Engineering superpositions of coherent states in coherent optical pulses through cavity-assisted interaction

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We propose a scheme to engineer quantum superpositions of coherent states ("Schrödinger-cat states") of propagating optical pulses. Multidimensional and multipartite cat states can be generated simply by reflecting coherent optical pulses successively from a single-atom cavity. The influences of various sources of noise, including atomic spontaneous emission and pulse-shape distortion, are characterized through detailed numerical simulation, which demonstrates the practicality of this scheme within the reach of current experimental technology.

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Superpositions of classically distinguishable states ("Schrödinger-cat states"), as a distinct class of nonclassical states, have attracted extensive research interest recently in the context of a fundamental test of quantum mechanics and quantum information theory. For bosonic modes, cat states are typically referred to as quantum superpositions of coherent states. Such states are of critical importance for investigation of the decoherence process and the quantum-classical boundary [1], for a fundamental test of the quantum nonlocality [2], and for implementation of quantum computation and communication [3–6]. Significant theoretical and experimental efforts have been made for realization of such cat states in different physical systems [1,7–9]. Until now, such states have been successfully generated for phonon modes of a single trapped ion [1], and for microwave photon modes confined inside a superconducting cavity [7].

There is also great interest in generating Schrödinger-cat states for propagating optical pulses. The motivation consists of at least two aspects: first, for applications in the test of quantum nonlocality or in the implementation of quantum computation and communication, one needs to use cat states of propagating optical pulses; secondly, for propagating pulses, with assistance of linear optical devices (such as beam splitters), it is possible to generate a larger class of cat states targeted to different kinds of applications [3–6,10]. The proposals for generating cat states of optical pulses are typically based on either the Kerr nonlinearity or postselections from nonlinear detectors [11–13]. Although the Kerr nonlinearity in principle provides a method for deterministic generation of the cat states, it is well known that such nonlinearity in typical materials is too small to allow cat state generation from weak coherent pulses.

In this paper, we propose a scheme to engineer Schrödinger-cat states of propagating optical pulses based on the state-of-the-art cavity technology. It has been demonstrated that a single atom can be trapped for seconds inside a high-Q optical cavity working in the strong-coupling regime [14–16]. With such a setup, we can generate a large class of cat states simply by reflecting weak coherent pulses successively from a cavity mirror. With the aid of a few beam splitters, we can generate multipartite and multidimensional cat states, and the preparation of such states is a necessary step for several distinct applications, such as loop-hole-free detection of Bell inequalities with homodyne detections [17] and quantum coding and computation [18]. This scheme also extends an earlier photonic quantum computation proposal by Duan and Kimble [19] to the continuous variable regime, eliminating the requirement of using single-photon pulses as a computation resource. To characterize the influences of various sources of experimental noise on this scheme, we develop a numerical simulation method to quantify the noise effects due to the atomic spontaneous emission, the photon pulse-shape distortion, and the cavity mode-matching inefficiency. The numerical method enables us to find out a range of the cat-state amplitude achievable within our scheme. And the calculation shows that substantial cat states can be generated within the reach of the current technology.

First, let us briefly introduce the basic idea of this scheme. We consider an atom with three effective levels trapped inside an optical cavity. The level configuration is shown in Fig. 1, where $|0\rangle$ and $|1\rangle$ are levels in the ground-state manifold with different hyperfine spins. The transition from level $|1\rangle$ to $|e\rangle$ is resonantly coupled to a cavity mode a_c, which is resonantly driven by an input optical pulse prepared in a weak coherent state $|\alpha\rangle$. The transition between level $|0\rangle$ and $|e\rangle$ is decoupled from the cavity mode due to the large detuning from the hyperfine frequency. If the atom is prepared in the level $|0\rangle$, the input pulse is resonant with the bare cavity mode a_c, and after resonant reflection it will acquire a phase of $e^{i\pi}$ from standard quantum optics calculation [20]. The effective state of the pulse is then given by $|-\alpha\rangle$. However, if the atom is prepared in the level $|1\rangle$, due to the strong

