

Quantum-dot cluster-state computing with encoded qubits

Yaakov S. Weinstein,^{1,*} C. Stephen Hellberg,^{1,†} and Jeremy Levy^{2,‡}

¹Center for Computational Materials Science, Naval Research Laboratory, Washington, DC 20375, USA

²Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA

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A class of architectures is advanced for cluster-state quantum computation using quantum dots. These architectures include using single and multiple dots as logical qubits. Special attention is given to supercoherent qubits introduced by Bacon *et al.* [Phys. Rev. Lett. **87**, 247902 (2001)] for which we discuss the effects of various errors and present a means of error protection.

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Cluster-state computation [1] is a measurement-based scheme for universal quantum computation (QC) formally equivalent to the traditional circuit method [2]. The basis of the scheme is the cluster state [3], a highly entangled state on which measurements alone can perform universal QC. To create the cluster state one rotates all qubits in a lattice of at least two dimensions into the state $|+\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$, followed by an Ising interaction $\exp[-i(\pi/4)\sigma_z^j\sigma_z^k]$ between all nearest-neighbor qubits j, k and single-qubit σ_z^j rotations.

After creation of a cluster state, the desired algorithm is mapped onto the cluster-state lattice. Extra qubits are removed via measurement along z and QC is performed by measuring the remaining qubits along bases in the x - y plane. For certain gates, such as arbitrary single-qubit rotations, the outcome of a given measurement is necessary to determine in which basis future measurements should be performed [1]. In such cases the measurements must be performed in a specified order.

For some systems, measurements along the z axis may be easier than those in the x - y plane [4]. In this case one can rotate the qubit to be measured from the x - y plane along z and perform the measurement. For example, a process equivalent to measurement along an axis an angle ϕ away from positive x , is a $\pi/2$ rotation about the axis $\phi - 90^\circ$ and a measurement along z . Alternatively, all of the qubits can be rotated as long as they are rotated back after the measurement.

Due to the centrality of the Ising interaction in cluster-state creation, proposed implementations of cluster-state QC [5–10] have included little on systems with direct spin-spin coupling. However, these systems do allow for cluster-state computation [11]. In this work we discuss several cluster-state QC models for systems with Heisenberg interaction. We outline the hardware requirements and possible error operators in each model. Encoding logical qubits (LQs) into several Heisenberg coupled physical qubits allows for a natural Ising-like inter-LQ evolution, reduces hardware require-

ments, and allows for protection against various forms of errors and decoherence. While these models are applicable to any Heisenberg coupled system we refer specifically to quantum-dot implementations.

The first architecture we present for quantum-dot cluster-state computation is a two-dimensional array of quantum dots. As in [11] each physical qubit acts as a LQ. After initialization, the dots can be rotated into the state $|+\rangle$ using a global magnetic field about y . For a pair of dots, an Ising interaction can be performed by evolution of the Heisenberg coupling, $U_1 = \exp[-i(\pi/8)\sigma^1\sigma^2]$, applying a π rotation to one of the dots about z , and again applying U_1 . For an array of dots an Ising interaction between all nearest neighbors can be implemented by turning on all nearest-neighbor Heisenberg couplings and applying π_z rotations to every other dot. This requires individual addressability of the dots, via g -factor engineering or local magnetic fields, or alternative placement of two species of quantum dots (A and B) in which all dots of a species are addressed equivalently without effecting dots of the other species. Turning on all nearest-neighbor couplings at once introduces many-body terms into the Hamiltonian [12]. Instead, the cluster state can be built in four steps as in Fig. 1(a). The construction is such that no qubit undergoes more than one coupling at each step. After the coupling terms, single-dot (or species of dot) rotations are needed to complete the cluster state.

Readout of the individual quantum dots can be accomplished via any of the suggested single-dot measurement schemes [13]. Certain schemes may be limited to measurement along the z axis [4] in which case all of the qubits can be rotated (as explained above) using a global magnetic field.

Let us summarize the necessary components for this architecture. The hardware requirements are a two-dimensional lattice of individually addressable dots or alternative placement of two species of dots in which each species is addressed equivalently without affecting the other species. In addition, it is necessary to control the interactions between dots and perform single quantum-dot measurements.

