Decoherence in collective quantum memories for photons

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The influence of decoherence on quantum memories for photons based on atomic ensembles is discussed. It is shown that despite the large entanglement of the collective storage states, corresponding to single photons or nonclassical states of light, the sensitivity to decoherence does not scale with the number of atoms. This is due to the existence of equivalence classes of storage states, which have the same projection onto the relevant quasiparticle mode (dark-state polariton). Several decoherence processes resulting from uncorrelated individual reservoir couplings are analyzed in detail: single-atom spin flips and dephasing, atom loss, and motion of atoms. Furthermore, it is shown that the sensitivity to collective decoherence processes that affect all polariton modes with comparable strength does also not increase with the number of atoms.

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I. INTRODUCTION

One of the essential ingredients for quantum information processing with photons as information carrier [1,2] is a reliable quantum memory, capable of a faithful storage of the quantum state of photons. It plays a key role in network quantum computing [3], in long-distance, secure quantum communication, and quantum teleportation [4–8]. The application to teleportation is of particular interest because of its potentials for quantum information processing with linear optical elements [9,10]. While photons are one of the most easy to handle information carriers, atoms or similar systems like quantum dots are reliable and long-lived storage units. Furthermore, Raman transitions provide a controllable and decoherence insensitive way of coupling between light and atoms. The conceptually simplest and for processing purposes best suited storage system for photonic qubits are individual atoms. Here coherent techniques have been developed that allow a controlled transfer of quantum information from light to the atom and vice versa [3]. However, strong-coupling resonators are required in order to achieve reasonable fidelities for the transfer [11]. On the other hand, if atomic ensembles are used rather than individual atoms, no such requirements exist and coherent and reversible transfer techniques for individual photon wave packets [12–18] and cw light fields [19–23] have been proposed and in part experimentally implemented.

The substantially alleviated requirements for the light-matter interface in the case of atomic ensembles are due to the enhanced coupling between collective many-atom states and the radiation field. The corresponding collective excitations of the ensemble are highly entangled many-particle states if nonclassical states of light are stored. So while classical information encoded—e.g., in single-particle Raman coherences—can be rather robust against decoherence processes, this is not *a priori* clear for quantum correlations stored in the entire ensemble. In fact, one might naively expect that the lifetime of quantum correlations decreases with the number of atoms involved, in which case the system would be practically useless as a quantum memory. We therefore analyze in the present paper the influence of various decoherence mechanisms on the fidelity of the quantum memory. We show that each quantum state of the radiation field, stored in the atomic ensemble, corresponds to a whole class of many-particle states. These equivalence classes represent all states with the same projection onto specific quasiparticle modes: the dark-state polaritons. Because of them, the quantum memory does not show an enhanced sensitivity to *(a) individual* and *(b) homogeneous collective* decoherence processes when compared to single-particle storage units.

In order to simplify the discussion we will restrict ourselves to a quantum memory for a single-mode radiation field, realized, for example, in a weak-coupling resonator [13]. In doing so we do not need to take into account effects on the longitudinal profile of a stored pulse arising from atomic motion, which are, however, important in free-space configurations [14,24]. First we reexamine the adiabatic transfer scheme of [13,14] in terms of quasiparticles (dark and bright polaritons) in Sec. II. We will show that only specific quasiparticle modes are relevant for the storage in the adiabatic limit. In Sec. III we first discuss the effect of different *individual* decoherence mechanisms, such as random spin flips, dephasing of Raman coherences, loss of atoms, atomic motion, and imperfect preparation. It will be shown that the decoherence rate of the stored quantum state does not depend on the number of atoms in all of these cases. We then discuss the influence of *collective* dephasing processes. If these processes affect all collective modes with comparable strength, the decoherence rate in the ensemble case is again of the same order of magnitude as in the single-particle case. This is because excitations of any quasiparticle mode other than the relevant dark-polariton mode do not influence the quantum state of the readout light field, provided the readout is adiabatic. They may matter, however, if nonadiabatic couplings are taken into account. We will therefore discuss the effect of decoherence in the presence of nonadiabatic couplings in Sec. VI.

II. DARK AND BRIGHT POLARITONS: EQUIVALENCE CLASSES OF STORAGE STATES

Let us consider an ensemble of N three-level atoms with internal states $|a\rangle$, $|b\rangle$, and $|c\rangle$. The states are resonantly

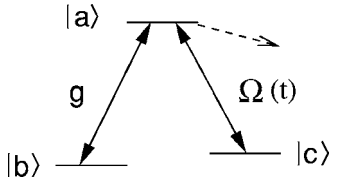

FIG. 1. Three-level atoms coupled to single-quantized resonator mode and classical control field of (real) Rabi frequency $\Omega(t)$; g-vacuum Rabi frequency; the dashed line indicates spontaneous decay.

coupled to a single-quantized mode of a resonator field with mode function e^{ik_0z} and a classical control field of Rabi frequency Ω with mode function $e^{i\mathbf{k}_1\cdot\mathbf{r}}$ as shown in Fig. 1. The dynamics of this system is described by the Hamiltonian ($E_b=\hbar\omega_b=0$)

$$H=\hbar\omega a^\dagger a+\hbar\omega_a\sum_{j=1}^{N}\sigma_{aa}^j+\hbar\omega_c\sum_{j=1}^{N}\sigma_{cc}^j+\hbar g\sum_{j=1}^{N}a\sigma_{ab}^j e^{ik_0z_j}+\hbar\Omega(t)e^{-i\nu t}\sum_{j=1}^{N}e^{i\mathbf{k}_1\cdot\mathbf{r}_j}\sigma_{ac}^j+\text{H.c.} \quad (1)$$

Here $\sigma_{\mu\nu}^j=|\mu\rangle_{jj}\langle\nu|$ is the flip operator of the jth atom and the vacuum Rabi frequency g is assumed to be equal for all atoms. For the time being we disregard atomic motion and thus the phase factors $e^{ik_0z_j}$ as well as $e^{i\mathbf{k}_1\cdot\mathbf{r}_j}$ will be absorbed into the definition of the atomic states $|a\rangle_j$ and $|c\rangle_j$. We will, however, come back to the issue of atomic motion in Sec. III.

When all atoms are initially prepared in level $|b\rangle$, the only states coupled by the interaction are the totally symmetric Dicke states [25] (after absorption of the spatial phase factors into the definition of the states)

$$|\mathbf{b}\rangle_N=|b_1,b_2,\ldots,b_N\rangle, \quad (2)$$

$$|\mathbf{a}^1\rangle_N=\frac{1}{\sqrt{N}}\sum_{j=1}^{N}|b_1,\ldots,a_j,\ldots,b_N\rangle, \quad (3)$$

$$|\mathbf{c}^1\rangle_N=\frac{1}{\sqrt{N}}\sum_{j=1}^{N}|b_1,\ldots,c_j,\ldots,b_N\rangle, \quad (4)$$

$$|\mathbf{c}^2\rangle_N=\binom{N}{2}^{-1/2}\sum_{i<j=1}^{N}|b_1,\ldots,c_i,\ldots,c_j,\ldots,b_N\rangle, \quad (5)$$

etc.

