# Nondissipative drag of superflow in a two-component Bose gas

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A microscopic theory of a nondissipative drag in a two-component superfluid Bose gas is developed. The expression for the drag current in the system with the components of different atomic masses, densities, and scattering lengths is derived. It is shown that the drag current is proportional to the square root of the gas parameter. The temperature dependence of the drag current is studied and it is shown that at temperature of order or smaller than the interaction energy the temperature reduction of the drag current is rather small. A possible way of measuring the drag factor is proposed. A toroidal system with the drag component confined in two half-ring wells separated by two Josephson barriers is considered. Under certain condition such a system can be treated as a Bose-Einstein counterpart of the Josephson charge qubit in an external magnetic field. It is shown that the measurement of the difference of number of atoms in two wells under a controlled evolution of the state of the qubit allows one to determine the drag factor.

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## I. INTRODUCTION

Macroscopic quantum coherence manifests itself in many specific phenomena. One of them is a nondissipative drag that takes place in superfluids and superconductors. The nondissipative drag, also known as the Andreev-Bashkin effect, was considered, for the first time, in Ref. [1], where a three velocity hydrodynamic model for $^3$He-$^4$He superfluid mixtures was developed. It was shown that superfluid behavior of such systems can be described under accounting the "drag" term in the free energy. This term is proportional to the scalar product of the superfluid velocities of two superfluid components. A similar situation may take place in mixtures of superfluids of $S_z=+1$ and $S_z=-1$ pairs in liquid $^3$He in the $A$-phase [2]. Among other objects, where the nondissipative drag may be important, are neutron stars, where the mixture of neutron and proton Cooper pair Bose condensates is believed to realize [3,4]. The possibility of realization of the nondissipative drag in superconductors was considered in [5]. The nondissipative drag in bilayer Bose systems was treated microscopically in [6,7] for a special case of two equivalent layers of charged bosons. The case of a bilayer system of neutral bosons was studied in [8] in the limit of small interlayer interaction.

The most promising systems where the nondissipative drag can be observed experimentally are two-component alkali metal vapors. In such systems the interaction between atoms of different species is of the same order as the interaction between atoms of the same specie and the effect is expected to be larger than in bilayers. In Bose mixtures the components are characterized by different densities, different masses of atoms, and different interaction parameters. In this paper we consider such a general case and obtain an analytical expression for the drag current for zero and finite temperatures.

In the system under consideration the drag force influences the dynamics of atoms in the drag component in the same manner as the vector potential of the electromagnetic field influences the dynamics of electrons in superconductors. In particular, in neutral superfluids with Josephson links the drag effect may induce the gradient of the phase of the order parameter in the bulk and, as a consequence, control the phase difference between weakly coupled parts of the system. Therefore, one can expect that the effect reveals itself in a modification of Josephson oscillations between weakly coupled Bose gases. In this paper we discuss possible ways for the observation of such a modification. We consider the Bose gas confined in a toroidal trap with two Josephson links. In the Fock regime [9] the low energy dynamics of the system can be described by the qubit model of general form (the model, where all three components of the pseudomagnetic field can be controlled independently). The parameters of the qubit Hamiltonian depend on the drag factor. The measurement of the state of the qubit under controlled evolution allows one to observe the effect caused by the nondissipative drag and determine the drag factor. In this paper we consider two particular schemes of the measurement. In the first scheme one should determine the time required to transform a reproducible initial state to a given final state. In the second scheme the geometrical (Berry) phase should be detected.

In Sec. II the microscopic theory of the nondissipative drag in two-component Bose gases is developed. In Sec. III a model of the Bose-Einstein qubit subjected by the drag force is formulated and the schemes of measurement of the drag factor are proposed. Conclusions are given in Sec. IV.