We can remove the necessity of performing logical operations with magnetic fields by encoding LQs into two physical qubits, one of each species of dot (where the species have different g factors). Again the dots are arranged in a two-dimensional lattice and placed in a magnetic field [14]. The logical qubit states are: $|0_L\rangle = |0_A 1_B\rangle$ and $|1_L\rangle = |1_A 0_B\rangle$. This

*Present address: Quantum Information Science Group, MITRE, Eatontown, NJ, 07724. Author to whom correspondence should be addressed. Electronic address: weinstein@mitre.org

†Electronic address: hellberg@dave.nrl.navy.mil

‡Electronic address: jlevy@pitt.edu

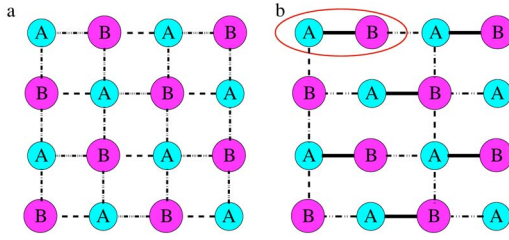


FIG. 1. (Color online) Two-species quantum-dot cluster architectures. (a) A two-species cluster where each logical qubit is one physical qubit. The cluster state for this architecture can be built in four coupling steps each requiring the application of U_1, π_z rotation on species A or B, and again U_1 . Four steps are necessary in order that a qubit will not interact with more than one neighbor at each step. The connecting lines show which dots are coupled at each step: dashed, dash-dot, dash-dot-dot, and dash-dot-dot-dot, the order of the steps being unimportant. (b) Each logical qubit is composed of two physical qubits, one from each species A and B (joined by solid lines). To build a cluster state one uses the intra-LQ coupling to create the $|+\rangle$ state on each qubit, and three coupling steps (dashed, dash-dot, and dash-dot-dot lines) each consisting of inter-LQ coupling and single LQ z rotations.

encoding allows for the performance of single LQ x rotations via the Heisenberg coupling between the physical qubits within the LQ (the intra-LQ coupling) and z rotations by simply waiting and allowing the Zeeman term to evolve. Rotations about $\hat{z}$ require a time $\sim 1/B_z \Delta g$, where B_z is the strength of the magnetic field and $\Delta g = g_A - g_B$ is the difference in species g factor (an alternate method to perform z rotations is given in [15]). Inter-LQ interactions are done via the Heisenberg coupling connecting two LQs. The latter operation takes part of the system state outside the LQ subspace. However, a logical π_z rotation on one of the LQs refocuses the unwanted part of the evolution leaving a logical Ising coupling, $\sigma_{z_L}^j \sigma_{z_L}^k$. In addition, this encoding protects against collective dephasing errors [16].

With the above encoding a cluster state can be created as follows. Rotate all LQs into the $|+\rangle$ state via intra-LQ couplings. The Ising coupling between LQs, described above, is applied in three steps so as not to introduce multiple couplings on one physical qubit. Each step requires logical π_z rotations on alternate LQs only. Finally, single LQ rotations are done using the Zeeman interaction. The required lattice for this architecture and the scheme for cluster-state construction is shown in Fig. 1(b). Measurement of the LQs can be done by measuring the individual quantum dots directly [14], via singlet-triplet measurements [17].

To summarize, the requirements for cluster-state computation using LQs encoded in two physical qubits are two species of quantum dots placed alternately in a two-dimensional lattice. The two-species g factors should be different enough such that LQ z rotations can be done within a reasonable time. In addition, it is necessary to control the interactions between dots and perform either single-dot or singlet-triplet measurements.

The previous cluster-state constructions all require the utilization of more than one species of physical qubit. By incorporating more advanced LQs we can remove this neces-

sity, perform computations using just the Heisenberg interaction and singlet-triplet measurement, while making the LQs robust against a variety of errors. In addition, cluster-state construction becomes easier since the Ising interaction is the natural coupling between the LQs. An example of this type of decoherence free subspace (DFS) is the three physical-qubit encoding discussed in Ref. [18]. This encoding provides DFS protection against collective errors. The inter-LQ Hamiltonian for this three-qubit encoding is $\sigma_{z_L}^j \sigma_{z_L}^k$ plus single LQ z rotations which can be modified by intra-LQ couplings.

A more robust encoding is the supercoherent qubit (SQ) of Ref. [19]. SQs minimize decoherence by forcing all reasonable interactions with the environment to overcome an energy gap between the LQ subspace and other states of the system. Environment-induced collective and single physical-qubit local errors must overcome this energy gap.

A SQ in its idle state consists of four physical qubits with equal nonzero Heisenberg coupling (set to 1 for simplicity) between all physical-qubit pairs. An orthogonal basis for the doubly degenerate ground state comprises of singlet or triplet states between physical qubits 1,2 and physical qubits 3,4,

$$\begin{aligned}
 |0_L\rangle &= \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{1,2} \otimes (-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_{3,4} \\
 |1_L\rangle &= \frac{1}{\sqrt{3}}(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle) - \frac{1}{2\sqrt{3}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_{1,2} \\
 &\quad \otimes (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_{3,4}.
 \end{aligned} \tag{1}$$

The state $|0_L\rangle$ incorporates the singlet state between spins 1 and 2 and spins 3 and 4. The state $|1_L\rangle$ incorporates triplet states between the same spins. We will refer to the pairs of spins 1-2 and 3-4 as “singlet” pairs since they are in a singlet state when the LQ is in the logical state $|0_L\rangle$. Measurement of the SQ is done via singlet-triplet measurement schemes [17]. Initialization of the SQ is done by raising or lowering one of the couplings, thereby breaking the ground-state degeneracy, and waiting for the system to decay [20].

