}(\Omega-\Delta)^2\right)\right]. \tag{7}$$

III. ENTANGLEMENT BETWEEN ATOMS AND CAVITY OR BETWEEN TWO ATOMS

In order to quantify the degree of entanglement, several measures [18] of entanglement have been introduced for both pure and mixed quantum states. In this paper, we adopt the concurrence to calculate the entanglement between atom and cavities or between two optical cavities with the phase decoherence. The concurrence related to the density operator ρ of a mixed state is defined by [19]

$$C(\rho) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{8}$$

where the $\lambda_i (i=1,2,3,4)$ are the square roots of the eigenvalues in decreasing order of magnitude of the "spin-flipped" density operator R,

$$R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{9}$$

where the asterisk indicates complex conjugation. The concurrence varies from $C=0$ for an unentangled state to $C=1$ for a maximally entangled state.

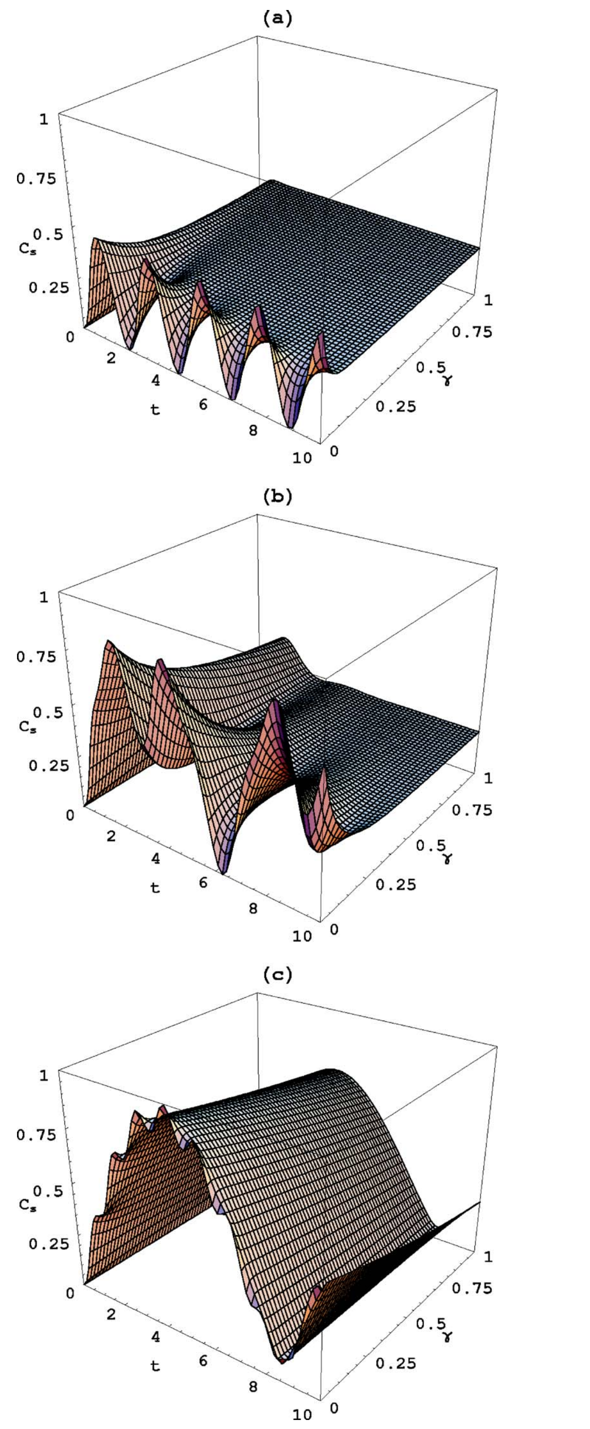

FIG. 1. (Color online) The concurrence C_s is plotted as a function of the time t and phase damping rate γ for three different cases, (a) $\Delta=0g$; (b) $\Delta=g$; (c) $\Delta=5g$. In the three cases, the units of t and γ are both $1/g$.

