PHYSICAL REVIEW A **72**, 020101(R) (2005)

Investigation of the Casimir force between metal and semiconductor test bodies

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(Received 23 January 2005; published 11 August 2005)

The measurement of the Casimir force between a large gold coated sphere and single crystal silicon plate is performed with an atomic force microscope. A rigorous statistical comparison of data with theory is done, without use of the concept of root-mean-square deviation, and excellent agreement is obtained. The Casimir force between metal and semiconductor is demonstrated to be qualitatively different than between two similar or dissimilar metals, which opens opportunities for applications in nanotechnology.

DOI: 10.1103/PhysRevA.72.020101 PACS number(s): 12.20.Fv, 12.20.Ds, 68.37.Ps

In this paper we present the results of the experimental and theoretical investigation of the Casimir force acting between a gold coated sphere and a single crystal silicon plate. The Casimir force is determined by the alteration of zero-point oscillations of the electromagnetic field due to the presence of material boundaries (see the original paper [1] and monographs [2–4]). The Casimir force is in fact the limiting case of the van der Waals force when the separation between the test bodies becomes large enough for retardation to be included. Historically, most measurements of the Casimir force were performed between dielectrics (see Ref. [5] for review), and anomalous behavior in silicon has been reported [6]. In the last few years many precise experiments using metallic test bodies have been done and the results were compared with theory, taking into account different corrections to the Casimir force [5,7–12]. The obtained results have been found to be of prime importance in the physics of nano- and micromechanical systems [13] and for testing predictions of extra-dimensional models and other theoretical schemes beyond the standard model [12].

To gain a better insight of the role of the Casimir effect in nanotechnology, it is important to understand the effect of semiconductor test bodies. These materials are central to the fabrication and design of nano- and microdevices and provide a wide variety of electrical properties that may influence the Casimir force. Until now, however, no precise experiments on the Casimir effect with semiconductor bodies have been performed (see the discussion on the importance of this subject in Ref. [14]). Below we demonstrate that the ratio of the Casimir forces between Au and Si test bodies to that from Au-Au decreases with the increase of separation. This is qualitatively different from the case when Si is replaced with some metal of lower conductivity than Au, where the same ratio is practically constant or increases with separation. Another important point of this paper is the comparison between the measurement data and theory without the use of the concept of the root-mean-square deviation widely employed in previous experiments on the Casimir effect. As was shown in Ref. [15], this approach may be inadequate when the measured force rapidly changes with separation distance, though no better approach was suggested.

One test body is a sphere attached to a cantilever of an atomic force microscope (AFM). The sphere is coated with an Au layer of 105 nm thickness. The diameter of the sphere was measured using a scanning electron microscope to be $2R=202.6\pm0.3$ μm. The other test body is a 5×10 mm^2 single crystal silicon Si$\langle100\rangle$ plate. The nominal resistivity of the Si plate was 0.01–0.001 Ω cm. Using the four-probe technique, we measured its resistivity to be $\rho=0.0035$ Ω cm. Note that for all frequencies contributing to the Casimir force, this Si plate, unlike metals, has a large absorption typical of semiconductors (metallic resistivities are usually two or three orders of magnitude lower). The Casimir force acting between the Au sphere and Si plate was measured by means of the improved setup previously used in Ref. [9] for two Au test bodies. The main improvements and innovations implemented in this experiment are: We now use a much higher vacuum 2×10^{-7} Torr (instead of 3×10^{-2} Torr in Ref. [9]) to maintain the chemical purity of the Si surface, which oxidizes rapidly to SiO_2. In addition, this vacuum system is oil-free, consisting of oil-free mechanical pumps, turbo pumps, and ion pumps to prevent contamination. To reduce the influence of mechanical noise during data acquisition, only the ion pump is used to maintain the lowest pressures.