The interaction part of Eq. (1) only connects states within certain subspaces distinguished by the number of atoms, N, and by the excitation number n, which corresponds to the maximum number of photons that can occur. Due to the symmetry of the interaction, there is no coupling between subspaces of different excitation. Classes with different atom number are coupled through the decay out of the excited state or through atom losses. The couplings within the subsystems corresponding to a single and a double excitation are shown in Fig. 2.

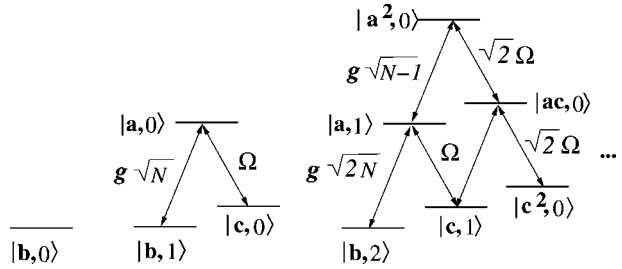

FIG. 2. Coupling of bare eigenstates of atom plus cavity system for at most two photons.

It is important to note that the matrix element between $|\mathbf{b},1\rangle$ and $|\mathbf{a}^1,0\rangle$ is enhanced by a factor $\sqrt{N}$ in an ensemble system, while that between $|\mathbf{a}^1,0\rangle$ and $|\mathbf{c}^1,0\rangle$ is the same as in the single-atom case:

$$|\langle\mathbf{b},1|H|\mathbf{a}^1,0\rangle|=\hbar g\sqrt{N}, \quad (6)$$

$$|\langle\mathbf{a}^1,0|H|\mathbf{c}^1,0\rangle|=\hbar\Omega(t). \quad (7)$$

In the following we will restrict ourselves to two-photon resonance—i.e., $\omega=\omega_a-\omega_c-\nu$. Furthermore, for simplicity, single-photon resonance is assumed. This is sufficient since we are not interested in the fidelity of the transfer process itself. The influence of a finite two-photon detuning on the transfer process is discussed in detail in [26]. In the case of two-photon resonance, the interaction of the N-atom system with the quantized radiation mode has a family of dark states—i.e., adiabatic eigenstates with vanishing component of the excited states $|a\rangle_j$:

$$|D,n\rangle_N=\sum_{k=0}^{n}\xi_{nk}(-\sin\theta)^k(\cos\theta)^{n-k}|\mathbf{c}^k,n-k\rangle_N,$$

$$\xi_{nk}\equiv\sqrt{\frac{n!}{k!(n-k)!}},\quad \tan\theta(t)\equiv\frac{g\sqrt{N}}{\Omega(t)}. \quad (8)$$

As can be seen from Eqs. (6) and (7) the minimum energy splitting of the dark states to other states, belonging to the same sub-system, is given by $\hbar g\sqrt{N}$. Thus adiabatic evolution occurs if

$$g\sqrt{N}T\gg 1, \quad (9)$$

where T is the characteristic time of changes. One recognizes that for a sufficiently large number of atoms this condition is much less stringent than the strong-coupling condition $gT\gg 1$ of single-atom cavity QED. It should also be noted that although the dark states $|D,n\rangle_N$ for different n and N are degenerate, there is no transition between them, even if nonadiabatic corrections are taken into account, due to the symmetry of the interaction Hamiltonian.

As can be seen from Eq. (8), adiabatically rotating the mixing angle θ from 0 to $\pi/2$ leads to a complete and reversible transfer of arbitrary photonic states to a collective atomic excitation, if the maximum number of photons, $n_{\max}$, is less than the number of atoms, N. If the initial quantum state of the single-mode light field is described by the density matrix $\rho_f=\Sigma_{n,m}^{n_{\max}}\rho_{nm}|n\rangle\langle m|$, the transfer process generates a

quantum state of collective excitations according to

$$\sum_{n,m}^{n_{\max}} \rho_{nm}|n\rangle\langle m| \otimes |\mathbf{b}\rangle_{NN}\langle\mathbf{b}|$$

$$\downarrow$$

$$\sum_{n,m}^{n_{\max}} \rho_{nm}|D,n\rangle_{NN}\langle D,m|$$

$$\downarrow$$

$$|0\rangle\langle 0| \otimes \sum_{n,m}^{n_{\max}} \rho_{nm}|\mathbf{c}^n\rangle_{NN}\langle\mathbf{c}^m|. \tag{10}$$

The dark states of the N-atom system can be identified as quasiparticle excitations in the space of atoms and cavity mode: the so-called dark-state polaritons Ψ [14]:

$$|D,n\rangle_N = \frac{1}{\sqrt{n!}}(\Psi^\dagger)^n|\mathbf{b},0\rangle_N. \tag{11}$$

Here $|\mathbf{b}\rangle_N$ is the total ground state of the N-atom system and $|0\rangle$ the vacuum state of the cavity mode. The dark-state polariton, defined as

$$\Psi = \cos\,\theta(t)a - \sin\,\theta(t)\frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{bc}^j, \tag{12}$$

is a superposition of the resonator mode and the collective spin, corresponding to the ground-state transition $|b\rangle \leftrightarrow |c\rangle$. Associated with the dark polariton is a bright polariton

$$\Phi_0 = \sin\,\theta(t)a + \cos\,\theta(t)\frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{bc}^j. \tag{13}$$

To obtain a complete set of operators in the space of the cavity mode and the N atoms in internal states $|b\rangle$ and $|c\rangle$, we also need to introduce the operators

$$\Phi_l = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{bc}^j \exp\left\{2\pi i\frac{lj}{N}\right\}, \quad l = 1,\ldots,N-1. \tag{14}$$

We will also refer to the Φ_l's as bright polaritons. In the limit of small atomic excitations the polariton operators obey approximately bosonic commutation relations

$$[\Psi,\Psi^\dagger] = \cos^2\,\theta + \sin^2\,\theta\frac{1}{N}\sum_{j=1}^{N}(\sigma_{bb}^j - \sigma_{cc}^j) = 1 + O\left(\frac{n_c}{N}\right), \tag{15}$$

$$[\Phi_0,\Phi_0^\dagger] = \sin^2\,\theta + \cos^2\,\theta\frac{1}{N}\sum_{j=1}^{N}(\sigma_{bb}^j - \sigma_{cc}^j) = 1 + O\left(\frac{n_c}{N}\right), \tag{16}$$

$$[\Phi_l,\Phi_m^\dagger] = \delta_{lm}\frac{1}{N}\sum_{j=1}^{N}(\sigma_{bb}^j - \sigma_{cc}^j) = \delta_{lm} + O\left(\frac{n_c}{N}\right), \tag{17}$$

where $n_c = \langle\Sigma_j\sigma_{cc}^j\rangle \ll N$ is the total population in level $|c\rangle$. Polariton operators of different type commute in lowest order of n_c/N:

$$[\Phi_i,\Psi^{(\dagger)}] = O\left(\frac{n_c}{N}\right). \tag{18}$$

It should be noted that the dark and bright polariton operators are explicitly time dependent through the mixing angle $\theta(t)$.