Single SQ rotations are performed by changing the Heisenberg coupling strength between appropriate pairs of qubits. Changing a coupling between a qubit and its “singlet” pair, 1 and 2, for example, performs a logical z rotation. Changing a coupling between a qubit and one that is not its pair, for example, 2 and 3, performs a rotation about the axis in the x - z plane 120° from the z axis [18–20]. Combinations of these operations are sufficient to perform any $SU(2)$ rotation.

To perform inter-SQ interactions Ref. [19] considers eight physical qubits with equal Heisenberg coupling between all pairs. In this way interactions between the four-qubit SQs can be implemented and universal quantum computation can proceed without leaving the LQ subspace. The problem with this coupling scheme is its realization. As already noted in [19], building two coupled SQs would be a daunting task and scaling up will only add complications.

A simplified scheme for coupling SQs is to perform the inter-SQ couplings adiabatically [18]; doing so insures that

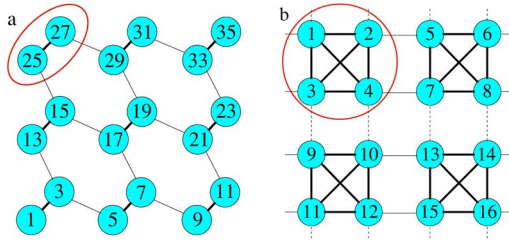


FIG. 2. (Color online) Proposed layouts for a two-dimensional cluster state made of supercoherent qubits (SQs). The thick lines represent the always-on exchange interaction which exist between all pairs within a single SQ. These are modified to perform single LQ rotations. The thin lines represent exchange interactions between SQs which are turned on to perform inter-SQ coupling. Two couplings are necessary to perform these interactions. (a) Two-layer architecture in which each layer consists of two of the four physical qubits per SQ. Only the top layer (odd numbers) is shown. The two even-numbered physical qubits are placed directly beneath the odd-numbered ones. Couplings exist between all pairs within a SQ. (b) Four SQs on a two-dimensional lattice requiring two steps to cluster-state creation. First, inter-SQ couplings are turned on between nearest horizontal SQs (solid lines) creating a cluster state across each row of the lattice. Then a SWAP is applied between physical qubits at the ends of a diagonal within each SQ (for example, 1 and 4) and the coupling is turned on between horizontal nearest neighbors (dashed lines). Another SWAP and single SQ z rotations (done by intra-SQ coupling) complete the required inter-SQ evolution.

the system returns to the logical subspace when the couplings between SQs are turned off. This condition is not difficult to satisfy. Adiabatic evolution is a requirement for *all* approaches using spins in quantum dots [21]. A coupling between one physical qubit in each LQ does not induce any evolution of the SQ, since SQs are robust against local operations. Instead, coupling between pairs of physical qubits are necessary. The pairs which give Ising coupling between SQs are those that are in a singlet (triplet) in the state $|0_L\rangle$ ($|1_L\rangle$), such as 1 and 2. When pairs of such qubits in two SQs are coupled the resulting SQ coupling is diagonal consisting of $\sigma_{z_L}^j \sigma_{z_L}^k$ and equal single SQ logical σ_{z_L} rotations. This is because the coupling breaks the degenerate eigenvalues into LQ eigenstates. The state does not leak out of the SQ subspace due to the gap between the SQ subspace and the rest of the system eigenvalues [Fig. 3(a)]. The two-SQ interaction plus equal logical z rotations on each of the two SQs implement an Ising interaction which, combined with arbitrary single SQ rotations, allow for a universal set of gates [22].

To create the cluster state we need to arrange the SQs in two dimensions. This is complicated by the fact that only couplings between certain pairs of physical qubits lead to a SQ diagonal coupling. We suggest two designs which allow the appropriate qubits to be coupled. The first requires arranging dots in two layers. The top layer of such an architecture is shown in Fig. 2(a) and the second layer is placed directly underneath. Each SQ consists of two physical qubits on each layer with identical couplings between all physical qubits.