The explicit analytical expression of the concurrence $C_s(t)$ characterizing the entanglement in $\rho_s(t)$ can be obtained as

$$C_s(t)=(A^2+B^2)^{1/2},$$

$$A=\frac{\Delta^2}{4\Omega^2}-\frac{1}{4}+\frac{1}{4}\left(1-\frac{\Delta^2}{\Omega^2}\right)\cos\Omega t\,\exp\left(-\frac{\gamma t}{2}\Omega^2\right),$$

$$B=\frac{1}{2}\left(1-\frac{\Delta}{\Omega}\right)\sin(\Omega+\Delta)t/2\,\exp\left(-\frac{\gamma t}{8}(\Omega+\Delta)^2\right)-\frac{1}{2}\left(1+\frac{\Delta}{\Omega}\right)\sin(\Omega-\Delta)t/2\,\exp\left(-\frac{\gamma t}{8}(\Omega-\Delta)^2\right). \tag{10}$$

In Fig. 1, we plot the concurrence C_s as the function of time t and phase decoherence rate γ for three values of the detuning parameter Δ. It is shown from Fig. 1(a) that the concurrence is always larger than zero for any time $t>0$ in the case with $\gamma\neq 0$, which means that the phase decoherence does not completely destroy the entanglement but generates a stationary entangled state of two atoms. We also see that the concurrence is not larger that 0.5, which is different with the nonresonant case [see Figs. 1(b) and 1(c)]. From Fig. 1(c), we can observe that two atoms can get very large entanglement in the large detuning case and the influence of phase decoherence on the entanglement generation of two atoms is strongly dependent on the detuning. The relation between entanglement generation of two atoms and the detuning for two different values of the phase decoherence rate is displayed in Fig. 2. Similar to the result that the phase decoherence suppresses the revival and collapse of Rabi oscillation in Ref. [17], it is shown that the phase decoherence suppress the oscillation of entanglement in this case. From Eq. (10), it is easy to verify that $C_s(\infty)=2g^2/(\Delta^2+8g^2)$ in the case with $\gamma\neq 0$, which means that the entanglement of stationary state decreases with the increase of the detuning.

In the large detuning limit, i.e., $g/|\Delta|\ll 1$, the population of the single mode cavity field will be very small in the time evolution, which leads to very small entanglement between atoms and cavity. Next, we investigate how entanglement is distributed in system (1). By tracing out the degree of freedom of atom 2 in $\rho(t)$, we obtain the reduced density matrix $\rho_{s1}(t)$ describing the subsystem of atom 1 and the cavity field,

$$\rho_{s1}(t)=\rho_{s10e}|0\rangle\langle 0|\otimes|e\rangle\langle e|+\rho_{s10g}|0\rangle\langle 0|\otimes|g\rangle\langle g|+\rho_{s11g}|1\rangle\times\langle 1|\otimes|g\rangle\langle g|+(\rho_{s1eg}|0\rangle\langle 1|\otimes|e\rangle\langle g|+\mathrm{H.c.}),$$

$$\rho_{s10e}=\left\{\frac{1}{8}\left[3+\frac{\Delta^2}{\Omega^2}+\left(1-\frac{\Delta^2}{\Omega^2}\right)e^{-(\gamma t/2)\Omega^2}\cos\Omega t\right]+\frac{1}{4}\left(1-\frac{\Delta}{\Omega}\right)e^{-(\gamma t/8)(\Omega+\Delta)^2}\cos\frac{(\Omega+\Delta)t}{2}+\frac{1}{4}\left(1+\frac{\Delta}{\Omega}\right)e^{-(\gamma t/8)(\Omega-\Delta)^2}\cos\frac{(\Omega-\Delta)t}{2}\right\},$$

$$\rho_{s10g}=\left\{\frac{1}{8}\left[3+\frac{\Delta^2}{\Omega^2}+\left(1-\frac{\Delta^2}{\Omega^2}\right)e^{-(\gamma t/2)\Omega^2}\cos\Omega t\right]-\frac{1}{4}\left(1-\frac{\Delta}{\Omega}\right)e^{-(\gamma t/8)(\Omega+\Delta)^2}\cos\frac{(\Omega+\Delta)t}{2}-\frac{1}{4}\left(1+\frac{\Delta}{\Omega}\right)e^{-(\gamma t/8)(\Omega-\Delta)^2}\cos\frac{(\Omega-\Delta)t}{2}\right\},$$