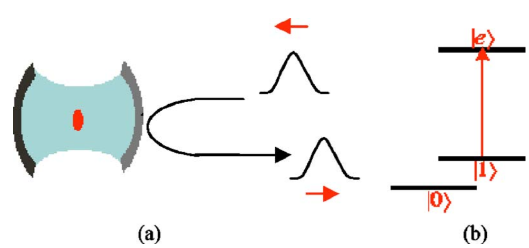

FIG. 1. (Color online) (a) Schematic setup for generation of cat states by reflecting a coherent optical pulse from a single-atom cavity. (b) The relevant level structure of the atom trapped in the cavity.

atom-cavity coupling, the frequency of the dressed cavity mode is significantly detuned from the center frequency of the input pulse. In this case, one would expect intuitively that the coupling between the atom-cavity system and the input pulse does not play an important role here, and the reflection is then similar to the reflection from a mirror, which keeps the pulse shape and phase unchanged. So the pulse will remain in the same state $|\alpha\rangle$ after the reflection, given that the amplitude α of the input pulse is not too large. Our numerical calculation confirms this expectation.

In order to generate a Schrödinger-cat state, we simply prepare the trapped atom in a superposition state $(|0\rangle+|1\rangle)/\sqrt{2}$. Then, we reflect a coherent pulse $|\alpha\rangle$ with this single-atom cavity. Following the above analysis, the final atom-photon state will become an entangled state,

$$|\Psi_c\rangle = (|0\rangle|-\alpha\rangle + |1\rangle|\alpha\rangle)/\sqrt{2}. \tag{1}$$

This entangled state can be experimentally verified through a homodyne detection of the state of the reflected photon pulse [21], correlated with a measurement of the atomic state in the basis $\{|\pm\rangle=(|0\rangle\pm|1\rangle)/\sqrt{2}\}$. With homodyne detections, one can also measure the Wigner function of the optical field with quantum state tomography [22], which fully characterizes the nonclassicality of the cat state.

With some extensions to the above method, we can generate more complicated types of cat states. First, by bouncing a series of coherent pulses (say, n pulses), each initially in the state $|\alpha\rangle$, successively from the same single-atom cavity, one will get the state $(|0\rangle|-\alpha\rangle^{\otimes n}+|1\rangle|\alpha\rangle^{\otimes n})/\sqrt{2}$, which yields entangled multipartite cat states $(|-\alpha\rangle^{\otimes n}\pm|\alpha\rangle^{\otimes n})$ (unnormalized) for the pulses after a projective measurement of the atomic state in the basis $\{|\pm\rangle\}$. Secondly, after generation of the state $(|-\alpha\rangle+|\alpha\rangle)$ for the pulse, one can transfer it to the state $(|\alpha\rangle+|3\alpha\rangle)$ through a simple linear optical manipulation (for instance, by interfering this pulse with another phase-locked stronger laser pulse at an unbalanced beam splitter, one can shift up each coherent component of the cat state by an amplitude of 2α). Then, if we reflect this pulse again from the same cavity, we will get a state $(|-3\alpha\rangle+|-\alpha\rangle+|\alpha\rangle+|3\alpha\rangle)$ for the pulse conditioned on a measurement of the atom giving the $|+\rangle$ state. It is straightforward to extend this idea to generate the multidimensional cat states $\sum_{i=-n}^{n+1}|(2i-1)\alpha\rangle$, and such states have important applications for continuous-variable quantum coding [18] and loop-hole-free detection of the Bell inequalities with efficient homodyne measurements [17].

In the above, we have presented the basic idea for preparation of the cat state (1) and described its various extensions. To understand and characterize this process better, however, we need a more detailed theoretical modeling for the interaction between the cavity atom and the light pulse. First, we want to know how large the amplitude of the cat state can be. If the amplitude $|\alpha|$ is too large, one would expect that the quantum field of an optical cavity cannot significantly change the property of a strong pulse, therefore the output state would be different from the state described by Eq. (1). Second, in practice, experiments always suffer various kinds of noise or imperfections, such as the photon loss due to the atomic spontaneous emission and the mirror scattering, the inherent pulse shape distortion induced by the reflection from the cavity, and the random variation of the cavity-mode–atom coupling rate caused by the thermal atomic motion. One needs to characterize the influence of these sources of noise on the generation of cat states.