An alternative is to arrange the SQs in a two-dimensional

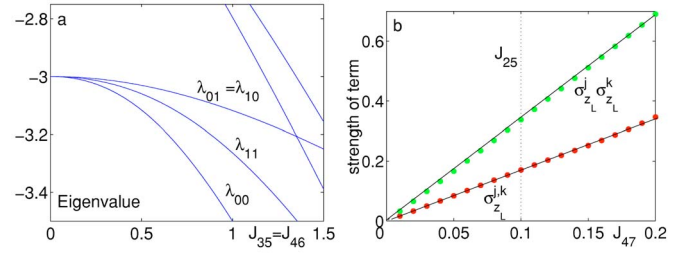


FIG. 3. (Color online) (a) Eigenvalues of the two-SQ system during coupling. As long as the J_{25} and J_{47} [using labels of Fig. 2(b) and assuming the singlet pairs are vertical] couplings are performed adiabatically and are below ≈ 1.4 the system will remain in the logical qubit subspace due to the observed gap between the logical qubit subspace and the rest of the system states. Under coupling the nondegeneracy is broken into the eigenstates of the logical computational basis leading to a logical diagonal coupling. (b) The effect of unequal couplings between physical qubits when implementing an inter-SQ coupling. While $J_{25}=0.1$ is held constant (vertical line) the effect of changing J_{47} is a linear change in strength of the one-SQ $\sigma_{z_L}^j$ terms and the two-SQ $\sigma_{z_L}^j \sigma_{z_L}^k$ term.

lattice as in Fig. 2(b). We use this arrangement to show how to implement the Ising interaction between all SQs. Assume that the singlet pairs are horizontal (1-2, 3-4). Then the vertical couplings between the SQs (3-9, 4-10) can be turned on to implement the Ising interaction (plus single SQ z rotations). We then apply a SWAP operation between dots on the diagonal of each SQ (1-4, 5-8) thereby swapping the state of those dots. The SWAP is implemented by increasing the Heisenberg interaction between those qubits for a time $t = \pi\Delta J$ where ΔJ is the amount the coupling is changed from its original value. The singlets are now vertical (1-3, 2-4) and turning on horizontal couplings (2-5, 4-7) implements an Ising interaction between the horizontally arranged SQs. Another SWAP between physical qubits along the diagonal of the SQs completes the interactions.

To summarize, encoding logical qubits into three and four physical qubits allows for cluster-state computation with only one species of quantum dot. These logical qubits are robust against a variety of decoherence mechanisms and allow for universal cluster-state computation without using magnetic fields to manipulate qubits. The challenge is the arrangement of LQs in two dimensions and the ability to change the coupling strength between physical qubits.

The three- and four-qubit encodings are a powerful means of protecting quantum information against a variety of errors in both circuit model and cluster-state quantum computation. We now present a basic examination of possible errors in cluster-state QC when performed with these architectures and possible correction methods.

The three- and four-qubit encodings require equal couplings between multiple qubits. This is exceedingly difficult because the coupling strength depends exponentially on the spacing between the electrons [23]. For the four-qubit encoding the challenge is greater since one cannot put the dots at equal distances from each other in just two dimensions. Methods of dealing with this latter issue will be addressed elsewhere. The error due to an unequal coupling in an LQ is a constant, linear, single-LQ rotation similar to the chemical

shift term in liquid-state nuclear magnetic resonance (NMR) [25]. A possible error-correcting technique is open-loop control or bang-bang operations [24]. As in NMR decoupling schemes, logical π pulses can be applied to refocus unwanted evolution. A hardware approach is to digitize the interaction between qubits by enforcing minimum and maximum coupling strengths and a smooth transition between them. A method of achieving a maximum coupling is by moving electrons transversely in channels [23]. This forces the electrons to remain a certain minimum distance apart. Couplings can be made equal by bringing neighboring electrons to this minimum separation.

Inter-LQ couplings provide another possible source of errors. For both the three- and four-physical-qubit encodings there is the problem of insuring that the coupling is exactly zero when required. This can be done to some accuracy through electrostatic gates and improved by bang-bang decoupling.

Inter-SQ evolution requires two adiabatic couplings between pairs of physical qubits. Errors that may occur include the ability to turn on the couplings simultaneously and the possibility that the coupling strengths are unequal. As SQ are immune to single physical-qubit errors, one inter-SQ cou-

pling will not effect the state of the SQs. Thus, the task of turning the couplings on at the same time is only an issue during the ramp up when the couplings may be unequal. For unequal coupling strengths the one-SQ $\sigma_{z_L}^j$ terms and the two-SQ $\sigma_{z_L}^j \sigma_{z_L}^k$ terms depend linearly on the difference in coupling strengths [Fig. 3(b)]. No other terms are introduced because of the inequality. This error can be corrected with LQ π pulses or by enforcing a maximum (and equal) coupling.

In conclusion, we have suggested several ways of achieving cluster-state quantum computation using quantum dots. The architectures range from having one to four physical qubits per logical qubit. We have also analyzed possible errors for the three- and four-qubit encodings and suggested ways of fixing them. Which architecture is most practical depends on the particular physical implementation.

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