The collective storage state corresponding to a coherent state of light factorizes as can be seen quite easily:

$$e^{|\alpha|^2/2}|\mathbf{b},\alpha\rangle_N = e^{\alpha a^\dagger}|\mathbf{b},0\rangle_N$$

$$\downarrow$$

$$e^{\alpha\Psi^\dagger}|\mathbf{b},0\rangle_N$$

$$\downarrow$$

$$\exp\left\{-\frac{\alpha}{\sqrt{N}}\sum_j\sigma_{cb}^j\right\}|\mathbf{b},0\rangle_N = \prod_j\left(1 - \frac{\alpha\sigma_{cb}^j}{\sqrt{N}}\right)|\mathbf{b},0\rangle_N. \tag{19}$$

On the other hand, storage states corresponding to nonclassical states of light, such as Fock states, are maximally entangled N-particle states $|\mathbf{c}^n\rangle_N$, as can be seen from Eqs. (4) and (5). These states are known to be rather sensitive to decoherence processes. For example, if for an initial state $|\mathbf{c}^1\rangle_N$ the atom number one undergoes a transition from level $|b\rangle$ to an auxiliary state, say, $|d\rangle$, the resulting state is almost orthogonal to the original one

$$|c_1,b_2,\ldots,b_N\rangle + |b_1,c_2,\ldots,b_N\rangle + \cdots|b_1,b_2,\ldots,c_N\rangle$$

$$\downarrow$$

$$|c_1,b_2,\ldots,b_N\rangle + |d_1,c_2,\ldots,b_N\rangle + \cdots|d_1,b_2,\ldots,c_N\rangle. \tag{20}$$

If p denotes the probability of one atom to undergo a transition from $|b\rangle$ to $|d\rangle$ due to environmental interactions, the total probability P_{error} to end up in an orthogonal state scales as $P_{\text{error}} \sim 1-(1-p)^N \sim pN$. Thus one might naively expect that for the storage of a single photon the collective quantum memory will have an N times enhanced sensitivity to decoherence as compared to a single-atom device. In some applications of maximally entangled N-atom states like Eqs. (4) and (5), such as subshot noise spectroscopy suggested in [27], this is indeed the case. Also, in the proposed application of light storage for efficient photon counting [28] or in the proposed quantum gate based on dipole blockade [29], a single atom spin-flip can mimic an additional photon and thus there is an N times enhanced sensitivity to single-atom

errors. We will now show that this conclusion is generally not correct for the quantum memory.

From the inverse relation between the polariton modes and the field mode

$$a = \cos\theta(t)\Psi + \sin\theta(t)\Phi_0, \tag{21}$$

one recognizes that for the resonator field only excitations of the dark polariton Ψ and the bright polariton Φ_0 matter. Furthermore, if after the storage of photon states in the atomic system the electromagnetic excitations are read out by rotating θ back from $\pi/2$ to 0, only excitations in the dark polariton mode are relevant. I.e., if W denotes the total density operator of the combined atom-cavity system after the writing process, only the reduced density operator

$$\rho = \mathrm{Tr}_\Phi\{W\} \tag{22}$$

is relevant for the storage. Here Tr_Φ denotes the partial trace over all bright-polariton excitations. For this reason all states of the total system that have the same projection onto the dark-polariton modes but an arbitrary number of excitations in any bright-polariton mode are *equivalent* from the point of view of storage. I.e., there exist equivalence classes of the form

$$|D,n\rangle_N \triangleq \sum_{i,j,\ldots}\sum_{k,l,\ldots} C^{k,l,\ldots}_{i,j\ldots}(\Phi_i^\dagger)^k(\Phi_j^\dagger)^l\cdots(\Psi^\dagger)^n|\mathbf{b},0\rangle_N. \tag{23}$$

It is important to note that any perturbation that acts only onto bright-polariton modes does also not destroy superpositions of storage states, since $[\Phi_l,\Psi]=0$:

$$\Phi_l^\dagger\sum_n \alpha_n|D,n\rangle_N = \Phi_l^\dagger\sum_n \frac{\alpha_n}{\sqrt{n!}}(\Psi^\dagger)^n|\mathbf{b},0\rangle_N$$
$$= \sum_n \frac{\alpha_n}{\sqrt{n!}}(\Psi^\dagger)^n\{\Phi_l^\dagger|\mathbf{b},0\rangle_N\}. \tag{24}$$

Likewise all dark states with the same number of excitations, n, but with a different number of atoms, $N\gg n$, are equivalent, because in the adiabatic readout process dark-polariton operators that correspond to different N have the same asymptotic mapping $\Psi\to a$ for $\theta\to 0$:

$$|D,n\rangle_N \triangleq |D,n\rangle_{N'}, \quad \text{if } N,N'\gg n. \tag{25}$$

This will be important later on when discussing the effect of atom losses from the system. The importance of the equivalence classes stems from the fact that unwanted interactions with the environment, which lead only to transitions within the equivalence classes and which do not destroy the relative phase between them, do not affect the fidelity of the quantum memory.

In order to understand the adiabatic dynamics of the system we will now take into account losses from the excited state $|a\rangle$ outside the three-level system. To this end we introduce a superoperator $\mathcal{L}$ acting on the atomic density operator ρ projected to the three states $\{|a\rangle,|b\rangle,|c\rangle\}$:

$$\mathcal{L}\rho = -\frac{i}{\hbar}[H,\rho] - \frac{\gamma}{2}\sum_{j=1}^{N}(\sigma_{aa}^j\rho + \rho\sigma_{aa}^j). \tag{26}$$

Here H is the Hamiltonian (1). Adiabatically eliminating the excited states, assuming two-photon resonance, and expressing $\mathcal{L}$ in terms of the polariton operators yields in a rotating frame

$$\mathcal{L}\rho = -\frac{i}{\hbar}[H_{\mathrm{eff}},\rho] - \frac{\Omega^2(t)}{2\gamma}\sum_{l=1}^{N-1}(\Phi_l^\dagger\Phi_l\rho + \rho\Phi_l^\dagger\Phi_l - 2\Phi_l\rho\Phi_l^\dagger), \tag{27}$$

$$H_{\mathrm{eff}} = \hbar\omega_c\left(\Psi^\dagger\Psi + \sum_{l=0}^{N-1}\Phi_l^\dagger\Phi_l\right). \tag{28}$$

We here see two important points: First of all, the adiabatic dynamics does not couple different polariton modes. Second, all bright-polariton excitations Φ_l, $l=1,2,\ldots,N-1$, decay by optical pumping—i.e., by excitation to the excited state and successive spontaneous emission if $\Omega\neq 0$—while the dark polaritons Ψ as well as the bright polaritons Φ_0 are immune to spontaneous emission.

