# Schemes for generating the cluster states in microwave cavity QED

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(Received 1 March 2005; published 12 July 2005)

We propose two experimental schemes to generate the cluster states in the context of microwave cavity quantum electrodynamics (QED). In the first scheme to prepare many cavities into the cluster states, we encode the vacuum state and one-photon state of the microwave cavity as the logic zero and one of the qubits. The second scheme is to prepare many atoms into the cluster states, where qubits are represented by the states of Rydberg atoms. Both schemes require the resonant atom-cavity interaction so that the quantum dynamics operates at a high speed, which is important in view of decoherence.

DOI: 10.1103/PhysRevA.72.013809 PACS number(s): 42.50.Dv

## I. INTRODUCTION

An entangled state of two or more particles is not only a key ingredient for the tests of quantum nonlocality [1–3], but also a basic resource in achieving tasks of quantum information processing, such as quantum cryptography [4], quantum dense coding [5], and quantum teleportation [6]. Most of the research in quantum information processing is based on quantum entanglement of two qubits. Recently, there has been much interest in quantum entanglement of many qubits. It has become clear that for the system shared by three and more parties, there are several inequivalent classes of entangled states [7]. In Ref. [8], Briegel *et al.* introduced a class of entangled states, the so-called cluster states. It has been shown that cluster states can be regarded as a resource for Greenberger-Horne-Zeilinger (GHZ) states [8] and are more immune to decoherence than GHZ states [9]. In Ref. [10], the proof of Bell's theorem without the inequalities was given for cluster states, and a new Bell inequality is considered, which is maximally violated by the four-qubit cluster state and is not violated by the four-qubit GHZ state. More interestingly, cluster states have been shown to constitute a universal resource for quantum computation with assistance by local measurement only [11]. It has been shown that cluster states can be produced through the Ising interaction [8].

Microwave cavity QED, with Rydberg atoms crossing superconducting cavities, offers an almost ideal system for the implementation of quantum information processing [13]. In the context of cavity QED, numerous theoretical schemes for generating entangled states of many atoms and nonclassical states of cavity fields have been proposed [14], which led to experimental realization of the Einstein-Podolsky-Rosen (EPR) state [15] of two atoms, the GHZ state [16] of three parties (two atoms plus one cavity mode), the Schrödinger cat state [17], and the Fock state [18] of a single-mode cavity field.

In this paper, we propose two cavity QED schemes to generate the cluster states. The first scheme is to prepare many cavities into the cluster states, in which we encode the vacuum state and one-photon state of a microwave cavity as the logic zero and one of the qubits. The second one is to prepare many atoms into the cluster states, in which qubits are represented by the states of Rydberg atoms. Both schemes are based on the resonant atom-cavity interaction, so that the quantum dynamics operates at a high speed, which is important in view of decoherence. The paper is organized as follows. In Sec. II, we give a brief introduction to cluster states. In Sec. III, we first propose a scheme to prepare many cavities into one-dimensional cluster states, then demonstrate how to construct a two-dimensional cluster state from one-dimensional cluster states. In Sec. IV, a scheme is proposed to prepare many atoms into the cluster states. A conclusion is given in Sec. V.

## II. A BRIEF INTRODUCTION TO CLUSTER STATES

A cluster state is a multipartite entangled state which has special features suitable for implementing a quantum computer on a network. It is necessary to give a brief review of the definition of the cluster state of the qubits positioned at specific sites of a lattice structure [8,11]. For any site $a$ of the lattice, one defines the operator

$$K^{(a)} = \sigma_x^a \bigotimes_{b \in \mathrm{ngbh}(a)} \sigma_z^{(b)} \tag{1}$$

where ngbb($a$) is the set of all the neighbors of the site $a$. The operators $\{K^{(a)}, a \in \text{lattice}\}$ form a complete family of commuting operators on the lattice. A cluster state is any of their common eigenstates