The input to the cavity is a coherent optical pulse, whose state $|\alpha\rangle_{\rm in}$ can be described by $|\alpha\rangle_{\rm in}=\exp[-|\alpha|^2/2]\times\exp[\alpha\int_0^T f_{\rm in}^*(t)a_{\rm in}^\dagger(t)dt]|{\rm vac}\rangle$, where $a_{\rm in}^\dagger(t)$ is a one-dimensional quantum field operator with the standard commutation relation $[a_{\rm in}(t),a_{\rm in}^\dagger(t')]=\delta(t-t')$, $f_{\rm in}(t)$ describes the input pulse shape with the normalization $\int_0^T|f_{\rm in}(t)|^2dt=1$ (T is the pulse duration), and $|{\rm vac}\rangle$ represents the vacuum state for all the optical modes. The average photon number of the pulse is given by $|\alpha|^2$. The input pulse drives the cavity mode a_c through the Langevin equation [20]

$$\dot{a}_c = -i[a_c,H] - \frac{\kappa}{2}a_c - \sqrt{\kappa}a_{\rm in}(t), \tag{2}$$

where κ is the cavity decay rate, and the Hamiltonian H describes the atom-cavity interaction with the form

$$H = \hbar g(|e\rangle\langle 1|a_c + |1\rangle\langle e|a_c^\dagger). \tag{3}$$

Here, g is the atom-cavity coupling rate. The cavity output field $a_{\rm out}$ is connected to the input through the input-output relation

$$a_{\rm out}(t) = a_{\rm in}(t) + \sqrt{\kappa}a_c(t). \tag{4}$$

We need to find out the quantum state of the cavity output field $a_{\rm out}$ through the series of Eqs. (2)–(4). As they are nonlinear operator equations with infinite modes, it is hard to solve them even numerically. For the case of a single-photon pulse input, a numerical method based on the mode discretization and expansion has been developed in Refs. [19,23]. But that method does not work if the photon number of the input pulse is larger than 1, as in the case of the present work. To attack this problem, we propose a variational method based on the following observation: if the atom is in the state $|0\rangle$, the Hamiltonian (3) does not play a role, and Eqs. (2) and (4) become linear, from which we observe that the state $|\phi_0\rangle_{\rm out}$ of the output field can be exactly written as $|\phi_0\rangle_{\rm out}=\exp[-|\alpha_0|^2/2]\exp[\alpha_0\int_0^T f_{\rm out}^{(0)}(t)a_{\rm out}^\dagger(t)dt]|{\rm vac}\rangle$. The normalized shape function can be expressed as

$$f_{\rm out}^{(0)}(t) = -\int \frac{\frac{\kappa}{2}+i\omega}{\frac{\kappa}{2}-i\omega}\exp[i\omega t]f_{\rm in}(\omega)d\omega,$$

where $f_{\rm in}(\omega)$ is the Fourier transform of $f_{\rm in}(t)$. The output optical field is still in an effective single-mode coherent state, but with the mode shape function $f_{\rm out}^{(0)}(t)$ in general different from the input shape $f_{\rm in}(t)$. If the atom is in the state $|1\rangle$, it is reasonable to make the ansatz that the output optical field is also in an effective single-mode coherent state $|\phi_1\rangle_{\rm out}=\exp[-|\alpha_1|^2/2]\exp[\alpha_1\int_0^T f_{\rm out}^{(1)*}(t)a_{\rm out}^\dagger(t)dt]|{\rm vac}\rangle$, but with probably a different normalized mode shape function $f_{\rm out}^{(1)}(t)$. In general, the amplitude α_1 can be different from α (actually

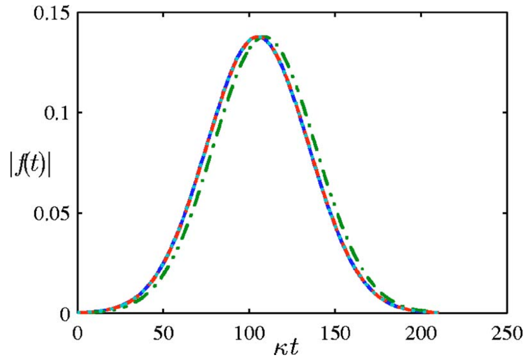

FIG. 2. (Color online) Pulse-shape functions for the input and output pulses. The solid curve shows the shape of the input pulse. The dashed-dotted, dashed, and dotted curves correspond to the output pulses with $g=0$ (for the atom in the level $|0\rangle$), $g/\kappa=3$, and $g/\kappa=6$, respectively. In the calculation, we assumed $\gamma_s=\kappa$.