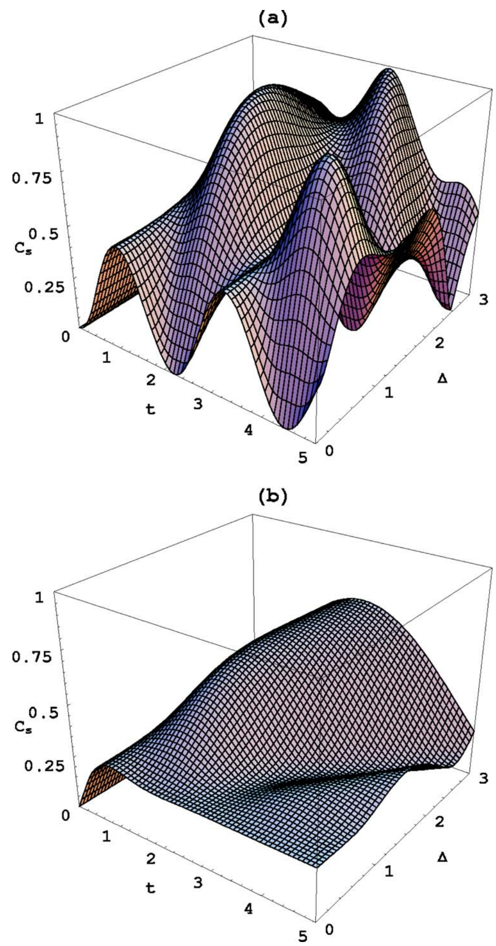

FIG. 2. (Color online) The concurrence C_s is plotted as a function of the time t and the detuning Δ in two different cases, (a) $\gamma=0/g$; (b) $\gamma=0.5/g$. In the cases, the units of t and Δ are $1/g$ and g, respectively.

$$\rho_{s11g}=\frac{2g^2}{\Omega^2}(1-e^{-(\gamma t/2)\Omega^2}\cos\Omega t),$$

$$\rho_{s1eg}=\frac{g}{2\Omega}\left(\frac{\Delta}{\Omega}-\frac{\Delta}{\Omega}e^{-(\gamma t/2)\Omega^2}\cos\Omega t+ie^{-(\gamma t/2)\Omega^2}\sin\Omega t+e^{(it/2)(\Omega-\Delta)-(\gamma t/8)(\Omega-\Delta)^2}-e^{-(it/2)(\Omega+\Delta)-(\gamma t/8)(\Omega+\Delta)^2}\right). \tag{11}$$

In the basis $\{|1\rangle|e\rangle,|1\rangle|g\rangle,|0\rangle|e\rangle,|0\rangle|g\rangle\}$, $\rho_{s1}(t)$ can be also regarded as a two-qubit mixed state. The concurrence C_{s1} measuring the entanglement of such a subsystem can be written as follows:

$$C_{s1}(t)=|\mu+\nu|,$$

$$\mu=\frac{g\Delta}{\Omega^2}-\frac{g\Delta}{\Omega^2}e^{-(\gamma t/2)\Omega^2}\cos\Omega t+\frac{ig}{\Omega}e^{-(\gamma t/2)\Omega^2}\sin\Omega t,$$

$$\nu=\frac{g}{\Omega}e^{(it/2)(\Omega-\Delta)-(\gamma t/8)(\Omega-\Delta)^2}-\frac{g}{\Omega}e^{-(it/2)(\Omega+\Delta)-(\gamma t/8)(\Omega+\Delta)^2}, \tag{12}$$

where $|x|$ gives the absolute value of x. Similarly, we can also investigate the entanglement of subsystem containing only cavity field and atom 2 by tracing out the degree of freedom of the atom 1. The corresponding concurrence C_{s2} is obtained,

$$C_{s2}=|\mu-\nu|. \tag{13}$$

In the resonance case, i.e., $\Delta=0$, C_{s1} and C_{s2} reduce to

$$C_{s1}=\left|\frac{\sqrt{2}}{2}\sin\sqrt{2}gt[\exp(-\gamma g^2t)+\cos\sqrt{2}gt\exp(-4\gamma g^2t)]\right| \tag{14}$$

and

$$C_{s2}=\left|\frac{\sqrt{2}}{2}\sin\sqrt{2}gt[\exp(-\gamma g^2t)-\cos\sqrt{2}gt\exp(-4\gamma g^2t)]\right|. \tag{15}$$

Equations (14) and (15) indicate that, in the case with $\Delta=0$ and $\gamma\neq 0$, there are not any entanglements between the cavity field and atom 1 or between the cavity field and atom 2 as the time approach to infinite. However, in the nonresonance case, i.e., $\Delta\neq 0$ and $\gamma\neq 0$, $C_{s1}(\infty)=C_{s2}(\infty)=|g\Delta/\Omega^2|$.