$$K_a|\psi_\kappa\rangle_C = (-1)^{\kappa_a}|\psi_\kappa\rangle_C \tag{2}$$

with $\kappa_a \in \{0,1\}$. A cluster state is completely specified by the eigenvalue equation (2) and it can be shown that all states are equally suitable for quantum computation [12]. For simplicity, we consider the state associated with all eigenvalues being 1. The compact notation for such cluster states built on the $d$-dimensional lattice structure $C$ is given by (without being normalized)

$$\bigotimes_{a \in C}\left(|0\rangle_c + |1\rangle_c \bigotimes_{\gamma \in \Gamma} \sigma_z^{(c+\gamma)}\right) \tag{3}$$

with the choice $\Gamma=\{1\}$ for $d=1$, $\Gamma=\{(1,0),(0,1)\}$ for $d=2$, and $\Gamma=\{(1,0,0),(0,1,0),(0,0,1)\}$ for $d=3$.

It has been shown that the cluster states can be produced through the Ising interaction [8]. However, an ideal Ising interaction is difficult to obtain experimentally. In Ref. [19], Browne *et al.* showed that the projective operator

 

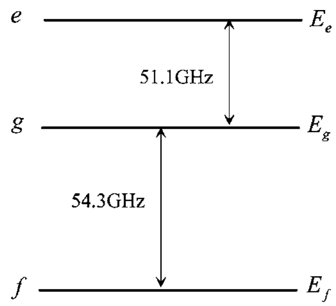


FIG. 1. This figure shows the electronic levels of the three-level atom in the energy representation.

$$P = |0\rangle\langle 0|\langle 0| + |1\rangle\langle 1|\langle 1| \tag{4}$$

can be used to combine two one-dimensional cluster states into a new two-dimensional cluster state. A linear optical scheme was also proposed to realize such a projective operator [19].

## III. GENERATION OF THE CLUSTER STATES OF MANY CAVITIES

In this section, we first propose a scheme to prepare many cavities into one-dimensional cluster states, then demonstrate how to realize the project operator (4), which can be used to construct a two-dimensional cluster state from one-dimensional cluster states. In this scheme, we encode the vacuum state and one-photon state of a microwave cavity as the logic zero and one of the qubits.

We now consider the resonant interaction of the Rydberg atom with a cavity field. The atomic levels of the Rydberg atom are labeled by $|f\rangle$, $|g\rangle$, and $|e\rangle$ (see Fig. 1). The $|e\rangle \leftrightarrow |g\rangle$ and $|f\rangle \leftrightarrow |g\rangle$ transitions are at 51.1 and 54.3 GHz, respectively. Thus, we can choose the frequencies of the cavity mode in a way that only the levels $|e\rangle$ and $|g\rangle$ are appropriately affected by the cavity field. The transition frequency between the states $|g\rangle$ and $|f\rangle$ is highly detuned from the cavity frequency and thus the state $|f\rangle$ is not affected during the atom-cavity interaction. We assume that the $|e\rangle \leftrightarrow |g\rangle$ transition is coupled to the cavity mode with the vacuum Rabi oscillation frequency $\Omega$. In the interaction picture, the effective interaction Hamiltonian for the atom-cavity system under the dipole and rotating wave approximation can be written as follows:

$$H = i\Omega(a^\dagger |g\rangle\langle e| - a|e\rangle\langle g|), \tag{5}$$

where $a$ and $a^\dagger$ are annihilation and creation operators for the cavity mode, and the cavity mode is assumed to be resonant with the corresponding atomic transitions. Under domination of the Hamiltonian (5), the time evolution of the relevant states of the joint atom+cavity-field system is given by

$$|g\rangle|0\rangle \to |g\rangle|0\rangle,$$

$$|g\rangle|n\rangle \to \cos(\sqrt{n}\Omega t)|g\rangle|n\rangle - \sin(\sqrt{n}\Omega t)|e\rangle|n-1\rangle,$$

$$|e\rangle|n-1\rangle \to \cos(\sqrt{n}\Omega t)|e\rangle|n-1\rangle + \sin(\sqrt{n}\Omega t)|g\rangle|n\rangle, \tag{6}$$

where the interaction time $t$ can be controlled by using a velocity selector.