III. INFLUENCE OF SINGLE-PARTICLE DECOHERENCE ON STORAGE FIDELITY

In order to discuss the influence of decoherence processes on the fidelity of the collective quantum memory, we have to distinguish between *individual* and *collective* reservoir couplings. The first is characterized by an interaction of all particles to distinguishable and independent reservoirs. The corresponding interaction Hamiltonian has the structure

$$H_{\mathrm{int}}^{(\mathrm{ind})} = \sum_{j=1}^{N}\sum_k g_{jk}\sigma_{\mu\nu}^j b_{jk} + \mathrm{H.a.}, \tag{29}$$

where the b_{jk} are reservoir operators corresponding to the jth atom. $[b_{jk},b_{il}]=0$, if $j\neq i$ and the unperturbed reservoir density operator factorizes $\rho_{\mathrm{res}}=\Pi_{j=1}^N\rho_{\mathrm{res}}^{(j)}$. The second reservoir-coupling model is characterized by collective variables, defined through the unitary transformation

$$\Sigma_{\mu\nu}^l = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\eta_{lj}\sigma_{\mu\nu}^j, \quad l=1,\ldots,N, \tag{30}$$

where η_{lj} is a phase factor obeying

$$\sum_{l=1}^{N}\eta_{jl}^*\eta_{li} = N\delta_{ij}. \tag{31}$$

The interaction Hamiltonian of a collective reservoir interaction is of the form

$$H_{\mathrm{int}}^{(\mathrm{coll})} = \sum_{l=1}^{N}\sum_k f_{lk}\Sigma_{\mu\nu}^l B_{lk} + \mathrm{H.a.}, \tag{32}$$

where B_{lk} is a reservoir operator corresponding to the collective mode l. Reservoir operators corresponding to different

modes are independent; i.e., they commute and the unperturbed density matrix factorizes in the collective modes. In reality reservoir interactions are of neither of the two types but somewhere in between; i.e., the individual or collective reservoirs do have some correlations. Many decoherence phenomena can, however, be described by one or the other model, and we will restrict ourselves to either one of the two. Since for dilute systems such as atomic vapors the individual reservoir model is more appropriate, we will discuss this case in detail in the present section. The effect of collective decoherence processes will be considered in the following one.

A. Imperfect preparation

In the description of the storage process given in the last section we had assumed that every atom in the ensemble was prepared in the ground state $|b\rangle$. If the ensemble is large enough, the probability to find an atom, e.g., in state $|c\rangle$ will, however, be non-negligible—e.g., in the presence of a finite-temperature reservoir. Naively one might expect that any atom left in state $|c\rangle$ after preparation of the ensemble would mimic a stored photon. This would require one to make the initial probability to find an atom in state $|c\rangle$ small compared to $1/N$. This is indeed not the case. It is rather sufficient that the initial probability of excitation of a dark-state polariton is small compared to unity. If we consider, e.g., as initial state a thermal state of temperature $\beta=1/k_BT$ ($\theta=\pi/2$),

$$\rho_0 = \frac{1}{Z}\exp\left\{-\beta\hbar\omega_c\left(\Psi^\dagger\Psi + \sum_{l=1}^{N-1}\Phi_l^\dagger\Phi_l\right)\right\}, \tag{33}$$

with Z being the statistical sum, the mean number of initially excited dark-state polaritons is independent of N and given by

$$\langle\Psi^\dagger\Psi\rangle = \frac{e^{-\beta\hbar\omega_c}}{1-e^{-\beta\hbar\omega_c}} = \frac{1}{N}\sum_{j=1}^{N}\langle\sigma_{cc}^j\rangle. \tag{34}$$

Thus, if the probability that an atom is initially in level $|c\rangle$ is small compared to unity, the number of initial dark-polariton excitations is small compared to unity as well.

B. Random spin flips and dephasing

On the level of individual atoms the storage occurs within the two-state system consisting of $|b\rangle$ and $|c\rangle$. If we assume that all other atomic states including $|a\rangle$ are energetically much higher, we may safely neglect decoherence processes involving the excitation of those states. Then decoherence caused by individual and independent reservoir interactions can be described by the action of the two-level Pauli operators

$$X_j = \sigma_{bc}^j + \sigma_{cb}^j, \quad Z_j = [\sigma_{bc}^j, \sigma_{cb}^j], \quad Y_j = i\sigma_{bc}^j - i\sigma_{cb}^j. \tag{35}$$

X_j describes a symmetric spin flip of the jth atom, Z_j a phase flip, and Y_j a combination of both. Any single-atom error can be expressed in terms of these, and we will restrict the discussion here to the action of X_j (symmetric spin flip), $X_j + iY_j$ (asymmetric spin flip), and Z_j (phase flip).

Inverting relations (12), (13), and (14) one easily finds a representation of σ_{cb}^j in terms of polaritons:

$$\begin{aligned}\sigma_{cb}^j &= \frac{1}{\sqrt{N}}\left(\sum_{l=1}^{N-1}\exp\left\{-2\pi i\frac{lj}{N}\right\}\Phi_l^\dagger - \Psi^\dagger\right)\\ &= \frac{1}{\sqrt{N}}\left(\sum_{l=1}^{N-1}\eta_{jl}\Phi_l^\dagger - \Psi^\dagger\right)\\ &= \frac{X_j + iY_j}{2}.\end{aligned} \tag{36}$$

Here and in the following $\theta=\pi/2$ is assumed, unless stated otherwise, which corresponds to the case of a completed transfer from the radiation field to the ensemble. Furthermore,

$$X_j = \frac{1}{\sqrt{N}}\left\{\sum_{l=1}^{N-1}(\eta_{jl}\Phi_l^\dagger + \eta_{jl}^*\Phi_l) - \Psi - \Psi^\dagger\right\} \tag{37}$$

and

$$Z_j = \frac{1}{N}\left[\sum_{l=1}^{N-1}\eta_{jl}^*\Phi_l - \Psi, \sum_{m=1}^{N-1}\eta_{jm}\Phi_m^\dagger - \Psi^\dagger\right]. \tag{38}$$

One recognizes at this point that applying the approximate commutation relations (15)–(18), which have been obtained with the assumption $\sigma_{bb}^j \approx 1$ and $\sigma_{cc}^j \approx 0$, would lead to $Z_j = \mathbf{1}_j$. Thus care must be taken when using operators quadratic in the polaritons such as Z_j.