In Figs. 3 and 4, the concurrence C_{s1} and C_{s2} are plotted as the functions of the time t for three different values of detuning parameter. It is shown that, although the optimal values of concurrence C_{s1} and C_{s2} in the case of $\Delta\neq 0$ with phase decoherence are smaller than those without phase decoherence, the phase decoherence does not completely destroy the entanglement. We also see that the individual atom and cavity field can not get maximally entangled. It is easy to prove that the optimal value of concurrence can not exceed $3\sqrt{6}/8$. In the case $\Delta\neq 0$ and $\gamma\neq 0$, the atom and field is almost entangled for $t>0$, which is different from the case of $\Delta=0$ and $\gamma=0$. C_s, C_{s1}, and C_{s2} are displayed as the functions of the time for $\Delta=10g$ in Fig. 5. From Fig. 5, we can see that the concurrence C_s characterizing the pairwise entanglement between two atoms can achieve a very large value even in the presence of phase decoherence, which is similar to the case without phase decoherence. It can be verified that, in the large detuning limit, the maximal value of the concurrence between atoms arrives near 1 in the time evolution even in the presence of phase decoherence. Moreover, we show that C_{s1} and C_{s2} exhibit very different dynamical behavior with or without phase decoherence. In Fig. 5(a), the pairwise entanglement between the cavity field and atom 1 or between the cavity field and atom 2 exhibits synchronous oscillation, which is different from Fig. 5(b).

Furthermore, we discuss how much pairwise entanglement between the various subsystems can be achieved if the initial atom 1 is in a thermal state. We assume that the initial atom 1 is in the state $\rho_A(0)=\delta|g\rangle\langle g|+(1-\delta)|e\rangle\langle e|$, where $0\leqslant\delta\leqslant 1$, atom 2 is in the ground state, and the cavity field is still in the vacuum state $|0\rangle$. Our calculation shows that C_s'

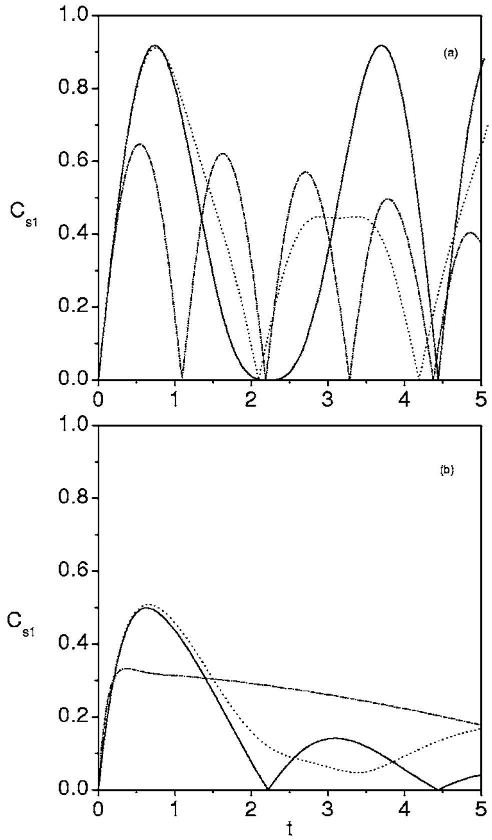

FIG. 3. The concurrence C_{s1} is plotted as a function of time t, (a) $\gamma=0/g$; solid line: $\Delta=0g$; dotted line: $\Delta=g$; dash-dotted line: $\Delta=5g$; (b) $\gamma=0.5/g$; solid line: $\Delta=0g$; dotted line: $\Delta=g$; dash-dotted line: $\Delta=5g$. The unit of t is $1/g$.

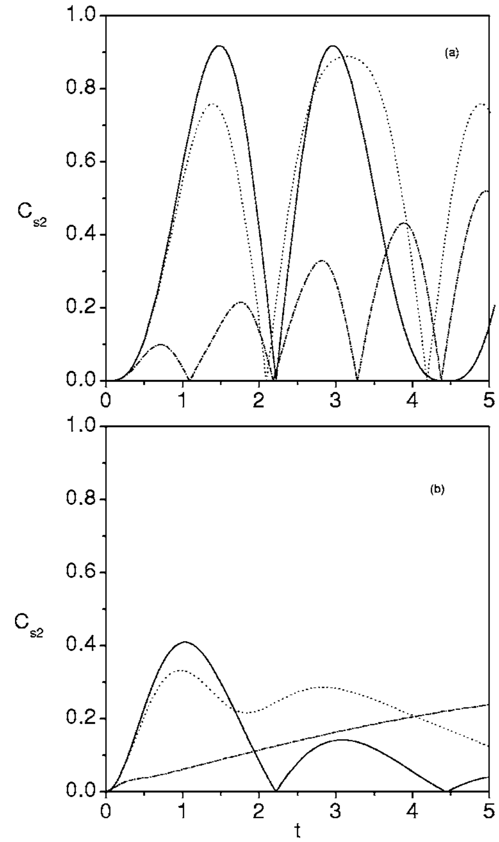

FIG. 4. The concurrence C_{s2} is plotted as a function of time t, (a) $\gamma=0/g$; solid line: $\Delta=0g$; dotted line: $\Delta=g$; dash-dotted line: $\Delta=5g$; (b) $\gamma=0.5/g$; solid line: $\Delta=0g$; dotted line: $\Delta=g$; dash-dotted line: $\Delta=5g$. The unit of t is $1/g$.