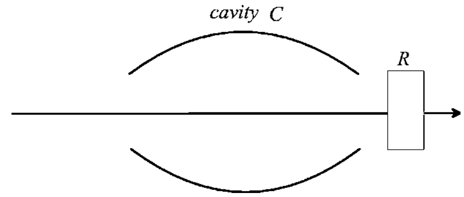


FIG. 2. The basic element to prepare many cavities into one-dimensional cluster states, which consists of one high-$Q$ cavity $C$ and one Ramsey zone $R$.

To explain the main idea of the scheme for preparing many cavities into one-dimensional cluster states, we introduce a simple building block (see Fig. 2) which describes a three-level Rydberg atom sequentially passing through one cavity and one Ramsey zone. In order to demonstrate the functionality of this basic block, we assume that the Rydberg atom, before entering into the cavity, is in the state of the form

$$\frac{1}{\sqrt{2}}(|f\rangle|\Psi_1\rangle + |e\rangle|\Psi_2\rangle), \tag{7}$$

where we assume that the Rydberg atom is entangled with other subsystems, and $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are arbitrary normalized wave functions of the subsystems. The cavity field is initially prepared in the vacuum state. Thus as the atom enters the cavity, the state of the system (atom+cavity) is

$$\frac{1}{\sqrt{2}}(|f\rangle|\Psi_1\rangle + |e\rangle|\Psi_2\rangle)|0\rangle_c. \tag{8}$$

After the atom passes through the cavity, the state of the system evolves into

$$\frac{1}{\sqrt{2}}[|f\rangle|0\rangle_c|\Psi_1\rangle + [\cos(\Omega t)|e\rangle|0\rangle_c + \sin(\Omega t)|g\rangle|1\rangle_c]|\Psi_2\rangle]. \tag{9}$$

If the interaction time $t$ between the atom and the cavity is chosen to satisfy $t = \pi/2\Omega$, the state of the system becomes

$$\frac{1}{\sqrt{2}}[|f\rangle|0\rangle_c|\Psi_1\rangle + |g\rangle|1\rangle_c|\Psi_2\rangle]. \tag{10}$$

After leaving the cavity $C$, the atom is sequentially subjected to two classical pulses in the Ramsey zone $R$. The first one is tuned to the transition $|e\rangle \leftrightarrow |g\rangle$. The amplitude and the phase of the classical field are chosen appropriately so that the atom undergoes the transition $|e\rangle \to |g\rangle$ and $|g\rangle \to |e\rangle$. The second one is tuned to the transition $|e\rangle \leftrightarrow |f\rangle$, which induces the transformation $|e\rangle \to (1/\sqrt{2})(|f\rangle - |e\rangle)$ and $|f\rangle \to (1/\sqrt{2})(|f\rangle + |e\rangle)$. Thus, the state (10) becomes

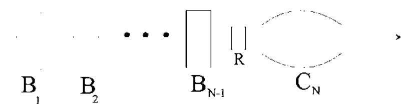


FIG. 3. This figure shows the required experimental setup to prepare $N$ cavities into cluster states. $B_i$ denotes the setup shown in Fig. 2 $(i=1,\ldots,N-1)$, and $C_N$ denotes the $N$th cavity. $R$ is the Ramsey zone.

$$\frac{1}{2}\left[|f\rangle(|0\rangle_c|\Psi_1\rangle+|1\rangle_c|\Psi_2\rangle)+\frac{1}{2}|e\rangle(|0\rangle_c|\Psi_1\rangle-|1\rangle_c|\Psi_2\rangle)\right], \tag{11}$$

which can be rewritten as follows:

$$\frac{1}{2}[|f\rangle+|e\rangle\sigma_c](|0\rangle_c|\Psi_1\rangle+|1\rangle_c|\Psi_2\rangle), \tag{12}$$

where $\sigma_c=|0\rangle_c\langle 0|-|1\rangle_c\langle 1|$.