1. Spin flip from $|b\rangle \to |c\rangle$

Consider a quantum memory initially in an ideal storage state W_0—i.e., without bright-polariton excitations. Suppose an atom undergoes a spin flip to the internal state $|c\rangle$ if it is initially in state $|b\rangle$. Such a spin-flip process, which could mimic a stored photon, can be described by the positive map

$$W_0 \to W_1 = \frac{\sigma_{cb}^j W_0 \sigma_{bc}^j}{\mathrm{Tr}\{\sigma_{cb}^j W_0 \sigma_{bc}^j\}}. \tag{39}$$

As noted in the previous section, only the reduced density operator traced over the bright-polariton modes is of relevance for the memory. Carrying out this trace yields

$$\begin{aligned}\mathrm{Tr}_\Phi(\sigma_{cb}^j W_0 \sigma_{bc}^j) = \frac{1}{N}\mathrm{Tr}_\Phi\left[\sum_{l,m}^{N-1}\eta_{jl}\Phi_l^\dagger W_0 \eta_{jm}^*\Phi_m\right] + \frac{1}{N}\Psi^\dagger\rho_0\Psi\\ - \frac{1}{N}\mathrm{Tr}_\Phi\left[\sum_{l}^{N-1}\eta_{jl}\Phi_l^\dagger W_0\Psi + \mathrm{H.a.}\right],\end{aligned} \tag{40}$$

where $\rho_0=\mathrm{Tr}_\phi\{W_0\}$. If we make use of the fact that the bright and dark polaritons commute in first order of $1/N$, we see that the last term in Eq. (40) vanishes, since there are no excitations of bright polaritons in the initial state W_0. For the same reason,

$$\mathrm{Tr}_\Phi\{\eta_{jl}\Phi_l^\dagger W_0 \eta_{jm}^* \Phi_m\} = \rho_0 \delta_{lm}, \tag{41}$$

and the first term in Eq. (40) evaluates to $(1-1/N)\rho_0$. Thus we arrive at

$$\mathrm{Tr}_\Phi(\sigma_{cb}^j W_0 \sigma_{bc}^j) = \left(1-\frac{1}{N}\right)\rho_0 + \frac{1}{N}\Psi^\dagger \rho_0 \Psi$$

and

$$\rho_1 = \mathrm{Tr}_\Phi\{W_1\} = \frac{\left(1-\frac{1}{N}\right)\rho_0 + \frac{1}{N}\Psi^\dagger \rho_0 \Psi}{1+\frac{1}{N}\langle \Psi^\dagger \Psi\rangle}. \tag{42}$$

One recognizes that the spin flip of an individual atom only causes an error of order $1/N$. This exactly compensates for the fact that the total probability of the N atoms is N times the probability of a single-atom spin flip.

From Eq. (42) one can easily calculate the fidelity of the quantum memory after a single spin-flip error, which for the case of an initial pure state $\rho_0 = |\psi_0\rangle\langle\psi_0|$ is defined as

$$f(|\psi_0\rangle) = \langle\psi_0|\rho_1|\psi_0\rangle = \mathrm{Tr}\{\rho_1\rho_0\}. \tag{43}$$

One finds, e.g., for a stored Fock-state $|n\rangle$ with $n \ll N$

$$f_{b\to c}(|n\rangle) = \frac{1-\frac{1}{N}}{1+\frac{n}{N}} = 1 - \frac{n+1}{N} + O\left(\frac{1}{N^2}\right), \tag{44}$$

while for a coherent state $|\alpha\rangle$ holds:

$$f_{b\to c}(|\alpha\rangle) = \frac{1-\frac{1}{N}+\frac{|\alpha|^2}{N}}{1+\frac{|\alpha|^2}{N}} = 1 - \frac{1}{N} + O\left(\frac{1}{N^2}\right). \tag{45}$$

The difference of the two results reflects the general property of nonclassical states to be more sensitive to decoherence than classical ones.

An alternative way of demonstrating that spin-flip errors do not depend on the number of atoms is to consider the Liouville operator $\mathcal{L}$ describing uncorrelated spin flips with rate Γ:

$$\dot{W} = \mathcal{L}W = \sum_{j=1}^{N} \mathcal{L}_j W, \tag{46}$$

$$\mathcal{L}_j W = -\frac{\Gamma}{2}(\sigma_{bc}^j \sigma_{cb}^j W + W\sigma_{bc}^j \sigma_{cb}^j - 2\sigma_{cb}^j W \sigma_{bc}^j). \tag{47}$$

Substituting expression (36) yields after tracing over the bright-polariton excitations,

$$\mathcal{L}\rho = -\frac{\Gamma}{2}(\Psi^\dagger\Psi\rho + \rho\Psi^\dagger\Psi - 2\Psi^\dagger\rho\Psi). \tag{48}$$

One recognizes that the decoherence rate of the reduced density operator of the ensemble of atoms due to spin flips is the same as for a single atom.

2. Symmetric spin flip

If instead of the asymmetric spin flip $\sigma_{cb}^j = X_j + iY_j$ a symmetric flip happens, Eq. (39) attains the form

$$W_0 \to W_1 = \frac{X_j W_0 X_j}{\mathrm{Tr}\{X_j W_0 X_j\}}. \tag{49}$$

We here have kept the normalization denominator although $\mathrm{Tr}\{X_j W_0 X_j\} = 1$ because we want to make use of the approximate commutation relations between dark and bright polaritons, which hold only to first order in $1/N$. Thus both the numerator and denominator in Eq. (49) have to be expanded in the same way to keep the normalization. Carrying out the trace over the bright polaritons yields

$$\begin{aligned}\mathrm{Tr}_\Phi(X_j W_0 X_j) &= \frac{1}{N}\mathrm{Tr}_\Phi\left[\sum_{l,m}^{N-1}(\eta_{jl}\Phi_l^\dagger + \text{H.a.})W_0(\eta_{jm}\Phi_m^\dagger + \text{H.a.})\right] \\ &+ \frac{1}{N}(\Psi^\dagger + \Psi)\rho_0(\Psi^\dagger + \Psi) \\ &- \frac{1}{N}\mathrm{Tr}_\Phi\left[\sum_{l}^{N-1}(\eta_{jl}\Phi_l^\dagger + \eta_{jl}^*\Phi_l)W_0(\Psi + \Psi^\dagger) + \text{H.a.}\right].\end{aligned} \tag{50}$$