$|\alpha_1|^2<|\alpha|^2$) because of the atomic spontaneous-emission loss. Due to that loss, some of the photons are scattered to other directions, so we have a weaker output field. To find out the functional form of $f_{\rm out}^{(1)}(t)$, we note that under the above ansatz, the expectation value of the input-output equation (4) leads to

$$\alpha_1 f_{\rm out}^{(1)}(t) = \alpha f_{\rm in}(t) + \sqrt{\kappa}\langle a_c(t)\rangle. \tag{5}$$

The expectation value of the cavity mode operator $a_c(t)$ can be found by solving the corresponding master equation for the atom-cavity-mode density operator ρ,

$$\dot{\rho} = -\frac{i}{\hbar}[H_{\rm eff},\rho] + \frac{\kappa}{2}(2a_c\rho a_c^\dagger - a_c^\dagger a_c\rho - \rho a_c^\dagger a_c) + \frac{\gamma_s}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-), \tag{6}$$

where $\sigma_-=|1\rangle\langle e|$ and $\sigma_+=|e\rangle\langle 1|$ are the atomic lowering and raising operators, and the effective Hamiltonian is given by $H_{\rm eff}=\hbar(g\sigma_+a_c+i\sqrt{\kappa}\langle a_{\rm in}\rangle a_c)+{\rm H.c.}$ Compared with the Hamiltonian (3), $H_{\rm eff}$ has two extra terms $i\hbar\sqrt{\kappa}\langle a_{\rm in}\rangle a_c+{\rm H.c.}$ to account for the driving from the input pulse. After that correction, the cavity decay and the atomic spontaneous-emission loss can then be described by the last two terms of the master equation (6), where γ_s denotes the atomic spontaneous-emission rate. The density operator ρ can be solved from the master equation (6) with standard numerical methods, from which we can calculate the expectation value $\langle a_c(t)\rangle={\rm tr}[\rho a_c(t)]$. Then, following Eq. (5), we can determine the output amplitude α_1 and its pulse shape $f_{\rm out}^{(1)}(t)$.

With the above method, we calculate the pulse shapes $f_{\rm out}^{(0)}(t)$ and $f_{\rm out}^{(1)}(t)$ of the output optical field with the atom in the state $|0\rangle$ and $|1\rangle$, respectively. For this calculation, we take a Gaussian shape for the input pulse with $f_{\rm in}(t) \propto \exp[-(t-T/2)^2/(T/5)^2]$, where T characterizes the pulse duration. The results are shown in Fig. 2, which demonstrates that the shape functions $f_{\rm out}^{(0)}(t)$ and $f_{\rm out}^{(1)}(t)$ of the output pulses overlap very well with $f_{\rm in}(t)$ of the input pulse when the pulse duration satisfies $T\gg 1/\kappa$. Furthermore, the global phase factors of $f_{\rm out}^{(0)}(t)$ and $f_{\rm out}^{(1)}(t)$ are given by -1 and 1, respectively, which confirms our previous expectation: if the atom is initially prepared in a superposition state $(|0\rangle+|1\rangle)/\sqrt{2}$, the final atom-photon state will be the desired entangled state $|\Psi_c\rangle$ as shown in Eq. (1), where $|-\alpha\rangle$ and $|\alpha\rangle$ are coherent states of the output mode with the mode-shape function $-f_{\rm out}^{(0)}(t)\approx f_{\rm out}^{(1)}(t)\approx f_{\rm in}(t)$. In the same figure, we have also shown the output shape $f_{\rm out}^{(1)}(t)$ for different atom-photon coupling rates g. Both the phase and the amplitude of $f_{\rm out}^{(1)}(t)$ are very insensitive to random variation of g within a certain range. For instance, even if g varies by a factor of 2 from 6κ to 3κ (which is the typical variation range of g caused by the atomic thermal motion), the change in $f_{\rm out}^{(1)}(t)$ is negligible ($<10^{-4}$) [23].