A special passivation procedure is used to prepare the Si surface. First nanostrip (a combination of H_2O_2 and H_2SO_4) is used to clean the surface of organics and other contaminants. This oxidizes the surface. Then we use 49% HF to etch SiO_2 and hydrogen to terminate the surface. The termination is stable for more than two weeks under the vacuum conditions described above [16]. The bottom of the Si plate is coated with about 100 nm of Au and used for the electrical contact. It was checked to be ohmic in nature. The above steps were necessary to keep the residual potential difference low, constant, and independent of separation distance.

The next improvement is the reduction of the uncertainty in the determination of absolute separation distances z down

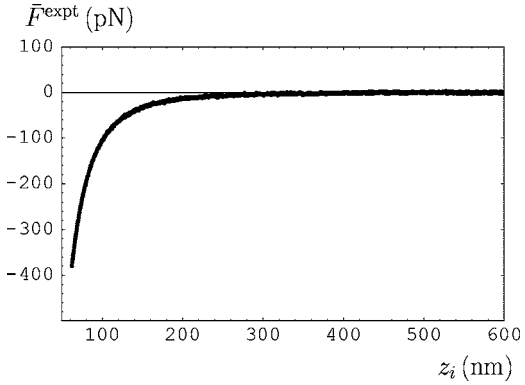

FIG. 1. The mean measured Casimir force as a function of separation between the Si plate and the Au sphere.

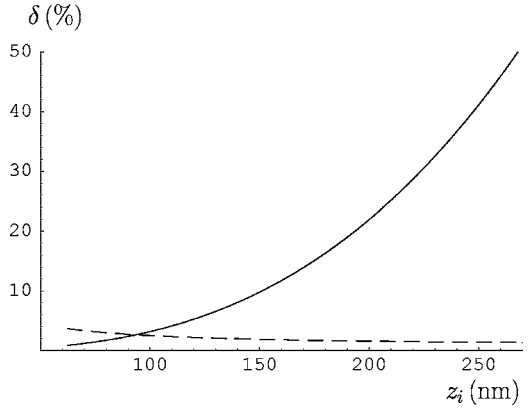

FIG. 2. The total relative experimental δ^{expt} (solid curve) and theoretical δ^{theor} (dashed curve) errors as a function of plate-sphere separation.

to $\Delta z=0.8$ nm (in comparison with 1 nm in Ref. [9]). To achieve this aim, here we use a piezo capable of traveling a distance of 6 μm from initial separation to contact of the two surfaces (previously [9] the movement of the plate to large separations was done mechanically and the piezo movement was used only at short separations of less than 2 μm). All 6 μm of piezo movement are calibrated interferometrically. As a result, the error in the piezo calibration practically does not contribute to Δz. Then different dc voltages between +0.2 to −0.4 V were applied to the plate and the electric force was measured. The electric force measurement with each voltage was repeated five times and the average value was used to fit the exact electrostatic force-distance relation [9] to determine the separation on contact of the two surfaces z_0. The resulting value, which is not zero due to the roughness of surfaces, is $z_0=32.1\pm0.8$ nm. The error in z_0 completely determines the error Δz of all measured absolute separations z. The values of z are found independently, without fitting to the theoretical expression for the Casimir force.

The same procedure also allowed an independent determination of the residual potential difference V_0 at different separations. The V_0 was determined to be $V_0=-0.114\pm0.002$ V and independent of the separation. This allowed us to confirm the absence of any contamination of the Si surface and the absence of localized charges (the presence of localized charges would lead to dipole and other multipolar electrostatic fields, resulting in a V_0 that varies with distance). The high conductivity of the Si plate used is important in preventing the formation of such charges.

Finally the Casimir force between the sphere and the plate as a function of distance is measured. The sphere is kept grounded while a compensating voltage corresponding to V_0 is applied to the plate to cancel the residual electrostatic force. The distance was varied from large to short separations by applying a continuous voltage to the piezo. The force data $F^{\text{expt}}(z_i)$ were collected at equal time intervals corresponding to separations z_i having a step size of 0.17 nm. This measurement was repeated for $n=65$ times.