Again the last term in Eq. (50) vanishes since there are no excitations of bright polaritons in the initial state W_0 and in the first term only the combination

$$\mathrm{Tr}_\Phi\{\eta_{jl}\Phi_l^\dagger W_0 \eta_{jm}^* \Phi_m\} = \rho_0 \delta_{lm} \tag{51}$$

remains, which evaluates to $(1-1/N)\rho_0$. This yields

$$\mathrm{Tr}_\Phi(X_j W_0 X_j) = \left(1-\frac{1}{N}\right)\rho_0 + \frac{1}{N}(\Psi^\dagger + \Psi)\rho_0(\Psi^\dagger + \Psi). \tag{52}$$

Thus we arrive at

$$\rho_1 = \frac{\left(1-\frac{1}{N}\right)\rho_0 + \frac{1}{N}(\Psi^\dagger + \Psi)\rho_0(\Psi^\dagger + \Psi)}{1-\frac{1}{N}+\frac{1}{N}\langle(\Psi^\dagger + \Psi)^2\rangle}, \tag{53}$$

which is similar to the case of an asymmetric spin flip, Eq. (42). Once again, it is seen that the collective quantum memory does not have an enhanced sensitivity to spin-flip errors as compared to a single-atom system.

The fidelity of the memory now reads for a stored Fock and coherent state after a symmetric spin flip

$$f_{b\leftrightarrow c}(|n\rangle) = 1 - \frac{2n+1}{N} + O\left(\frac{1}{N^2}\right), \tag{54}$$

$$f_{b\leftrightarrow c}(|\alpha\rangle) = 1 - \frac{1}{N} + O\left(\frac{1}{N^2}\right). \tag{55}$$

3. Phase flip

If after the preparation of an ideal storage state an atom undergoes a phase flip, the corresponding positive map would read

$$W_0 \to W_1 = \frac{Z_j W_0 Z_j}{\mathrm{Tr}\{Z_j W_0 Z_j\}}. \tag{56}$$

This map is, however, not a good starting point for further discussions because the approximations used when introducing bosonic polariton operators lead to $Z_j \equiv \mathbf{1}_j$. For this reason we follow a different approach and calculate the fidelity of the quantum memory directly. Consider an ideal storage state initially of the form

$$|\psi_0\rangle = \sum_{n=0}^{n_{\max}} c_n |D,n\rangle = \sum_{n=0}^{n_{\max}} \frac{c_n}{\sqrt{n!}} (\Psi^\dagger)^n |\mathbf{b},0\rangle, \tag{57}$$

where $n_{\max} \ll N$. If the kth atom undergoes a phase flip, the state changes according to

$$|\psi_0\rangle \to |\psi_1\rangle = \sum_{n=0}^{n_{\max}} c_n \frac{1}{\sqrt{n!}} (\tilde{\Psi}^\dagger)^n |\mathbf{b},0\rangle, \tag{58}$$

where

$$\tilde{\Psi} \equiv -\frac{1}{\sqrt{N}} \left[\sum_{j \neq k} \sigma_{bc}^j - \sigma_{bc}^k \right] = \Psi + \frac{2}{\sqrt{N}} \sigma_{bc}^k. \tag{59}$$

Using Eq. (36) this can be written in the form

$$\tilde{\Psi} = \left(1 - \frac{2}{N}\right) \Psi + \frac{2}{N} \sum_{l=1}^{N-1} \eta_{kl}^* \Phi_l. \tag{60}$$

This yields in lowest order of $1/N$

$$\tilde{\Psi}^n = \left(1 - \frac{2n}{N}\right) \Psi^n + \frac{2n}{N} \sum_l \eta_{kl}^* \Phi_l \Psi^{n-1} + O\left(\frac{1}{N^2}\right). \tag{61}$$

Tracing over the bright-polariton excitations leads to the reduced density operator

$$\rho_1 = \mathrm{Tr}_\Phi[|\psi_1\rangle\langle\psi_1|] = \sum_{n,m=0}^{n_{\max}} c_n^* c_m \left(1 - \frac{2(n+m)}{N}\right) |D,n\rangle\langle D,m| \tag{62}$$

and eventually to the fidelity

$$f_{\mathrm{deph}}(|\psi_0\rangle) = 1 - \frac{4\langle n\rangle}{N} + O\left(\frac{1}{N^2}\right), \tag{63}$$

where $\langle n\rangle = \langle \Psi^\dagger \Psi\rangle$ is the average number of dark-state polaritons in the initial state. One recognizes that a phase flip of a single atom leads to a fidelity reduction which is of the order of $1/N$. The term $1/N$ again compensates for the fact that in an N-atom ensemble the likelihood that one arbitrary atom undergoes a phase flip is N times the probability of a phase flip for a single atom. It is interesting to note that the fidelity only depends on the average dark-state polariton number. I.e., dephasing affects in lowest order of $1/N$ classical and nonclassical states in a similar way.

C. One-atom losses

Another important source of errors in a collective quantum memory is the loss of an atom from the ensemble. As discussed in Sec. II, all storage states corresponding to the same dark-state excitations in ensembles of different atom number are equivalent as long as the atom number is large compared to the relevant number of stored photons. We now calculate the fidelity of the quantum memory after loss of one atom. We consider again an ideal initial storage state

$$|\psi_0\rangle_N = \sum_{n=0}^{n_{\max}} c_n |D,n\rangle_N, \tag{64}$$

where the subscript N denotes the total number of atoms in the ensemble and $n_{\max} \ll N$. The loss of an atom, which, without loss of generality, can be taken to be the Nth atom, can be described by the partial trace over the degrees of freedom of that atom:

$$W_1 = \mathrm{Tr}_N\{|\psi_0\rangle\langle\psi_0|\}. \tag{65}$$

To carry out the trace let us first consider the case of a Fock state of n polaritons $|D,n\rangle_N$

$$|D,n\rangle_N = \binom{N}{n}^{-1/2} \sum_{j_1<\ldots<j_n}^{N} |b_1,\ldots,c_{j1},\ldots,c_{jn},\ldots,b_N\rangle. \tag{66}$$

Tracing over the Nth atom results in

$$\begin{aligned}\mathrm{Tr}_N\{|D,n\rangle_{NN}\langle D,n|\} &= \binom{N}{n}^{-1/2} \\ &\times \sum_{j_1<\ldots<j_n}^{N-1} |b_1,\ldots,c_{j1},\ldots,c_{jn},\ldots,b_{N-1}\rangle\langle\cdots| + \binom{N}{n}^{-1/2} \\ &\times \sum_{j_1<\ldots<j_{n-1}}^{N-1} ||b_1,\ldots,c_{j1},\ldots,c_{j(n-1)},\ldots,b_{N-1}\rangle\langle\cdots| \end{aligned} \tag{67}$$

$$= \frac{N-n}{N} |D,n\rangle_{N-1N-1}\langle D,n| + \frac{n}{N} |D,n-1\rangle_{N-1N-1}\langle D,n-1|. \tag{68}$$