## II. NONDISSIPATIVE DRAG IN A TWO-COMPONENT BOSE SYSTEM: MICROSCOPIC DERIVATION

Let us consider a uniform two-component atomic Bose gas in a Bose-Einstein condensed state. We will study the most general situation where the densities of atoms in each component are different from one another ($n_1 \neq n_2$), the at-

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oms of each components have different masses ($m_1 \neq m_2$), and the interaction between atoms is described by three different scattering lengths ($a_{11} \neq a_{22} \neq a_{12}$). The Hamiltonian of the system can be presented in the form

$$H = \sum_{i=1,2} (E_i - \mu_i N_i) + \frac{1}{2} \sum_{i,i'=1,2} E_{ii'}^{int}, \qquad (1)$$

where

$$E_i = \int d^3 r \frac{\hbar^2}{2m_i} [\nabla \hat{\Psi}_i^\dagger(\mathbf{r})] \cdot \nabla \hat{\Psi}_i(\mathbf{r}) \qquad (2)$$

is the kinetic energy,

$$E_{ii'}^{int} = \int d^3 r \hat{\Psi}_i^\dagger(\mathbf{r}) \hat{\Psi}_{i'}^\dagger(\mathbf{r}) \gamma_{ii'} \hat{\Psi}_{i'}(\mathbf{r}) \hat{\Psi}_i(\mathbf{r}) \qquad (3)$$

is the energy of interaction, $\gamma_{ii}=4\pi\hbar^2 a_{ii}/m_i$ and $\gamma_{12}=2\pi\hbar^2(m_1+m_2)a_{12}/(m_1 m_2)$ are the interaction parameters, and $\mu_i$ are the chemical potentials.

For the further analysis it is convenient to use the density and phase operator approach (see, for instance, [10,11]). The approach is based on the following representation for the Bose field operators:

$$\hat{\Psi}_i(\mathbf{r}) = \exp[i\varphi_i(\mathbf{r}) + i\hat{\varphi}_i(\mathbf{r})]\sqrt{n_i + \hat{n}_i(\mathbf{r})}, \qquad (4)$$

$$\hat{\Psi}_i^\dagger(\mathbf{r}) = \sqrt{n_i + \hat{n}_i(\mathbf{r})}\exp[-i\varphi_i(\mathbf{r}) - i\hat{\varphi}_i(\mathbf{r})], \qquad (5)$$

where $\hat{n}_i$ and $\hat{\varphi}_i$ are the density and phase fluctuation operators, $\varphi_i(\mathbf{r})$ are the $c$-number terms of the phase operators, which are connected with the superfluid velocities by the relation $\mathbf{v}_i = \hbar \nabla \varphi_i / m_i$. In what follows we specify the case of the superfluid velocities independent of $\mathbf{r}$.

Substituting Eqs. (4) and (5) into Eq. (1) and expanding it in series in powers of $\hat{n}_i$ and $\nabla\hat{\varphi}_i$ we present the Hamiltonian of the system in the following form

$$H = H_0 + H_2 + \cdots. \qquad (6)$$

In Eq. (6) the term

$$H_0 = V\left( \sum_{i=1,2} \left[ \frac{1}{2} m_i n_i \mathbf{v}_i^2 + \frac{\gamma_{ii}}{2} n_i^2 - \mu_i n_i \right] + \gamma_{12} n_1 n_2 \right) \qquad (7)$$

does not contain the operator part. Here $V$ is the volume of the system. The minimization conditions for the Hamiltonian $H_0$ yield the equations

$$\frac{1}{2} m_i \mathbf{v}_i^2 + \gamma_{ii} n_i + \gamma_{12} n_{3-i} - \mu_i = 0 \quad (i = 1,2). \qquad (8)$$

Under the conditions (8) the terms, linear in the density fluctuation operators, vanish in the Hamiltonian. Taking into account the $\nabla \cdot [n_i \nabla \varphi_i(\mathbf{r})] = 0$, we find that the terms, linear in the phase fluctuation operators, vanish in the Hamiltonian as well.

The part of the Hamiltonian quadratic in $\nabla\hat{\varphi}_i$ and $\hat{n}_i$ operators reads as