We now turn to a determination of experimental errors and precision. First the experimental points were analyzed for the presence of so-called "outlying" results using the statistical criteria of Ref. [17]. It was found that none of the $n=65$ sets of measurements are outlying and all of them can be used in error analysis. To find the random error the mean values of the measured force over all sets of measurements $\bar{F}^{\text{expt}}(z_i)$ are calculated at all points z_i $(1\leqslant i\leqslant 3164)$. The mean experimental force as a function of separation for the distance range 62.33 to 600.04 nm is shown in Fig. 1. An estimate for the variance of this mean $s_{\bar{F}}(z_i)$ is not uniform, i.e., changes randomly when the separation changes less than $\Delta z=0.8$ nm. In this case, the best estimate for a variance is calculated by a statistical procedure [18] in the theory of repeated measurements (see Ref. [19] for details). Then the variance is approximately the same for all z_i and equal to $s_{\bar{F}}=1.5$ pN.

Using the Student's t distribution with a number of degrees of freedom $f=n-1=64$ and choosing $\beta=0.95$ (the hypothesis is true at 95% confidence), we obtain $p=(1+\beta)/2=0.975$ and $t_p(f)=2.00$. Then for the confidence interval it follows $|\bar{F}^{\text{expt}}(z)-F(z)|\leqslant\Delta^{\text{rand}}F^{\text{expt}}\equiv s_{\bar{F}}t_p(f)\approx 3.0$ pN. Here $F(z)$ is the true value of the Casimir force at a separation z (this exact value can only be approached with complete knowledge of all possible corrections) and $\Delta^{\text{rand}}F^{\text{expt}}$ is the random absolute error of force measurements in the present experiment. It is almost two times smaller than the random error in the experiment of Ref. [9] with two gold test bodies.

The systematic errors are the same as in the experiment with two gold bodies (see the second paper in Ref. [9]). They are given by the error in force calibration (0.82 pN), by the noise when the calibration voltage is applied to the cantilever (0.55 pN), by the instrumental sensitivity (0.31 pN), and by the restrictions on computer resolution of data (0.12 pN). The combined systematic error in Ref. [9] was, however, overestimated. To obtain the best estimate for it, the difference between the experimental and true force values at each separation is assumed to be distributed uniformly. The resulting systematic error $\Delta^{\text{syst}}F^{\text{expt}}\approx 1.17$ pN at 95% confidence is given by the composition of N uniform distributions [17] (in contrast with 2.7 pN obtained in Ref. [9]). The total experimental error of the Casimir force $\Delta^{\text{tot}}F^{\text{expt}}\approx 3.33$ pN at 95% confidence is obtained from Ref. [17] by combining the above random and systematic errors. In Fig. 2 the relative error $\delta^{\text{expt}}=\Delta^{\text{tot}}F^{\text{expt}}/\bar{F}^{\text{expt}}$ is given by the solid curve as a function of separation. It is equal to only 0.87% at the short-

est separation of 62.33 nm and increases with an increase of separation.

For separations used here, the thermal corrections at $T=300$ K are not significant. As noted in Ref. [20], in this case one can use the Lifshitz formula at zero temperature [21]

$$F_c(z)=\frac{\hbar R}{2\pi}\int_0^\infty k_\perp dk_\perp \int_0^\infty d\xi\{\ln[1-r_\parallel^{(1)}r_\parallel^{(2)}e^{-2zq}] + \ln[1-r_\perp^{(1)}r_\perp^{(2)}e^{-2zq}]\}. \quad (1)$$

The reflection coefficients for two independent polarizations are given by

$$r_\parallel^{(p)}=\frac{\varepsilon^{(p)}(i\xi)q-k^{(p)}}{\varepsilon^{(p)}(i\xi)q+k^{(p)}}, \quad r_\perp^{(p)}=\frac{k^{(p)}-q}{k^{(p)}+q}, \quad (2)$$

where $q^2\equiv k_\perp^2+\xi^2/c^2$, ${k^{(p)}}^2\equiv k_\perp^2+\varepsilon^{(p)}(i\xi)\xi^2/c^2$, and $\varepsilon^{(p)}(\omega)$ is the dielectric permittivity of gold ($p=1$) and silicon ($p=2$).