Thus the fidelity of the quantum memory for a Fock state $|n\rangle$ after loss of a single atom is given by

$$f_{\mathrm{loss}}(|n\rangle) = 1 - \frac{n}{N}. \tag{69}$$

The decrease of the fidelity again scales only as $1/N$. This result could of course have been expected as the n excitations are equally distributed over all atoms. Thus removing one reduces the stored information only by the amount n/N. Generalizing the above result to nondiagonal elements leads after some calculation to

$$\mathrm{Tr}_N\{|D,n\rangle_{NN}\langle D,m|\} = \frac{\sqrt{(N-n)(N-m)}}{N}|D,n\rangle_{N-1N-1}\langle D,m| + \frac{\sqrt{nm}}{N}|D,n-1\rangle_{N-1N-1}\langle D,m-1|. \tag{70}$$

Thus the fidelity after the loss of an atom reads for the case of a general state

$$f_{\mathrm{loss}}(|\psi_0\rangle) = 1 - \frac{1}{N}(\langle\Psi^\dagger\Psi\rangle - \langle\Psi^\dagger\rangle\langle\Psi\rangle) + O\left(\frac{1}{N^2}\right). \tag{71}$$

If the initial storage state corresponds, e.g., to a coherent state, the second and third terms in Eq. (71) compensate each other, and the fidelity differs from unity only in order $1/N^2$. Here again the robustness of classical states becomes apparent.

D. Atomic motion

Until now it has been assumed that the atoms used in the quantum memory are at a fixed position during the entire storage time. Since the coupling of the atoms to the quantum and control fields contains, however, a spatial phase [see Eq. (1)], atomic motion results in an effective dephasing and will lead to a reduction of the fidelity. Recently Sun *et al.* have argued that inhomogeneities of the atom-light interaction strength or in the control field together with atomic motion lead to an increase of the characteristic decoherence rate by a factor $\sqrt{N}$ [30]. We thus will analyze the effect of atomic motion in the following in more detail. To this end we will follow the approach of Sec. III B 3 and describe the motion by the map of an initially ideal storage state $|\psi_0\rangle$, Eq. (57), according to

$$|\psi_0\rangle \to |\psi_1\rangle = \sum_n c_n \frac{1}{n!}[\check{\Psi}^\dagger(t)]^n|\mathbf{b},0\rangle, \tag{72}$$

where

$$\check{\Psi}^\dagger(t) = -\frac{1}{\sqrt{N}}\sum_j \sigma_{bc}^j \exp\{-i\Delta\vec{k}\cdot\vec{r}_j(t)\}, \tag{73}$$

with $\vec{r}_j(t)$ denoting the position of the jth atom at time t and $\Delta\vec{k}=\vec{k}_1-k_0\vec{e}_z$ is the wave-vector difference between control field and quantized mode. It should be noted that Eq. (73) is equivalent to a coupling field with inhomogeneous phase.

To reduce the effect of motion in an atomic vapor one could either reduce the temperature or use a buffer gas of sufficient density. In the latter case, which has been used in room-temperature gas-cell experiments [8,15], the free motion is replaced by a diffusion. In the following we will restrict the discussion to this important case. We thus can assume that the phase

$$\Delta\phi_j(t) \equiv \Delta\vec{k}\cdot\vec{r}_j(t) \tag{74}$$

follows a Wiener diffusion process [31]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta\phi_j(t) = \mu_j(t), \tag{75}$$

$$\overline{\mu_j(t)} = 0, \tag{76}$$

$$\overline{\mu_j(t)\mu_k(t')} = D\delta_{jk}\delta(t-t'), \tag{77}$$

with D being a characteristic diffusion rate. We now want to show that the decrease in fidelity due to the phase diffusion is only determined by D and independent of the number of atoms N. For this it is sufficient to consider an initial Fock state $|n=1\rangle$. Expressing the single-atom flip operators in terms of collective ones yields

$$\check{\Psi}^\dagger = \frac{1}{N}\sum_j \mathrm{e}^{i\Delta\phi_j}\left[-\sum_{l=1}^{N-1}\eta_{jl}\Phi_l^\dagger + \Psi^\dagger\right]. \tag{78}$$

With this one finds for an initial Fock state $W_0 = |D,1\rangle\langle D,1|$

$$W_1(t) = \overline{(\check{\Psi}^\dagger)|\mathbf{b},0\rangle\langle\mathbf{b},0|(\check{\Psi})}, \tag{79}$$

where the overbar denotes averaging over the phase diffusion process. From $W_1(t)$ we can calculate the fidelity of the quantum memory by first tracing over the bright polaritons and then sandwiching with the original state $|D,1\rangle$. This yields

$$\begin{aligned} f_{\mathrm{motion}}(|1\rangle) &= \overline{\langle D,1|\mathrm{Tr}_\Phi(W_1(t))|D,1\rangle}, \\ &= \overline{\left(\frac{1}{N}\sum_j \mathrm{e}^{i\Delta\phi_j}\right)\left(\frac{1}{N}\sum_k \mathrm{e}^{-i\Delta\phi_k}\right)} \\ &= \frac{1}{N} + \frac{1}{N^2}\sum_{j\neq k}\overline{\mathrm{e}^{i\Delta\phi_j}\mathrm{e}^{-i\Delta\phi_k}} \\ &= \frac{1}{N}[1+(N-1)\mathrm{e}^{-Dt}] \\ &\sim \mathrm{e}^{-Dt}. \end{aligned} \tag{80}$$

A generalization to an arbitrary fock state $|D,n\rangle$ leads to a fidelity decay proportional to $\exp\{-nDt\}$. One recognizes that the atomic motion causes a decay of the fidelity with a rate given only by the single-atom diffusion rate D. In contrast to the results of Sun, Yi, and You [30], we find that there is no enhancement of the decay with increasing number of atoms, which is again due to the existence of equivalence classes.