$$H_2 = \int d\mathbf{r} \Bigg( \sum_i \Bigg\{ \frac{\hbar^2}{2m_i} \left[ \frac{[\nabla \hat{n}_i(\mathbf{r})]^2}{4 n_i} + n_i [\nabla \hat{\varphi}_i(\mathbf{r})]^2 \right] + \frac{\hbar \mathbf{v}_i}{2} \cdot \{ \hat{n}_i(\mathbf{r}) \nabla \hat{\varphi}_i(\mathbf{r}) + [\nabla \hat{\varphi}_i(\mathbf{r})] \hat{n}_i(\mathbf{r}) \} + \frac{i\hbar^2}{2m_i} \{ [\nabla \hat{n}_i(\mathbf{r})] \cdot \nabla \hat{\varphi}_i(\mathbf{r}) - [\nabla \hat{\varphi}_i(\mathbf{r})] \cdot \nabla \hat{n}_i(\mathbf{r}) \} + \frac{\gamma_{ii}}{2} [\hat{n}_i(\mathbf{r})]^2 \Bigg\} + \gamma_{12} \hat{n}_1(\mathbf{r}) \hat{n}_2(\mathbf{r}) \Bigg). \qquad (9)$$

The quadratic part of the Hamiltonian determines the spectra of the elementary excitations. Hereafter we will neglect the higher order terms in the Hamiltonian (6). These terms describe the scattering of the quasiparticles and they can be omitted if the temperature is much smaller than the temperature of Bose-Einstein condensation.

Let us rewrite the quadratic part of the Hamiltonian in terms of the operators of creation and annihilation of the elementary excitations. As the first step, we use the substitution

$$\hat{n}_i(\mathbf{r}) = \sqrt{\frac{n_i}{V}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \sqrt{\frac{\epsilon_{ik}}{E_{ik}}} [b_i(\mathbf{k}) + b_i^\dagger(-\mathbf{k})], \qquad (10)$$

$$\hat{\varphi}_i(\mathbf{r}) = \frac{1}{2i} \sqrt{\frac{1}{n_i V}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \sqrt{\frac{E_{ik}}{\epsilon_{ik}}} [b_i(\mathbf{k}) - b_i^\dagger(-\mathbf{k})], \qquad (11)$$

where operators $b_i^\dagger$, $b_i$ satisfy the Bose commutation relations. Here $\epsilon_{ik} = \hbar^2 k^2 / 2m_i$ is the spectrum of free atoms, and

$$E_{lk} = \sqrt{\epsilon_{ik}(\epsilon_{ik} + 2\gamma_{ii} n_i)} \qquad (12)$$

is the spectrum of the elementary excitations at $\gamma_{12}=0$ and $\mathbf{v}_i = \mathbf{0}$. The substitution (10) and (11) reduces the Hamiltonian (9) to the form quadratic in $b_i^\dagger$ and $b_i$ operators:

$$H_2 = \sum_{i\mathbf{k}} \left[ \mathcal{E}_i(\mathbf{k}) \left( b_i^\dagger(\mathbf{k}) b_i(\mathbf{k}) + \frac{1}{2} \right) - \frac{1}{2} \epsilon_{ik} \right] + \sum_{\mathbf{k}} g_k [b_1^\dagger(\mathbf{k}) b_2(\mathbf{k}) + b_1(\mathbf{k}) b_2(-\mathbf{k}) + \text{H.c.}]. \qquad (13)$$

Here

$$\mathcal{E}_i(\mathbf{k}) = E_{ik} + \hbar \mathbf{k} \cdot \mathbf{v}_i \qquad (14)$$

and

$$g_k = \gamma_{12} \sqrt{\frac{\epsilon_{1k} \epsilon_{2k} n_1 n_2}{E_{1k} E_{2k}}}. \qquad (15)$$

The Hamiltonian (15) contains nondiagonal in Bose creation and annihilation operator terms and it can be diagonalized using the standard procedure of $u$-$v$ transformation [13]. The result is

$$H_2=\sum_{\mathbf{k}}\left[\sum_{\lambda=\alpha,\beta}\mathcal{E}_\lambda(\mathbf{k})\left(\beta^\dagger_\lambda(\mathbf{k})\beta_\lambda(\mathbf{k})+\frac{1}{2}\right)-\frac{1}{2}\sum_{i=1,2}\epsilon_{ik}\right], \tag{16}$$

where $\beta^\dagger_\lambda(k)$ and $\beta_\lambda(k)$ are the operators of creation and annihilation of elementary excitations.

