Influence of the asymmetry of the potential on the dynamics of a two-level superconducting quantum interference device qubit

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We analyze the interaction of a two-level superconducting quantum interference device (SQUID) qubit with a classical microwave pulse. The rf-SQUID is characterized by an asymmetric double well potential that gives rise to diagonal matrix elements. The diagonal matrix elements are accounted for in the interaction of the microwave pulse with the SQUID. We present analytical results that correctly describe the system's dynamics.

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Solid state systems that make use of the Josephson effect are potential candidates for carrying out quantum computations [1,2]. A particular scheme of such systems is based on magnetic flux states in superconducting quantum interference devices (SQUIDs) [3–25]. In some of these schemes [4–7,10–14,23], the SQUID qubit, which is the basic element of a SQUID quantum computer, is based on a two-level system manipulated by external fields. The interaction of two-level SQUID qubits with both classical [5,14] and quantized [6,7,10–13,23] fields has been analyzed theoretically.

In this work we study the interaction of a two-level SQUID qubit with a classical microwave pulsed field. The rf-SQUID is characterized by an asymmetric double well potential. The asymmetry of the potential gives rise to diagonal matrix elements that should be accounted for in the interaction of the SQUID with the microwave pulse. The main approximation that we make is that the dynamical behavior of the system can be well described by the evolution of the two levels of the SQUID qubit. We include the diagonal matrix elements in the analysis of the interaction of the SQUID qubit with the microwave pulse and present analytical results, within a generalized rotating wave approximation. This inclusion is found to be important for the correct description of the system's dynamics.

Our model consists of a rf-SQUID which interacts with a microwave field. The rf-SQUID is made of a superconducting ring interrupted by a Josephson tunnel junction. The SQUID Hamiltonian is given by [26]

$$H_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x), \tag{1}$$

with the potential of the SQUID being

$$V(x) = \frac{1}{2}m\omega_{LC}^2(x-x')^2 - \frac{1}{4\pi^2}m\omega_{LC}^2\beta\cos(2\pi x), \tag{2}$$

where $x=\Phi/\Phi_0$, $m=C\Phi_0^2$, $\omega_{LC}=1/\sqrt{LC}$, $\beta=2\pi LI_c/\Phi_0$, and $x'=\Phi_x/\Phi_0$. Here, Φ is the total magnetic flux in the ring, L is the ring inductance, Φ_x is an externally applied magnetic flux to the SQUID, I_c is the critical current of the junction, C is the junction capacitance, and $\Phi_0=h/2e$ is the flux quantum. We describe a realistic SQUID system and use the same parameters as in the work of Zhou *et al.* [8,14], i.e., $L=100$ pH, $C=40$ fF, and $I_c=3.95$ μA, leading to $\omega_{LC}=5\times10^{11}$ rad/s and $\beta=1.2$. We also take $x'=-0.501$. The form of the potential for these parameters is asymmetric, and is shown in Fig. 1. The first two eigenenergies of the system are also indicated in the same figure. These have been obtained by solving the time-independent Schrödinger equation, using the Hamiltonian H_0 with the above parameters. The values of the two lowest energies of the system are $\hbar\omega_0=7.819\,84$ meV and $\hbar\omega_1=7.901\,83$ meV [24].

The interaction between the SQUID and microwave pulse, which is considered as linearly polarized electromagnetic field with their magnetic field perpendicular to the plane of the SQUID ring, is described by the time-dependent potential

$$V_{\mathrm{int}}(x,t) = m\omega_{LC}^2(x-x')\varepsilon f(t)\cos(\omega t). \tag{3}$$

Here, ε, is the field amplitude (in units of Φ_x/Φ_0), $f(t)$ is the dimensionless pulse envelope and ω is the field angular fre-

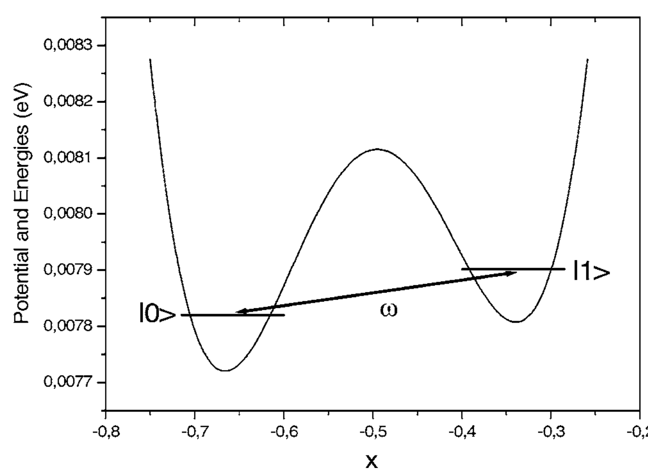

FIG. 1. The potential energy (solid curve) and the lowest two eigenenergies $|0\rangle$, $|1\rangle$ (horizontal lines) of the SQUID, in eV. The arrow-headed line denotes the coupling of the microwave field.