Now we demonstrate how to prepare many cavities in one-dimensional cluster states by arranging the building blocks as shown in Fig. 3 A Rydberg atom is sequentially sent through $(N-1)$ building blocks $B_i$ $(i=1,\ldots,N-1)$ and one cavity $C_N$. We assume that all cavities are initially prepared in the vacuum states and the atom is in the superposition state $(|f\rangle+|e\rangle)/\sqrt{2}$, i.e., the initial state of the system is

$$\frac{1}{\sqrt{2}}(|f\rangle+|e\rangle)|0\rangle_{c_1}|0\rangle_{c_2}\cdots|0\rangle_{c_N}. \tag{13}$$

After the atom passes through $(N-1)$ building blocks, based on Eq. (12), the state (13) becomes

$$\frac{1}{\sqrt{2^N}}[|f\rangle+|e\rangle\sigma_{cN-1}](|0\rangle_{cN-1}+|1\rangle_{cN-1}\sigma_{cN-2})\cdots(|0\rangle_{c2}+|1\rangle_{c2}\sigma_{c1})(|0\rangle_{c1}+|1\rangle_{c1})|0\rangle_{cN}. \tag{14}$$

Before the atom enters the $N$th cavity, it is subjected to one classical pulse in the Ramsey zone $R$, which induces the transformation $|f\rangle\to|g\rangle$ and $|g\rangle\to|f\rangle$. The state (14) becomes

$$\frac{1}{\sqrt{2^N}}[|g\rangle+|e\rangle\sigma_{cN-1}](|0\rangle_{cN-1}+|1\rangle_{cN-1}\sigma_{cN-2})\cdots(|0\rangle_{c2}+|1\rangle_{c2}\sigma_{c1})(|0\rangle_{c1}+|1\rangle_{c1})|0\rangle_{cN}. \tag{15}$$

Thus, after the atom passes through the $N$th cavity with the duration $\tau=\pi/2\Omega$, the quantum state (15) becomes

$$\frac{1}{\sqrt{2^N}}[|0\rangle_{cN}+|1\rangle_{cN}\sigma_{cN-1}](|0\rangle_{cN-1}+|1\rangle_{cN-1}\sigma_{cN-2})\cdots(|0\rangle_{c2}+|1\rangle_{c2}\sigma_{c1})(|0\rangle_{c1}+|1\rangle_{c1}), \tag{16}$$

where we have neglected the atomic state, since the atom and cavity fields are disentangled. If we encode the vacuum state and one-photon state as the logic zero and one of the qubits, Eq. (16) is exactly the expected one-dimensional cluster state (3).

So far, we have only considered the generation of one-dimensional cluster states, which are insufficient for a one-way quantum computer. In the following, we will consider how to implement the projective operator (4), which can be used to combine two one-dimensional cluster states into a new two-dimensional cluster state. The experimental setup is shown in Fig. 4. We assume that the cavity $i$ $(i=1,2)$ is in the state

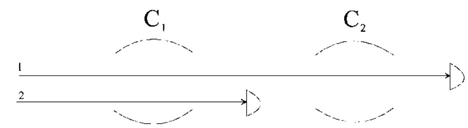


FIG. 4. Experimental setup to realize the project operator (4) of two cavity modes. After atom 1 sequentially passes through cavity 1 and 2, atom 2 is sent through cavity 1.

$$|\Phi_i\rangle=\frac{1}{\sqrt{2}}(|0\rangle_{ci}|\Psi_{i1}\rangle+|1\rangle_{ci}|\Psi_{i2}\rangle), \tag{17}$$

where we assume that the $i$th cavity field is entangled with other subsystems, and $|\Psi_{i1}\rangle$ and $|\Psi_{i2}\rangle$ are arbitrary normalized wave functions of the subsystems. In order to implement the expected projector operator (4), we send the atom 1 in the superposition state $(|f\rangle_1+|g\rangle_1)/\sqrt{2}$ sequentially through two cavities 1 and 2. If the interaction times between the atom and two cavity fields are the same at $\pi/\Omega$, the time evolution can be written as follows:

$$\frac{1}{\sqrt{2}}|\Phi_1\rangle|\Phi_2\rangle(|f\rangle_1+|g\rangle_1)\xrightarrow{\text{cavity 1}}\frac{1}{2}[|0\rangle_{c1}(|f\rangle_1+|g\rangle_1)|\Psi_{11}\rangle+|1\rangle_{c1}(|f\rangle_1-|g\rangle_1)|\Psi_{12}\rangle]|\Phi_2\rangle\xrightarrow{\text{cavity 2}}\frac{1}{2\sqrt{2}}[(|f\rangle_1+|g\rangle_1)\times(|0\rangle_{c1}|0\rangle_{c2}|\Psi_{11}\rangle|\Psi_{21}\rangle+|1\rangle_{c1}|1\rangle_{c2}|\Psi_{12}\rangle|\Psi_{22}\rangle)+(|f\rangle_1-|g\rangle_1)(|0\rangle_{c1}|1\rangle_{c2}|\Psi_{11}\rangle|\Psi_{22}\rangle+|1\rangle_{c1}|0\rangle_{c2}|\Psi_{12}\rangle\times|\Psi_{21}\rangle)]. \tag{18}$$

After atom 1 exits from cavity 2, one detects whether the atom is in the state $(|g\rangle_1+|f\rangle_1)/\sqrt{2}$. This detection process can be implemented by passing the atom through the classical microwave field zone and field ionization counters. If the atom is in the expected state, the state of the two cavity modes is projected into

$$\frac{1}{\sqrt{2}}(|0\rangle_{c1}|0\rangle_{c2}|\Psi_{11}\rangle|\Psi_{21}\rangle+|1\rangle_{c1}|1\rangle_{c2}|\Psi_{12}\rangle|\Psi_{22}\rangle). \tag{19}$$

Now we send the atom 2 prepared in the state $|g\rangle_2$ through the cavity 1. If the interaction is chosen to be $\pi/2\Omega$, the state (19) becomes

$$\frac{1}{\sqrt{2}}(|g\rangle_2|0\rangle_{c2}|\Psi_{11}\rangle|\Psi_{21}\rangle-|e\rangle_2|1\rangle_{c2}|\Psi_{12}\rangle|\Psi_{22}\rangle), \tag{20}$$

where we have neglected the vacuum state of cavity 1. After the atom exits from the cavity 1, one detects whether the atom is in the state $(|g\rangle_2\pm|e\rangle_2)/\sqrt{2}$. If the atom is in the state $(|g\rangle_2-|e\rangle_2)/\sqrt{2}$, the state (20) is projected into

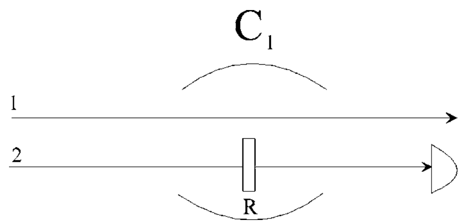


FIG. 5. The basic operation to prepare many atoms into one-dimensional cluster states. Two atoms are sent through the cavity one by one. $R$ denotes that a short pulse of classical field is applied to the atom 2 midway while it passes through the cavity.

$$\frac{1}{\sqrt{2}}(|0\rangle_{c2}|\Psi_{11}\rangle|\Psi_{21}\rangle + |1\rangle_{c2}|\Psi_{12}\rangle|\Psi_{22}\rangle). \tag{21}$$

If the atom is in the state $(|g\rangle_2+|e\rangle_2)/\sqrt{2}$, the state (20) is projected into

$$\frac{1}{\sqrt{2}}(|0\rangle_{c2}|\Psi_{11}\rangle|\Psi_{21}\rangle - |1\rangle_{c2}|\Psi_{12}\rangle|\Psi_{22}\rangle),$$

which can be transferred into Eq. (21) by local operation. It is noticed that Eq. (21) can be rewritten as follows:

$$(|0\rangle_{c2c1}\langle 0|_{c2}\langle 0| + |1\rangle_{c2c1}\langle 1|_{c2}\langle 1|)|\Phi_1\rangle|\Phi_2\rangle, \tag{22}$$

which shows that the setup shown in Fig. 4 can be used for the project operator (4). The success probability of the scheme is 1/2. Thus by combining the setups shown in Figs. 3 and 4, we can generate the two-dimensional cluster state step by step.