$=(1-\delta)C_s$, $C'_{s1}=(1-\delta)C_{s1}$, and $C'_{s2}=(1-\delta)C_{s2}$. This means that even if the initial atom 1 is in a maximally mixed state $\frac{1}{2}|g\rangle\langle g|+\frac{1}{2}|e\rangle\langle e|$, it can still entangle various subsystems.

In what follows, we discuss the nonlocality of two atoms in this system. Bell's inequality test with entangled atoms inside a cavity has been extensively studied [20]. The nonlocal property of two atoms can be characterized by the maximal violation of Bell's inequality. The most commonly discussed Bell's inequality is the CHSH inequality [21,22]. The CHSH operator reads

$$\hat{B}=\vec{a}\cdot\vec{\sigma}\otimes(\vec{b}+\vec{b'})\cdot\vec{\sigma}+\vec{a'}\cdot\vec{\sigma}\otimes(\vec{b}-\vec{b'})\cdot\vec{\sigma}, \quad (16)$$

where $\vec{a},\vec{a'},\vec{b},\vec{b'}$ are unit vectors. In the above notation, the Bell inequality reads

$$|\langle\hat{B}\rangle|\leq 2. \quad (17)$$

The maximal amount of Bell's violation of a state ρ is given by [23]

$$\mathcal{B}=2\sqrt{\lambda+\tilde{\lambda}}, \quad (18)$$

where λ and $\tilde{\lambda}$ are the two largest eigenvalues of $T_\rho^\dagger T_\rho$. The matrix T_ρ is determined completely by the correlation functions being a 3×3 matrix whose elements are $(T_\rho)_{nm}$ $=\mathrm{Tr}(\rho\sigma_n\otimes\sigma_m)$. Here, $\sigma_1\equiv\sigma_x$, $\sigma_2\equiv\sigma_y$, and $\sigma_3\equiv\sigma_z$ denote the usual Pauli matrices. We call the quantity $\mathcal{B}$ the maximal violation measure, which indicates the Bell violation when $\mathcal{B}>2$ and the maximal violation when $\mathcal{B}=2\sqrt{2}$. For the density operator ρ_s in Eq. (7) characterizing the time evolution of two atoms, $\lambda+\tilde{\lambda}$ can be written as follows:

$$\lambda+\tilde{\lambda}=\varsigma+\max[\varsigma,\zeta], \quad (19)$$

where

$$\varsigma=\frac{4g^4}{\Omega^4}(1-e^{-(\gamma t/2)\Omega^2}\cos\Omega t)^2 +\frac{1}{4}\left[\left(1-\frac{\Delta}{\Omega}\right)e^{-(\gamma t/8)(\Omega+\Delta)^2}\sin\frac{(\Omega+\Delta)t}{2} -\left(1+\frac{\Delta}{\Omega}\right)e^{-(\gamma t/8)(\Omega-\Delta)^2}\sin\frac{(\Omega-\Delta)t}{2}\right]^2, \quad (20)$$

and

$$\zeta=\left(\frac{\Delta^2+4g^2}{\Omega^2}+\frac{4g^2}{\Omega^2}e^{-(\gamma t/2)\Omega^2}\cos\Omega t\right)^2. \quad (21)$$

From Eqs. (18)–(21), it is easy to see the violation of Bell's inequality for two atoms. Similarly, one can also obtain the analytical expressions of maximal violation of Bell's in-

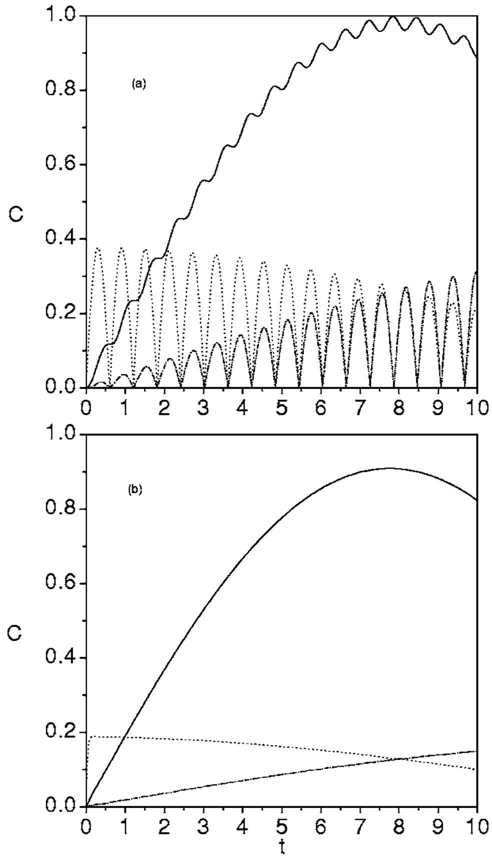

FIG. 5. The concurrences C_s (solid line), C_{s1} (dotted line), and C_{s2} (dash-dotted line) are plotted as a function of time t with $\Delta = 10g$: (a) $\gamma = 0/g$; (b) $\gamma = 0.5/g$.