$\varepsilon^{(1)}(i\xi)$ was found [5] by means of the dispersion relation from the imaginary part of $\varepsilon^{(1)}(\omega)$ obtained using the complex refractive index from tables [22]. The same procedure was used for single crystal Si. Since the optical properties of Si at low frequencies depend on the concentration of charge carriers, the tabulated data in Ref. [22], obtained for a sample of high resistivity $\rho_0=1000\ \Omega$ cm, should be adapted for the silicon plate used in our experiment with a resistivity $\rho=0.0035\ \Omega$ cm. This is achieved [22] by adding the imaginary part of the Drude dielectric function to the imaginary part of the dielectric permittivity obtained using the data from tables. In doing so the plasma frequency for Si at a resistivity ρ is found from $\omega_p^{\mathrm{Si}}=2\sqrt{\pi}/\sqrt{\varepsilon_0\rho\tau^{\mathrm{Si}}}=6.37\times10^{14}$ rad/s, where ε_0 is the dielectric permittivity of vacuum, $\tau^{\mathrm{Si}}=1/\gamma^{\mathrm{Si}}=10^{-13}$ s [22] is the Si relaxation time, and γ^{Si} is the relaxation parameter (note that change of ω_p even by a factor of 1.5 leads to less than a 1% change in the Casimir force magnitudes within the entire separation region). Within our range of characteristic frequences there are only negligible differences in the values of $\varepsilon^{(1)}(i\xi)$ found for the sample of resistivity ρ, used in this experiment, as compared to Si with much higher resistivity ρ_0, as in the tables [22]. Thus, the relatively high conductivity of our Si plate plays an important role in avoiding charging but, at the same time, the sample demonstrates the typical semiconductor frequency dependence of $\varepsilon(i\xi)$ within the frequency range contributing to the force.

For comparison of the theoretical results with the experiment, one should take into account the surface roughness corrections. The topography of the Au coating on the sphere and of the Si plate was measured using an AFM. It was found that roughness is mostly represented by the stochastically distributed distortions with the typical heights 11–20 nm on the sphere and 0.3–0.6 nm on the Si plate. There are also rare pointlike peaks on the sphere with the heights up to 25 nm. Denoting by $v_k^{(p)}$ the fractions of the surface area with roughness height $h_k^{(p)}$ ($p=1$ for a sphere and $p=2$ for a plate), one can find the zero roughness levels $H_0^{(1)}\approx15.35$ nm, $H_0^{(2)}\approx0.545$ nm. In the additive approach the theoretical Casimir force, including both finite conductivity and surface roughness corrections, can be calculated as [5,8,9,12]

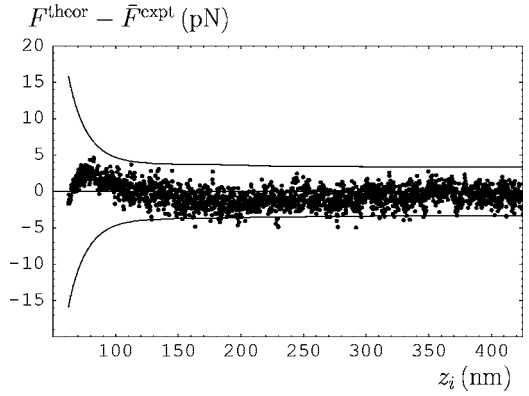

FIG. 3. The 95% confidence intervals (solid curves) and the differences between the theoretical and mean measured Casimir forces vs plate-sphere separation.

$$F^{\mathrm{theor}}(z_i)=\sum_{k,j} v_k^{(1)}v_j^{(2)}F_c(\tilde{z}_i), \quad (3)$$

where $\tilde{z}_i=z_i+H_0^{(1)}+H_0^{(2)}-h_k^{(1)}-h_j^{(2)}$, and the values of F_c are obtained from Eq. (1). As it was demonstrated in Refs. [9,12], for such values of roughness the diffraction-type contributions [23,24] are negligible.