## IV. CAVITY QED GENERATION OF THE CLUSTER STATES OF MANY ATOMS

In this section, we first present a scheme to use a cavity to prepare many atoms in one-dimensional cluster states, and then show how to construct the project operator (4).

The basic building block is shown in Fig. 5, which requires two three-level Rydberg atoms sequentially passing through one cavity and one Ramsey zone. To demonstrate the functionality of the basic block, we assume that, before entering into the cavity, the cavity field is prepared in the state of the form

$$\frac{1}{\sqrt{2}}(|0\rangle_c|\Psi_1\rangle + |1\rangle_c|\Psi_2\rangle), \tag{23}$$

where we assume that the cavity field is entangled with other subsystems, and $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are arbitrary normalized wave functions of the subsystems. We send the atoms 1 prepared in the superposition state $(|f\rangle_1+|g\rangle_1)/\sqrt{2}$ through the cavity. If the interaction time is chosen to be $\pi/\Omega$, the state of the system becomes

$$\frac{1}{2}[|0\rangle_c(|f\rangle_1 + |g\rangle_1)|\Psi_1\rangle + |1\rangle_c(|f\rangle_1 - |g\rangle_1)|\Psi_2\rangle]. \tag{24}$$

After exiting from the cavity, the atom 1 is subjected to one classical pulse in the Ramsey zone $R$, which induces the transformation $|g\rangle_1 \rightarrow (1/\sqrt{2})(|f\rangle_1-|g\rangle_1)$ and $|f\rangle_1 \rightarrow (1/\sqrt{2})(|f\rangle_1+|g\rangle_1)$. The state (24) becomes

$$\frac{1}{\sqrt{2}}[|0\rangle_c|f\rangle_1|\Psi_1\rangle + |1\rangle_c|g\rangle_1|\psi_2\rangle]. \tag{25}$$

Now we send one auxiliary atom 2 in the state $|g\rangle_2$ into the cavity. After the atom interacts with the cavity with the interaction time $\pi/2\Omega$, the state of the system becomes

$$\frac{1}{\sqrt{2}}[|g\rangle_2|f\rangle_1|\Psi_1\rangle + |e\rangle_2|g\rangle_1|\psi_2\rangle]|0\rangle_c. \tag{26}$$

At this time, a short strong classical pulse is applied to the atom 2, which induces the transformation $|g\rangle_2 \rightarrow (1/\sqrt{2})(|g\rangle_2+|e\rangle_2)$ and $|e\rangle_2 \rightarrow (1/\sqrt{2})(|g\rangle_2-|e\rangle_2)$. Here we assume that the classical pulse is strong enough so that, in this process, we can neglect contributions of atom-cavity interaction. Thus the state of the system becomes

$$\frac{1}{2}[|g\rangle_2(|f\rangle_1|\Psi_1\rangle + |g\rangle_1|\Psi_2\rangle) + |e\rangle_2(|f\rangle_1|\Psi_1\rangle - |g\rangle_1|\Psi_2\rangle)]|0\rangle_c. \tag{27}$$

Then atom 2 again interacts with the cavity for the duration $\pi/2\Omega$. The state of the system becomes

$$\frac{1}{2}[|0\rangle_c(|f\rangle_1|\Psi_1\rangle + |g\rangle_1|\Psi_2\rangle) + |1\rangle_c(|f\rangle_1|\Psi_1\rangle - |g\rangle_1|\Psi_2\rangle)], \tag{28}$$

which can be rewritten as follows:

$$\frac{1}{2}(|0\rangle_c + |1\rangle_c\sigma_1)(|f\rangle_1|\Psi_1\rangle + |g\rangle_1|\Psi_2\rangle), \tag{29}$$

where $\sigma_i=|f\rangle_1\langle f|-|g\rangle_1\langle g|$.