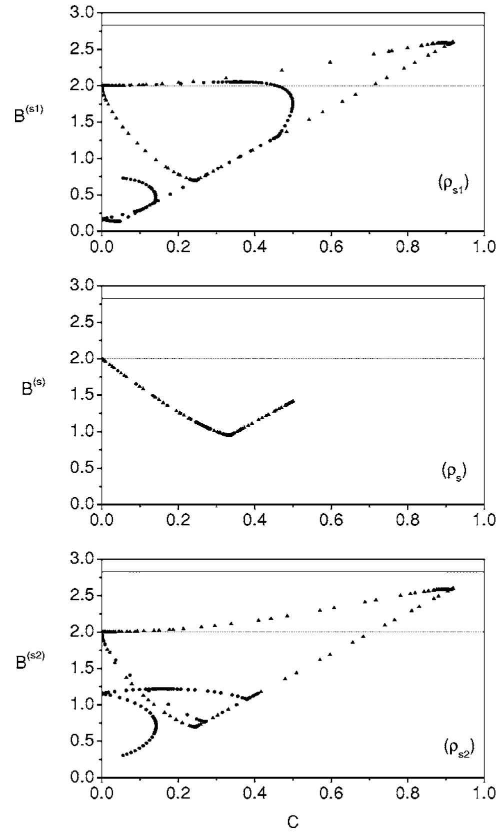

FIG. 6. The maximal violations versus concurrences for three subsystems ρ_s, ρ_{s1}, and ρ_{s2} during the time interval $t \in [0,4]$ are depicted with $\Delta = 0g$, and two different decoherence rates, i.e., $\gamma = 0/g$ (solid triangle) and $\gamma = 0.5/g$ (solid circle). The unit of t is $1/g$.

equality $\mathcal{B}^{(s1)}$ and $\mathcal{B}^{(s2)}$ of atom 1 and the cavity field or atom 2 and the cavity field, respectively. In the analysis of the Bell violation concerning the degree of freedom of the cavity field, the three components of dichotomous observables of the cavity field are analogously defined as $\sigma_x^{(c)} \equiv |1\rangle\langle 0| + |0\rangle\langle 1|$, $\sigma_y^{(c)} \equiv -i|1\rangle\langle 0| + i|0\rangle\langle 1|$, and $\sigma_z^{(c)} \equiv |1\rangle\langle 1| - |0\rangle\langle 0|$.

Recently, Verstraete *et al.* investigated the relations between the violation of the CHSH inequality and the concurrence for systems of two qubits [24]. They showed that the maximal value of $\mathcal{B}$ for given concurrence C is $2\sqrt{1+C^2}$, which can be achieved by the pure states and some Bell-diagonal states. If the given concurrence C is larger than $\sqrt{2}/2$, the minimal value of $\mathcal{B}$ is $2\sqrt{2}C$, which can be achieved by the maximal entangled mixed state. Furthermore, the entangled two-qubits state with the concurrence $C \leqslant \sqrt{2}/2$ may not violate any CHSH inequality, even after all possible local filtering operations, except their Bell-diagonal normal form does violate the CHSH inequalities [24]. In what follows, we attempt to reveal the relations between the entanglement and Bell violation in the system (1). First, we focus our attention on the resonant case. Our calculations show that two atoms cannot violate the CHSH inequality in this case, though two atoms get entangled in the time evolution. However, the Bell violation of the individual atom 1 or 2 and the cavity field can emerge in this case, whereas it can be destroyed by the decoherence. In Fig. 6, we display the Bell violation versus the concurrence for three subsystems in the resonant case. One interesting point noted from Fig. 6 is that the Bell violation and the concurrence does not satisfy the monotonous relation. It means that possibly the more violation, the less concurrence, or vice versa. For realizing the quantum information processing in the system of atoms coupling to cavity field, the large detuning case is often adopted to suppress the cavity decay [9,12]. In Fig. 7, the Bell violation versus the concurrence for three subsystems in the large detuning case is depicted. It is shown that very large violation of CHSH inequality can be achieved by two atoms in this case. Meanwhile, the Bell violations of the individual atom 1 or 2 and the cavity field deteriorate in the large detuning case. One may question whether there exists an infinitesimal time evolution in which two atoms can achieve more entanglement, but less violation. The answer is yes, and it can be confirmed by observing the middle graph in Fig. 7. The relation between the Bell violation and the entanglement may strongly depend on the choices of the observables to build the Bell inequality and the entanglement measure. So it is very interesting to study further whether this phenomenon is still valid or not for other forms of Bell's inequalities and other entanglement measures.