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quency. The dynamics of the system is governed by the time-dependent Schrödinger equation $i\hbar(\partial/\partial t)|\psi\rangle=(H_0+V_{\mathrm{int}})|\psi\rangle$. If the angular frequency of the microwave field is chosen near resonant with the transition $|0\rangle\leftrightarrow|1\rangle$ the dynamics of the system is governed by the interaction of the two lower states of the SQUID with the microwave pulse. This is usually referred to as the two-level system approximation. Then, the state vector of the system is expanded only in the two lowest eigenvectors of the Hamiltonian (1) as $|\psi(t)\rangle=a_0(t)|0\rangle+a_1(t)|1\rangle$. Substituting the Hamiltonian H_0+V_{int} and the state vector into the time-dependent Schrödinger equation we obtain a system of ordinary coupled differential equations for the expansion amplitudes $a_0(t)$ and $a_1(t)$ as

$$i\frac{d}{dt}a_0(t)=[\omega_0+\Omega_{00}f(t)\cos(\omega t)]a_0(t)+\Omega f(t)\cos(\omega t)a_1(t), \tag{4}$$

$$i\frac{d}{dt}a_1(t)=\Omega f(t)\cos(\omega t)a_0(t)+[\omega_1+\Omega_{11}f(t)\cos(\omega t)]a_1(t), \tag{5}$$

where $\Omega=x_{01}m\omega_{LC}^2\varepsilon/\hbar$, $\Omega_{00}=(x_{00}-x')m\omega_{LC}^2\varepsilon/\hbar$, $\Omega_{11}=(x_{11}-x')m\omega_{LC}^2\varepsilon/\hbar$, with $x_{jk}=\langle j|x|k\rangle$ and $j,k=0,1$. This system of differential equations must be solved in order to obtain the dynamics of the SQUID. For the system under study the matrix elements have the values $x_{00}=-6.568\,75\times10^{-1}$, $x_{11}=-3.491\,41\times10^{-1}$, and $x_{01}=4.418\,71\times10^{-4}$. We note that x_{00},x_{11} are nonzero due to the asymmetry of the potential of the SQUID.

Equations (4) and (5) are similar to those used for the description of a two-level molecule with permanent dipoles interacting with a laser field [27,28]. We follow a method used in molecular physics [28] and obtain an analytic solution of Eqs. (4) and (5). We make the following change of variables

$$b_j(t)=a_j(t)\exp\left[i\left(\omega_j t+\Omega_{jj}\int_0^t dt' f(t')\cos(\omega t')\right)\right],\quad j=0,1, \tag{6}$$

and obtain from Eqs. (4) and (5),

$$i\frac{d}{dt}b_0(t)=\Omega f(t)\cos(\omega t)\times\exp\left(-i(\omega_1-\omega_0)t-id\int_0^t dt' f(t')\cos(\omega t')\right)b_1(t), \tag{7}$$

$$i\frac{d}{dt}b_1(t)=\Omega f(t)\cos(\omega t)\times\exp\left(i(\omega_1-\omega_0)t+id\int_0^t dt' f(t')\cos(\omega t')\right)b_0(t), \tag{8}$$

with $d=\Omega_{11}-\Omega_{00}$. The integral $\int_0^t dt' f(t')\cos(\omega t')$ can be written as

$$\int_0^t dt' f(t')\cos(\omega t')=\frac{f(t)}{\omega}\sin(\omega t)-\frac{1}{\omega}\int_0^t dt'\frac{df(t')}{dt'}\sin(\omega t'). \tag{9}$$