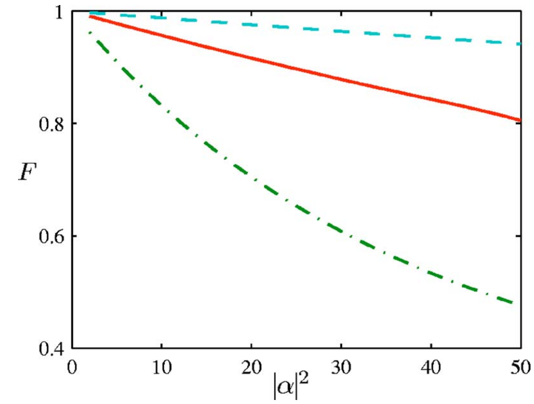

FIG. 3. (Color online) The cat-state fidelity shown as a function of the average input photon number $|\alpha|^2$ when the spontaneous-emission rate is set to zero ($\gamma_s=0$). Other parameters are $g/\kappa=3$ and $\kappa T=210$ for the solid curve; $g/\kappa=6$ and $\kappa T=210$ for the dotted curve (exactly overlapped with the solid curve); $g/\kappa=6$ and $\kappa T=100$ for the dash-dotted curve; and $g/\kappa=6$ and $\kappa T=400$ for the dashed curve.

To quantify the limit of the cat states that one can prepare and the influence of some practical noise, we introduce several quantities to measure the quality of the cat-state preparation. First, the distortion between the output and the input pulses can be measured by their pulse shape mismatching $\xi_1=1-\int f_{\rm in}(t)f_{\rm out}^{(1)*}(t)dt$ and $\xi_0=1+\int f_{\rm in}(t)f_{\rm out}^{(0)*}(t)dt$ [as $f_{\rm out}^{(0)}(t)$ has an opposite phase]. With typical experimental parameters, $\xi_0\gg\xi_1$, so ξ_0 has the dominant contribution to the imperfection of our scheme. Second, the effect of the spontaneous-emission loss can be quantified by the photon loss parameter $\eta=1-|\alpha_1|^2/|\alpha|^2$, which represents the fraction of the photons scattered to other directions instead of to the cavity output [24]. Both the pulse-shape distortion and the photon loss contribute to the imperfection of the final cat state, which can be characterized by the state fidelity. The ideal cat state is given by $|\Psi_c\rangle$ in Eq. (1), while with noise, the real state obtained is denoted by a density matrix $\rho_{\rm real}$. The fidelity, defined as $F\equiv\langle\Psi_c|\rho_{\rm real}|\Psi_c\rangle$, can be expressed by ξ_0 and η as

$$F \approx \left|\frac{e^{-|\alpha|^2(1-\sqrt{1-\eta})}+e^{-|\alpha|^2\xi_0}}{2}\right|^2, \tag{7}$$

where we have neglected the contribution of ξ_1 as $\xi_1\ll\xi_0$.

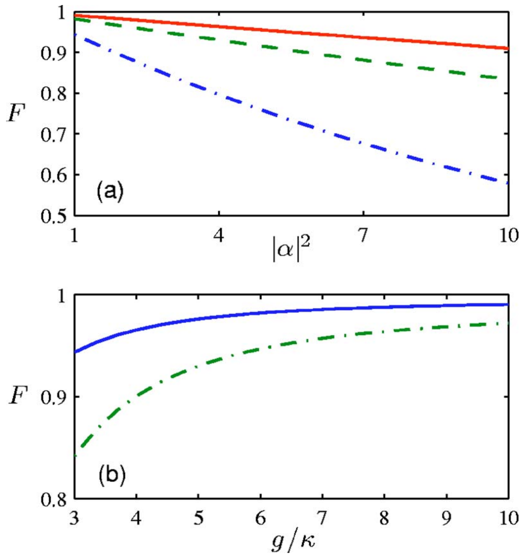

FIG. 4. (Color online) (a) The cat-state fidelity shown as a function of the average photon number $|\alpha|^2$ of the input pulse. The dash-dotted, dashed, and solid curves correspond to $g/\kappa=3$, $g/\kappa=6$, and $g/\kappa=10$, respectively. (b) The fidelity shown as a function of the coupling rate g. The solid and dashed curves correspond to the average input photon number $|\alpha|^2=1$ and $|\alpha|^2=3$, respectively. In both calculations for (a) and (b), we have taken $\gamma_s=\kappa$ and $\kappa T=210$.