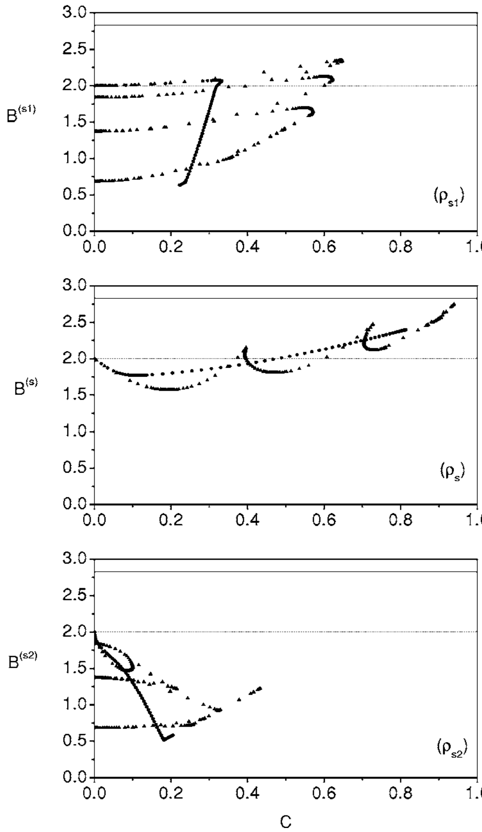

FIG. 7. The maximal violations versus concurrences for three subsystems ρ_s, ρ_{s1}, and ρ_{s2} during the time interval $t \in [0,4]$ are depicted with $\Delta = 5g$, and two different decoherence rates, i.e., $\gamma = 0/g$ (solid triangle) and $\gamma = 0.5/g$ (solid circle). The unit of t is $1/g$.

IV. GENUINE THREE-PARTITE ENTANGLEMENT IN THIS SYSTEM

In this section, we turn to investigate whether the genuine three-partite entanglement can appear or not in the evolution of this system. For discriminating the genuine three-partite entangled states from other entangled states or separable states, there have been many criterions such as the Mermin-Klyshko inequality [25,26] and state preparation fidelity [7,8], etc. The Mermin-Klyshko inequality and state preparation fidelity are related to two sufficient conditions that distinguish between genuinely N-partite entangled states and those in which only M particles are entangled ($M < N$). Here, we adopt the state preparation fidelity to study how the three-partite entangled states can be generated in this system. For a three-qubit state ρ, the state preparation fidelity F is defined as

$$F(\rho) = \langle \mathbf{GHZ} | \rho | \mathbf{GHZ} \rangle, \tag{22}$$

where $|\mathbf{GHZ}\rangle$ is the three-partite GHZ state. A sufficient condition for genuine three-partite entanglement is given by

$$F(\rho) > \frac{1}{2}. \tag{23}$$

Now we consider the state described by the density matrix $\rho(t)$ in Eq. (6) and choose the GHZ state $|\mathbf{GHZ}\rangle$ to be

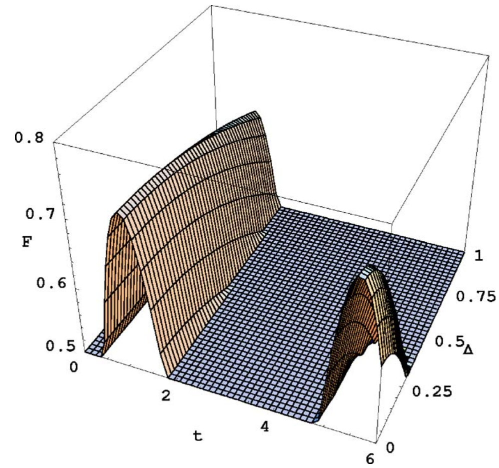

FIG. 8. (Color online) The state preparation fidelity F of the evolving state $\rho(t)$ is plotted as the function of the time t and the detuning Δ for the case without any phase decoherence, i.e. $\gamma = 0/g$ The units of$t$ and Δ are $1/g$ and g respectively.