IV. COLLECTIVE DECOHERENCE PROCESSES

In the previous section we have discussed the influence of decoherence processes due to individual reservoir interactions. This model describes a number of important processes relevant for dilute atomic vapors such as collisions with atoms of a background gas. On the other hand, even for dilute gases there are certain errors sources, such as fluctuating magnetic fields, which are of collective nature and are not covered by this model. We therefore discuss in this section collective reservoir interactions. For this we assume that each collective mode $\Sigma_{\mu\nu}^l$ as introduced in Eq. (30) couples to an independent reservoir. We furthermore restrict our-

selves to Markovian processes. In this case the dynamics of the N-atom density operator can be described by a Lindblad equation of the type

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho|_{\mathrm{decoh}} = \sum_{l=1}^{N} \frac{\gamma_l}{2}(2\Sigma^l \rho \Sigma^{l\dagger} - \rho\Sigma^{l\dagger}\Sigma^l - \Sigma^{l\dagger}\Sigma^l \rho), \quad (81)$$

where Σ^l and $\Sigma^{l\dagger}$ are the collective operators introduced in Eq. (30) and the lower indices have been suppressed for simplicity. In order to compare the effect of collective decoherence processes on the ensemble with that on an individual atom we express the collective operators in terms of single-atom variables. This yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho|_{\mathrm{decoh}} = \sum_{i,j=1}^{N} \frac{C_{ij}}{2}(2\sigma_j \rho \sigma_i^\dagger - \rho\sigma_i^\dagger\sigma_j - \sigma_i^\dagger\sigma_j\rho), \quad (82)$$

where $C_{ij} = 1/N\Sigma_{k=1}^{N} \eta_{ik}^* \eta_{jk}\gamma_k$.

Let $\gamma_{\max}$ be the largest collective decay rate. Then one easily verifies

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho|_{\mathrm{decoh}} \leqslant \gamma_{\max}\sum_{j=1}^{N}(2\sigma_j \rho \sigma_j^\dagger - \rho\sigma_j^\dagger\sigma_j - \sigma_j^\dagger\sigma_j\rho), \quad (83)$$

where relation (31) was used. A quantum state stored in an individual atom would decay with a rate slower than the maximum collective decay rate. Thus, if the collective decoherence rates of the relevant dark-state polariton modes are of the same order of magnitude as the maximum collective rate $\gamma_{\max}$, the decoherence time of the collective storage state is not much different from that of the quantum state stored in an individual atom. An example for the latter case is an isotropic and homogeneous, but fast fluctuating magnetic field. The fluctuating magnetic field causes a dephasing of the Zeeman coherences of every atom. If the magnetic field is isotropic and homogeneous, the resulting dephasing of the dark- and bright-polariton modes is the same. Thus an ensemble-based quantum memory is also as robust against collective decoherence processes as single atoms provided the decoherence affects all collective modes with comparable strength.

V. NONADIABATIC COUPLING AND DECOHERENCE

In Sec. II it was shown that for the retrieval of a stored quantum state of light only the reduced density operator (22) is relevant. For this reason all states of the system which have the same number of dark-state polariton excitations but an arbitrary number of excitations in the bright-state polariton modes belong to the same equivalence class and lead to the same result, provided the readout process is adiabatic. Due to decoherence, a large number of bright-state polaritons may be excited in the system after the storage period. Now the question arises what happens to these excitations if the readout process is not adiabatic. Even if there is only a weak nonadiabatic coupling between bright- and dark-state polariton modes, it may be sufficient to transfer some of the unwanted excitations into the dark-polariton mode. We will show in the following that only the bright-polariton mode Φ_0 can lead to nonadiabatic contributions to the readout signal. For this we consider the effective interaction Hamiltonian, Eq. (28), in a rotating frame and add the coupling of the quantized resonator mode a to free-space modes b_k:

$$H_{\mathrm{eff}} = \hbar\omega_c\left(\Psi^\dagger\Psi + \sum_{l=0}^{N-1}\Phi_l^\dagger\Phi_l\right) + \hbar\sum_k \kappa(\cos\theta\Psi^\dagger + \sin\theta\Phi_0)b_k + \mathrm{H.a.} \quad (84)$$

The equation of motion for the dark-state polariton operator, $\Psi = \cos\theta(t)a - \sin\theta(t)\Sigma_{bc}$, with $\Sigma_{bc} = \Sigma_j \sigma_{bc}^j/\sqrt{N}$, then reads

$$\dot{\Psi} = -\dot{\theta}(t)\sin\theta(t)a + \dot{\theta}(t)\Sigma_{bc} + \frac{i}{\hbar}[H,\Psi]$$
$$= -\dot{\theta}(t)\Phi_0 + i\kappa\cos\theta(t)\sum_k b_k. \quad (85)$$

Thus the dark polariton is coupled to the outside modes b_k and the bright-polariton operator $\Phi_0 = \sin\theta(t)a + \cos\theta(t)\Sigma_{bc}$ only. In a similar way one finds for Φ_0 and the b_k's:

$$\dot{\Phi}_0 = \dot{\theta}(t)\Psi, \quad (86)$$

$$\dot{b}_k = i\kappa a = i\kappa\cos\theta(t)\Psi + i\kappa\sin\theta(t)\Phi_0. \quad (87)$$

One recognizes that $\{\Psi, \Phi_0, b_k,\}$ are a closed set of coupled variables, even if nonadiabatic corrections are taken into account. Thus only decoherence-induced excitations generated in Φ_0 will influence the readout signal. All other $N-1$ bright-polariton modes remain uncoupled from the storage systems.

VI. SUMMARY

In the present paper we have studied the influence of individual and certain collective decoherence processes on the fidelity of a quantum memory for photons. Despite the fact that the atomic storage states, corresponding to nonclassical states of the radiation field, are entangled, the system shows no enhanced sensitivity to decoherence as compared, e.g., to single-atom storage systems if the error is caused by a coupling of the atoms to individual reservoirs. The same holds for collective reservoir couplings provided they are (approximately) homogeneous—i.e., affect all collective modes with similar strength. This is due to the existence of equivalence classes of storage states corresponding to the excitations of only one eigenmode of the system: the dark-state polariton. It was shown that all states with the same reduced density operator after tracing out the N bright-polariton modes reproduce the same quantum state of light in the readout process and are thus equivalent. For similar reasons no stringent requirements for preparation of the atomic system before the storage exist. It is sufficient that the number of atoms remaining in the storage level $|c\rangle$ after preparation of the ensemble is small compared to the total number of atoms, which can easily be achieved by optical pumping. It was shown, moreover, that the loss of an atom from the sample causes only an error of the order of $1/N$. Motion of atoms during the storage time causes an effective dephasing and thus leads to a decrease in fidelity. The corresponding error

is, however, again independent of the number of atoms, which is in contrast to the result of [30]. Finally since nonadiabatic effects only couple one of the bright-polariton modes to the dark-polariton moles, the potentially large number of excitations in the N bright-polariton modes caused by decoherence processes does not leak into the readout signal in a significant amount even if the readout process is not adiabatic.

The present paper proves that atomic ensembles are suitable systems for the storage of quantum states of the radiation field even in the presence of the most relevant decoherence processes.

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