The second term on the right-hand side of Eq. (9) can be omitted if the duration of the microwave pulse is much larger than $1/\omega$. Then, Eqs. (7) and (8) can be approximated by

$$i\frac{d}{dt}b_0(t)=\Omega f(t)\cos(\omega t)\times\exp\left(-i(\omega_1-\omega_0)t-i\frac{d}{\omega}f(t)\sin(\omega t)\right)b_1(t), \tag{10}$$

$$i\frac{d}{dt}b_1(t)=\Omega f(t)\cos(\omega t)\times\exp\left(i(\omega_1-\omega_0)t+i\frac{d}{\omega}f(t)\sin(\omega t)\right)b_0(t). \tag{11}$$

Using the relation $\exp(ix\sin\theta)=\sum_{n=-\infty}^{\infty}J_n(x)\exp(in\theta)$, where $J_n(\cdot)$ is the nth order ordinary Bessel function, Eqs. (10) and (11) are written as

$$i\frac{d}{dt}b_0(t)=\frac{\Omega}{2}f(t)\sum_{n=-\infty}^{\infty}J_n\left(\frac{d}{\omega}f(t)\right)e^{-i(\omega_1-\omega_0)t+i\omega t-in\omega t}b_1(t)+\frac{\Omega}{2}f(t)\sum_{n=-\infty}^{\infty}J_n\left(\frac{d}{\omega}f(t)\right)e^{-i(\omega_1-\omega_0)t-i\omega t-in\omega t}b_1(t), \tag{12}$$

$$i\frac{d}{dt}b_1(t)=\frac{\Omega}{2}f(t)\sum_{n=-\infty}^{\infty}J_n\left(\frac{d}{\omega}f(t)\right)e^{i(\omega_1-\omega_0)t+i\omega t+in\omega t}b_0(t)+\frac{\Omega}{2}f(t)\sum_{n=-\infty}^{\infty}J_n\left(\frac{d}{\omega}f(t)\right)e^{i(\omega_1-\omega_0)t-i\omega t+in\omega t}b_0(t). \tag{13}$$

As the angular frequency of the microwave field is chosen near resonant with the transition $|0\rangle\leftrightarrow|1\rangle$, i.e., $\omega_1-\omega_0\approx\omega$, then in the realistic limit that $\omega\gg\Omega$ only the term with $n=0$ is significant in the first (second) sum and the term with $n=-2$ is significant in the second (first) sum of Eq. (12)

[Eq. (13)]. The rest of the terms in the sums are highly off-resonant and can be omitted. This approximation is analogous to the rotating wave approximation in laser-matter interaction [29]. Under this approximation Eqs. (12) and (13) become

$$i\frac{d}{dt}b_0(t) = \frac{1}{2}K(t)e^{-i\Delta t}b_1(t), \tag{14}$$

$$i\frac{d}{dt}b_1(t) = \frac{1}{2}K(t)e^{i\Delta t}b_0(t), \tag{15}$$

where

$$K(t) = \Omega f(t)\left[J_0\left(\frac{d}{\omega}f(t)\right) + J_2\left(\frac{d}{\omega}f(t)\right)\right], \tag{16}$$

and $\Delta = \omega_1 - \omega_0 - \omega$. We note that in the derivation of Eqs. (14) and (15) we used the relation $J_{-2}(\cdot) = J_2(\cdot)$. The parameter $K(t)$ is the interaction parameter between the microwave field and the rf-SQUID (the generalized time-dependent Rabi frequency) that depends on the Rabi frequency Ω, the pulse envelope $f(t)$, but also on the difference of the interaction parameters arising from the diagonal matrix elements, d, and on the angular frequency of the applied microwave pulse, ω. In the limit that $d \to 0$ then $K(t) = \Omega f(t)$ which is the form of the interaction parameter if the diagonal matrix elements x_{00}, x_{11} are omitted. We note that the same limit also holds if $d/\omega \ll 1$. If, however, this is not the case then $K(t)$ is better approximated by Eq. (16).

We are particularly interested in the case of exact resonance of the microwave field with the $|0\rangle \leftrightarrow |1\rangle$ transition, i.e., the case that $\Delta = 0$. Then, the analytic solution of Eqs. (14) and (15) are

$$b_0(t) = \cos\left(\frac{1}{2}\int_0^t dt' K(t')\right)b_0(0) - i\sin\left(\frac{1}{2}\int_0^t dt' K(t')\right)b_1(0), \tag{17}$$

$$b_1(t) = -i\sin\left(\frac{1}{2}\int_0^t dt' K(t')\right)b_0(0) + \cos\left(\frac{1}{2}\int_0^t dt' K(t')\right)b_1(0). \tag{18}$$