Now we demonstrate how to prepare many atoms in one-dimensional cluster states. We sequentially send $2(N-1)$ atoms through the cavity, which is initially prepared in the superposition state $(|0\rangle_c+|1\rangle_c)/\sqrt{2}$. If the $(2i-1)$th and $(2i)$th atoms perform the operation shown in Fig. 5, and the $(2i)$th atom is auxiliary, the state of the system [one cavity mode $+(N-1)$ atoms] is given by

$$\frac{1}{\sqrt{2^N}}[|0\rangle_c + |1\rangle_c\sigma_{2N-3}](|f\rangle_{2N-3} + |g\rangle_{2N-3}\sigma_{2N-5}) \cdots (|f\rangle_3 + |g\rangle_3\sigma_1)(|f\rangle_1 + |g\rangle_1), \tag{30}$$

then we send the $(2N-1)$th atom through the cavity with the duration $\pi/2\Omega$. The state of the system becomes

$$\frac{1}{\sqrt{2^N}}[|g\rangle_{2N-1} + |e\rangle_{2N-1}\sigma_{2N-3}](|f\rangle_{2N-3} + |g\rangle_{2N-3}\sigma_{2N-5}) \cdots (|f\rangle_3 + |g\rangle_3\sigma_1)(|f\rangle_1 + |g\rangle_1). \tag{31}$$

If we encode the $|f\rangle$ and $|g\rangle$ as logic zero and one of the qubits, the state (31) is equal to the one-dimensional cluster state.

In the following, we demonstrate how to use one cavity to implement the project operator (4) of two atoms. We assume the cavity field is in the superposition state $(|0\rangle_c+|1\rangle_c)/\sqrt{2}$, and the atom $i$ $(i=1,2)$ is in the state

$$|\Phi_i\rangle = \frac{1}{\sqrt{2}}(|f\rangle_{ai}|\Psi_{i1}\rangle + |g\rangle_{ai}|\Psi_{i2}\rangle), \tag{32}$$

where the $i$th atom is entangled with other subsystems, and $|\Psi_{i1}\rangle$ and $|\Psi_{i2}\rangle$ are arbitrary normalized wave functions of the subsystems. In order to implement the expected projector operator, we send the atoms 1 and 2 sequentially through the cavity with the duration $\pi/\Omega$, and the state of the system becomes

$$\frac{1}{2\sqrt{2}}[(|0\rangle_c + |1\rangle_c)(|f\rangle_{a1}|f\rangle_{a2}|\Psi_{11}\rangle|\Psi_{21}\rangle + |g\rangle_{a1}|g\rangle_{a2}|\Psi_{12}\rangle|\Psi_{22}\rangle) + (|0\rangle_c - |1\rangle_c)(|f\rangle_{a1}|g\rangle_{a2}|\Psi_{11}\rangle|\Psi_{22}\rangle + |g\rangle_{a1}|f\rangle_{a2}|\Psi_{12}\rangle \times|\Psi_{21}\rangle)]. \tag{33}$$

Then one auxiliary atom in the state $|g\rangle$ is sent through the cavity with the duration $\pi/2\Omega$. If the atom 2 and the auxiliary atom are detected in the state $(|g\rangle+|e\rangle)/\sqrt{2}$, we obtain the state of the system

$$\frac{1}{\sqrt{2}}(|f\rangle_{a1}|\Psi_{11}\rangle|\Psi_{21}\rangle + |g\rangle_{a1}|\Psi_{12}\rangle|\Psi_{22}\rangle). \tag{34}$$

If the atom 2 is in the state $(|g\rangle-|e\rangle)/\sqrt{2}$ and the auxiliary atom is in the state $(|g\rangle+|e\rangle)/\sqrt{2}$, we can obtain

$$\frac{1}{\sqrt{2}}(|f\rangle_{a1}|\Psi_{11}\rangle|\Psi_{21}\rangle - |g\rangle_{a2}|\Psi_{12}\rangle|\Psi_{22}\rangle), \tag{35}$$

which can be transferred into Eq. (34) by local operation. It is noticed that Eq. (34) can be rewritten as follows:

$$(|f\rangle_{a1a1}\langle f|_{a2}\langle f| + |g\rangle_{a2a1}\langle g|_{a2}\langle g|)|\Phi_1\rangle|\Phi_2\rangle, \tag{36}$$

which demonstrates the implementation of project operator (4) of the two atoms. The success probability of the scheme is 1/2. Thus by combining the setups shown in Fig. 5 and the project operator, we can generate the two-dimensional cluster state step by step.