$$|\mathbf{GHZ}\rangle = \frac{1}{4}[(|0\rangle - i|1\rangle) \otimes (|g\rangle + |e\rangle) \otimes (|g\rangle - |e\rangle) - (|0\rangle + i|1\rangle) \otimes (|g\rangle - |e\rangle) \otimes (|g\rangle + |e\rangle)]. \tag{24}$$

Substituting the GHZ state $|\mathbf{GHZ}\rangle$ in Eq. (24) and the density matrix $\rho(t)$ in Eq. (6) into Eq. (22), we can obtain the state preparation fidelity F as follows:

$$F(\rho(t)) = \frac{1}{4} + \frac{g^2}{2\Omega^2}[1 - \cos(\Omega t) e^{-(\gamma t/2)\Omega^2}] + \mathrm{Re}\left(- \frac{ig}{2\Omega} (e^{(it/2)(\Omega - \Delta)} e^{-(\gamma t/8)(\Omega - \Delta)^2} - e^{-(it/2)(\Omega + \Delta)} e^{-(\gamma t/8)(\Omega + \Delta)^2}) \right), \tag{25}$$

where $\mathrm{Re}[x]$ gives the real part of a complex number x. In Fig. 8, we plot the state preparation fidelity F as the function of the time t and the detuning Δ in the case without any phase decoherence. It can be observed that F can exceed above $\frac{1}{2}$ in the evolution of this system both in the resonant case and the off-resonant case. This implies that the genuine three-partite entangled pure state can be achieved in this case. However, in the case with $\Delta \gg g$, the state preparation fidelity F cannot be larger than $\frac{1}{2}$. The physical reason is the cavity field initially in the vacuum state cannot be excited in the large detuning limit. So the degree of the freedom of the cavity field is always separable with the atoms in the large detuning limit. For clarifying the influence of the phase decoherence on the state preparation fidelity, we plot the state preparation fidelity F as the function of the time t and the decoherence rate γ in Fig. 9. It is shown that the two atoms and the cavity field which are in a three-partite mixed state can also become genuinely three-partite entangled even in the presence of phase decoherence. If the phase decoherence rate is large enough, the state preparation fidelity defined by

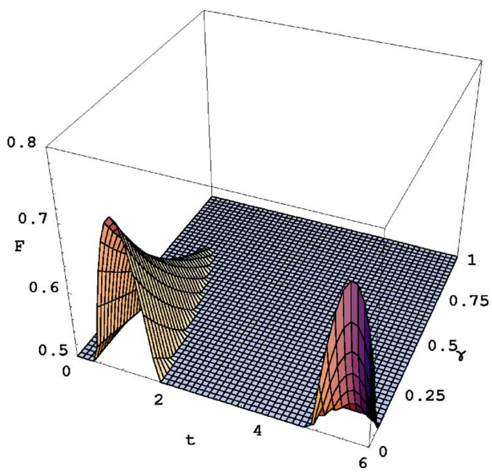

FIG. 9. (Color online) The state preparation fidelity F of the evolving state $\rho(t)$ is plotted as the function of the time t and the phase decoherence rate γ for the resonant case, i.e., $\Delta=0g$. The units of t and γ are both $1/g$.

Eqs. (22) and (24) cannot be larger than $\frac{1}{2}$ during the whole evolution. Since $F>\frac{1}{2}$ is only a sufficient condition that determining a genuinely three-partite entangled state, we need other criterions such as the violation of three-partite Bell inequalities to investigate further whether the genuine three-partite entanglement can be generated or not in the cases with very large decoherence rate.

V. CONCLUSION

In this paper, we investigate analytically the system of two two-level atoms coupled to a single mode optical cavity with the phase decoherence and calculate the entanglement between atoms and cavity field or between two atoms in the presence of phase decoherence. It is shown that in the resonance case (i) atom-field entanglement rapidly decays with phase decoherence and disappears in the stationary state, (ii) atom-atom entanglement is more robust against phase decoherence and survives in the stationary state. In the nonresonance case, the pairwise entanglement between atoms and cavity is sensitive with the detuning parameter and not completely destroyed by the phase decoherence. Furthermore, we show that even if the atom 1 is initially in a maximally mixed state, it can also be entangled with atom 2 initially prepared in ground state in this system. In addition, the nonlocality of two atoms is investigated, and the phenomenon that the more violation, the less entanglement, or vice versa, is revealed. Finally, we investigate whether the genuine three-partite entanglement can appear or not in the evolution of this system by making use of state preparation fidelity. It is shown that the two atoms and the cavity field which are in a three-partite mixed state can become genuinely three-partite entangled even in the presence of phase decoherence.

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