The two main errors in the theoretical Casimir force $\delta_m^{\mathrm{theor}}=\Delta^{(m)}F^{\mathrm{theor}}/|F^{\mathrm{theor}}|$ are due to the use of the proximity force theorem ($m=1$) and due to sample to sample variations of the optical data ($m=2$). As is concluded in Ref. [9], $\delta_1^{\mathrm{theor}}<z/R$ and $\delta_2^{\mathrm{theor}}\approx0.5\%$ (the other errors contained in the theoretical force due to the influence of patch potentials, spatial dispersion, and the finite size of the plate were shown [9,19] to be much smaller). In the absence of exact information, both random quantities are assumed to be distributed uniformly (i.e., can take any value with equal probability within the fixed intervals determined by the respective absolute errors; this assumption is the most conservative because the use of any other probability distribution rule leads to a smaller combined error). For this reason the resulting error $\delta_0^{\mathrm{theor}}$ at 95% confidence can be found once more from Ref. [17]. Another type of error in the theoretical Casimir force arises when one substitutes into Eqs. (1) and (3) the experimental data for separations z_i and sphere radius R. It is given by [25] $\delta_3^{\mathrm{theor}}\approx\Delta R/R+3\Delta z/z$ (here we do not use the additional fit [9] in order to decrease Δz because the comparison between theory and experiment is not based on the root-mean-square deviation).

To determine the total theoretical error of the Casimir force computations $\delta^{\mathrm{theor}}=\Delta^{\mathrm{tot}}F^{\mathrm{theor}}/|F^{\mathrm{theor}}|$, one should combine the errors $\delta_0^{\mathrm{theor}}$ and $\delta_3^{\mathrm{theor}}$. In doing so we take into account that these errors are described by nonuniform and uniform distributions, respectively. The quantity δ^{theor} as a function of separation is plotted in Fig. 2 as a dashed curve. Finally, combining the total experimental, $\Delta^{\mathrm{tot}}F^{\mathrm{expt}}$, and theoretical, $\Delta^{\mathrm{tot}}F^{\mathrm{theor}}$, errors in a conservative way, we obtain the resulting absolute error $\Xi(z)$ for the difference $F^{\mathrm{theor}}(z)-F^{\mathrm{expt}}(z)$.

Now we are in a position to compare theory with the experiment. In Fig. 3 the differences of the theoretical and mean experimental Casimir forces (shown in Fig. 1) are plotted. In the same figure the solid curves exhibit the confidence interval $[-\Xi(z),\Xi(z)]$ computed for any z at 95% confidence. As is seen from Fig. 3, almost all differences between the theoretical and experimental forces (not just 95% of them as is required by the accepted confidence) are well within the confidence interval, i.e., theory is in excellent agreement with data [we do not plot the results at $z>425$ nm as the force magnitudes there are less than $\Xi(z)$]. Quantitatively, the rigorous measure of agreement between theory and experiment is equal to $\Xi(z)/|F^{\mathrm{theor}}|$. This quantity results in the smallest value of 3.8% within the separation region from 75.8 to 81.5 nm. It is notable, however, that the actual difference between the theoretical and experimental force values are less than 1% of force magnitude within the separations from 62.33 to 69.98 nm. At the same time the rigorous measure of agreement in this interval varies between 4.15% and 3.9%.