## V. CONCLUSION

In summary, we have proposed two cavity QED schemes to generate the cluster states, which can be used to test quantum nonlocality and constitute a universal resource for quantum computation assisted by local measurement only. In the first scheme to prepare many cavities into the cluster states, we encode the vacuum state and the one-photon state of the microwave cavity as the logic zero and one of the qubits. The second one is to prepare many atoms in the cluster states, where qubits are represented by the states of Rydberg atoms. Both schemes require the resonant atom-cavity interaction so that the quantum dynamics operates at a high speed, which is important in view of decoherence.

We now give a brief discussion on the experimental feasibility of the proposed scheme within the microwave cavity QED. The scheme presented here requires (i) resonant interaction between the Rydberg atom and cavity mode, (ii) negligible cavity loss during the whole preparation process, (iii) no atomic spontaneous decay during the atom-cavity interactions, (iv) detection of atoms in given states, and (v) controlled interaction time between atom and cavity. Based on the microwave cavity QED experiments performed by Haroche and co-workers [13], the cavity can have a photon storage time of $T=1$ ms (corresponding to $Q=3\times10^8$), and the radiative time of the Rydberg atoms with the principal quantum numbers 49, 50, and 51 is about $3\times10^{-2}$ s. The coupling constant of the atoms to the cavity field is $\Omega/2\pi=25$ kHz. The detection process of the atom in a desired state can be implemented by passing the atom through the classical microwave field zone and field ionization counters. The interaction time between each of the atoms and the cavity can be controlled by using a velocity selector and applying Stark field adjustment in order to make the atom resonant with the field for the right amount of time. As shown in Secs. III and IV, the proposed schemes consist of a series of the basic building blocks. It is necessary to give a brief discussion on the experimental feasibility of the building blocks. An experimental realization of the building blocks shown in Fig. 2 is straightforward modification of a previous experiment demonstrating a quantum phase gate in cavity QED [20], in which a three-level Rydberg atom is sent through a hight-$Q$ cavity and a Ramsey zone. The difference between our scheme and experiment is that the atom is subjected to a different classical pulse in the Ramsey zone, which can be easily realized with the current experimental technology. To realize the projective the operator shown in Fig. 4, the basic operations are the quantum phase gate realized in experiment [20] and the Rabi oscillation indicating the resonant interaction of the atom and the cavity field. In Ref. [21], quantum Rabi oscillation and quantum memory with a single photon in the microwave cavity have been experimentally reported. The availability of an experimental configuration with two cavities can be considered as a natural development of the present configuration where only one cavity is present. Note also that some other interesting proposals require at least two cavities [22]. An experimental configuration of the building blocks shown in Fig. 5 is similar to a previous experiment generating an EPR state [15], which requires two three-level Rydberg atoms sequentially passing through one cavity. The first atom performs a quantum phase gate operation demonstrated in experiment [20], and the second atom is subjected to quantum Rabi oscillations to swap quantum information between the atom and the cavity field [21]. In order to realize these building blocks, the atom-cavity interaction times are at the order of $10^{-5}$ s. At this time scale, the times needed for the classical field pulse are negligible. Thus, the interaction times needed to implement these setups are at the order of $10^{-5}$ s, which is much shorter than the radiative time and the photon lifetime 1 ms in the present cavity. Therefore, based on cavity QED techniques, the schemes to realize basic building blocks in Secs. III and IV will be realizable in the near future. However, to combine these setups together to generate the cluster states is an experimental challenge.

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