If the system is initially in state $|0\rangle$ then $b_0(0) = 1$ and $b_1(0) = 0$. The probability for the system to be in state $|0\rangle$ or $|1\rangle$, $P_n(t) = |a_n(t)|^2 = |b_n(t)|^2$, with $n = 0, 1$ are in this case

$$P_0(t) = \cos^2\left(\frac{1}{2}\int_0^t dt' K(t')\right), \tag{19}$$

$$P_1(t) = \sin^2\left(\frac{1}{2}\int_0^t dt' K(t')\right). \tag{20}$$

In the case of a rectangular microwave pulse of duration T, then $f(t) = 1$ for $0 \leqslant t \leqslant T$ and zero elsewhere. Then, if $0 \leqslant t \leqslant T$ the probabilities become

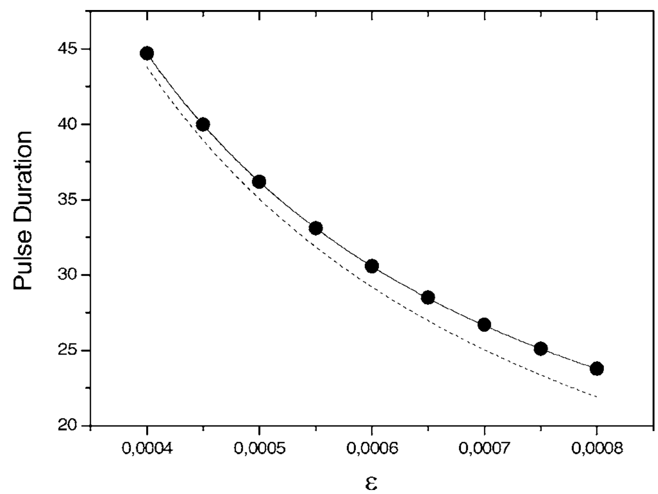

FIG. 2. The minimum microwave pulse duration (in ns) for the complete transition of the system from the initial state $|0\rangle$ to state $|1\rangle$ as a function of the normalized field amplitude ε. With circles we represent the results of the numerical solution of Eqs. (4) and (5). With solid curve we represent the analytical results of T_{inv} and with dashed curve the analytical results of T'_{inv}.

$$P_0(t) = \cos^2\left\{\frac{1}{2}\Omega\left[J_0\left(\frac{d}{\omega}\right) + J_2\left(\frac{d}{\omega}\right)\right]t\right\}, \tag{21}$$

$$P_1(t) = \sin^2\left\{\frac{1}{2}\Omega\left[J_0\left(\frac{d}{\omega}\right) + J_2\left(\frac{d}{\omega}\right)\right]t\right\}. \tag{22}$$

If the microwave pulse duration is chosen as

$$T_{\mathrm{inv}} = \frac{\pi}{\Omega\left[J_0\left(\frac{d}{\omega}\right) + J_2\left(\frac{d}{\omega}\right)\right]}, \tag{23}$$

then $P_0(T_{\mathrm{inv}}) = 0$ and $P_1(T_{\mathrm{inv}}) = 1$ so after the application of the microwave pulse the system is in state $|1\rangle$. We assess the analytical results of Eq. (23) by comparing them with the results of numerical solution of Eqs. (4) and (5). For the numerical solution a fourth-order Runge-Kutta method was used. The numerical and analytical results are shown in Fig. 2. It is clear that the analytical results reproduce the numerical results. For reasons of comparison we plot in the same figure the analytical result for $d = 0$, i.e., the analytical result if the diagonal matrix elements x_{00}, x_{11} are omitted, which gives $T'_{\mathrm{inv}} = \pi/\Omega$. The latter result cannot reproduce the numerical results and the difference between numerical and the analytical results of T'_{inv} becomes larger as the field amplitude increases.

In summary, we have studied the dynamical behavior of the interaction of a two-level SQUID qubit with a classical microwave pulse. The SQUID system is described by an asymmetric double well potential. This gives rise to diagonal matrix elements that should be included in the equations of motion, for the proper study of the interaction of the SQUID system with the microwave pulse. Including only the two relevant levels that compose the SQUID qubit in an expansion of the system's wave fuction, we present analytical results for the time evolution of the interaction of a two-level SQUID system with a classical microwave pulse. The analytical results are found to be in good agreement with the results of numerical simulations for realistic parameters of the system.

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