To conclude, we have performed a measurement of the Casimir force between large Au sphere and single crystal Si plate with the experimental relative error equal to 0.87% at the shortest separation. Data are found to be in excellent agreement with theory demonstrating that this measurement is both precise and accurate. At the same time, the uncertainties in the measurement of surface separations do not permit one to obtain the theoretical results of the same precision as the experimental ones at separations less than 100 nm. The case of metal-semiconductor test bodies appears to be quite different from the case of dissimilar metals Au-Cu [12] where no noticeable changes of the force magnitude were found in comparison with the Au-Au system. Here the ratio of the Casimir forces between Au and Si to Au-Au is 0.74 at the shortest separation. At a separation of 200 nm, it is only 0.63. This reduction can be understood physically from lower reflectivity of a semiconductor in comparison to a metal. The distance dependence of the above ratio is explained by the fact that the force between Au-Au bodies decreases with the increase of separation distance more slowly than between Au-Si bodies. We note that if our silicon plate were to behave as a metal instead of a semiconductor, the ratio under discussion would be practically constant or increase with the increase of separation. This behavior of a metal-semiconductor system in comparison with the case of two metals opens opportunities for the modulation of the Casimir force due to material properties in micro- and nanoelectromechanical systems.

This work was supported by the NSF Grant No. PHY0355092 and DOE Grant No. DE-FG02-04ER46131. G.L.K. and V.M.M. were also supported by Finep.

[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[2] P. W. Milonni, *The Quantum Vacuum* (Academic Press, San Diego, 1994).
[3] V. M. Mostepanenko and N. N. Trunov, *The Casimir Effect and its Applications* (Clarendon, Oxford, 1997).
[4] K. A. Milton, *The Casimir Effect* (World Scientific, Singapore, 2001).
[5] M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rep. **353**, 1 (2001).
[6] W. Arnold, S. Hunklinger, and K. Dransfeld, Phys. Rev. B **19**, 6049 (1979).
[7] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
[8] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998); G. L. Klimchitskaya *et al.*, Phys. Rev. A **60**, 3487 (1999).
[9] B. W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000); F. Chen *et al.*, Phys. Rev. A **69**, 022117 (2004).
[10] G. Bressi *et al.*, Phys. Rev. Lett. **88**, 041804 (2002).
[11] F. Chen *et al.*, Phys. Rev. Lett. **88**, 101801 (2002).
[12] R. S. Decca *et al.*, Phys. Rev. D **68**, 116003 (2003).
[13] H. B. Chan *et al.*, Science **291**, 1941 (2001).
[14] S. K. Lamoreaux, Rep. Prog. Phys. **68**, 201 (2005).
[15] T. Ederth, Phys. Rev. A **62**, 062104 (2000).
[16] D. Gräf *et al.*, J. Appl. Phys. **68**, 5155 (1990); K. Arima *et al.*, Appl. Phys. Lett. **76**, 463 (2000).
[17] S. G. Rabinovich, *Measurement Errors and Uncertainties* (Springer, New York, 2000).
[18] K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering* (Wiley, New York, 1965).
[19] R. S. Decca *et al.*, Ann. Phys. (N.Y.) **318**, 37 (2005).
[20] B. Geyer, G. L. Klimchitskaya, and V. M. Mostepanenko, Phys. Rev. A **67**, 062102 (2003); **70**, 016102 (2004).
[21] E. M. Lifshitz, Sov. Phys. JETP **2**, 73 (1956).
[22] *Handbook of Optical Constants of Solids*, edited by E. D. Palik (Academic, New York, 1985).
[23] T. Emig *et al.*, Phys. Rev. Lett. **87**, 260402 (2001).
[24] C. Genet *et al.*, Europhys. Lett. **62**, 484 (2003).
[25] D. Iannuzzi *et al.*, in *Quantum Field Theory Under the Influence of External Conditions*, edited by K. A. Milton (Rinton, Princeton, NJ, 2004).