First, let us examine the intrinsic limit to the amplitude of the cat state that one can prepare even if we neglect the influence of practical photon loss. This intrinsic limit comes from the fact that the quantum field of an optical cavity cannot affect the state of a strong optical field (i.e., with a large α) efficiently. For that purpose, we simply set the spontaneous-emission rate $\gamma_s=0$, and look at the state fidelity as a function of the cat-state amplitude α. The result is shown in Fig. 3, which reveals that given a certain fidelity requirement the maximal achievable cat amplitude $|\alpha|$ depends only on the pulse duration T if we completely neglect the photon loss noise. In general, a longer input pulse duration allows the generation of a larger cat state. In particular, the fidelity increases dramatically when we increase the pulse duration T as the shape distortion parameter ξ_0 significantly reduces for a pulse with a very narrow bandwidth.

We then take the influence of practical noise into account, and investigate, under typical experimental configurations, how large the amplitude of an achievable cat state can be. With the spontaneous-emission rate $\gamma_s=\kappa$, the state fidelity F is shown as a function of the cat amplitude in Fig. 4(a), and as a function of the coupling rate in Fig. 4(b). The fidelity increases with the coupling rate g and decreases with the cat amplitude α, as one would expect. We note that the photon loss from spontaneous decay reduces when the coupling rate g increases. Under a reasonable atom-cavity coupling rate $g\approx 10\kappa$ comparable with the current technology, a cat state with a remarkable amplitude $\alpha\approx 3.4$ (corresponding to an entangled state of about 10 photons) could be generated with a fidelity of 90%.

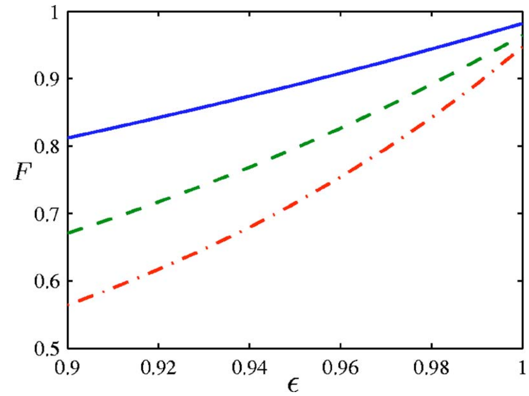

FIG. 5. (Color online) The cat state fidelity shown as a function of the mode-matching efficiency ϵ. The solid, dashed, and dash-dotted curves correspond to $|\alpha|^2=1$, $|\alpha|^2=2$, and $|\alpha|^2=3$, respectively. Other parameters are $\gamma_s=\kappa$, $g=6\kappa$, and $\kappa T=210$.

Another source of noise for cavity QED experiments is the mode-matching inefficiency between the intracavity field and the input-output beams. When the mode matching is not perfect, a portion of the input pulse will not be able to enter the cavity, so the state of the pulse will not be affected by the cavity-atom coupling efficiently. Independent of the atomic state, this portion of the light pulse will be directly reflected without phase flip. This will degrade the fidelity of our cat-state preparation. To quantify this effect, we note that if we neglect all the other imperfections, the cat-state fidelity from the mode-matching inefficiency can be described by

$$F \approx \left| \frac{1+e^{-2|\alpha|^2(1-\epsilon)}}{2} \right|^2,$$

where ϵ denotes the efficiency of the mode matching. If we take into account at the same time the other sources of noise that we considered above, the state fidelity will have a more complicated analytic expression. In Fig. 5, we show the state fidelity as a function of the mode-matching efficiency ϵ. In this calculation, we have also included the noise contributions from the atomic spontaneous emission and pulse-shape distortion. It is shown that for a cat state with a large amplitude, the fidelity could be quite sensitive to the mode-matching efficiency.

In summary, we have proposed a scheme to generate and control multipartite and high-dimensional Schrödinger-cat states for propagating optical pulses. The scheme is based on the state-of-the-art of the cavity technology. We have developed a variational calculation method which can be used to solve efficiently the interaction between the input-output quantum field and the cavity atom. This calculation technique enables us to quantitatively characterize the influence of various sources of practical noise on the performance of this scheme.

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