The energies $\mathcal{E}_\lambda(\mathbf{k})$ satisfy the equation

$$\det\begin{pmatrix}\mathbf{A}-\mathcal{E}\mathbf{I} & \mathbf{B}\\ \mathbf{B} & \mathbf{A}+\mathcal{E}\mathbf{I}\end{pmatrix}=0, \tag{17}$$

where

$$\mathbf{A}=\begin{pmatrix}\mathcal{E}_1(\mathbf{k}) & 0 & g_k & 0\\ 0 & \mathcal{E}_1(-\mathbf{k}) & 0 & g_k\\ g_k & 0 & \mathcal{E}_2(\mathbf{k}) & 0\\ 0 & g_k & 0 & \mathcal{E}_2(-\mathbf{k})\end{pmatrix}, \tag{18}$$

$$\mathbf{B}=\begin{pmatrix}0 & 0 & 0 & g_k\\ 0 & 0 & g_k & 0\\ 0 & g_k & 0 & 0\\ g_k & 0 & 0 & 0\end{pmatrix}, \tag{19}$$

and $\mathbf{I}$ is the identity matrix.

The densities of superfluid currents in two components can be obtained from the relation

$$\mathbf{j}_i=\frac{1}{V}\frac{\partial F}{\partial \mathbf{v}_i}, \tag{20}$$

where $F$ is the free energy of the system. Here the quantity $\mathbf{j}_i$ is defined as the density of the mass current.

The free energy of the system, described by the Hamiltonian (6), is given by the formula

$$F=H_0+\frac{1}{2}\sum_{\mathbf{k}}\left[\sum_{\lambda=\alpha,\beta}\mathcal{E}_\lambda(\mathbf{k})-\sum_{i=1,2}\epsilon_{ik}\right]+T\sum_{\mathbf{k}}\sum_{\lambda=\alpha,\beta}\ln\left[1-\exp\left(-\frac{\mathcal{E}_\lambda(\mathbf{k})}{T}\right)\right]. \tag{21}$$

The second term in Eq. (21) is the energy of the zero-point fluctuations and the third term is the standard temperature dependent part of the free energy for the gas of noninteracting elementary excitations.

We specify the case of small superfluid velocities (much smaller than the critical ones). In this case the currents can be approximated by the expressions linear in $\mathbf{v}_i$. To obtain these expressions we will find the free energy as series in $\mathbf{v}_i$, neglecting the terms higher than quadratic.

At $\mathbf{v}_1=\mathbf{v}_2=0$ the equation (17) is easily solved and the spectra are found to be

$$E_{\alpha(\beta)k}=\left(\frac{E_{1k}^2+E_{2k}^2}{2}\pm\sqrt{\frac{(E_{1k}^2-E_{2k}^2)^2}{4}+4\gamma_{12}^2 n_1n_2\epsilon_{1k}\epsilon_{2k}}\right)^{1/2}. \tag{22}$$

As required in the procedure [13], we take positive valued solutions of Eq. (17). The energies (22) should be real valued quantities. This requirement yields the common condition for the stability of the two-component system: $\gamma_{12}^2\leqslant\gamma_{11}\gamma_{22}$. If this condition were not fulfilled, spatial separation of two components (at positive $\gamma_{12}$) or a collapse (at negative $\gamma_{12}$) would take place.

At nonzero superfluid velocities we present the solutions of Eq. (17) as series in $\mathbf{v}_i$:

$$\mathcal{E}_\alpha(\mathbf{k})=E_{\alpha k}+\frac{1}{2}\hbar\mathbf{k}\cdot\mathbf{v}_1\left(1+\frac{E_{1k}^2-E_{2k}^2}{E_{\alpha k}^2-E_{\beta k}^2}\right)+\frac{1}{2}\hbar\mathbf{k}\cdot\mathbf{v}_2\left(1-\frac{E_{1k}^2-E_{2k}^2}{E_{\alpha k}^2-E_{\beta k}^2}\right)+\frac{2\gamma_{12}^2n_1n_2\epsilon_{1k}\epsilon_{2k}(3E_{\alpha k}^2+E_{\beta k}^2)}{E_{\alpha k}(E_{\alpha k}^2-E_{\beta k}^2)^3}\hbar^2(\mathbf{k}\cdot\mathbf{v}_1-\mathbf{k}\cdot\mathbf{v}_2)^2, \tag{23}$$

$$\mathcal{E}_\beta(\mathbf{k})=E_{\beta k}+\frac{1}{2}\hbar\mathbf{k}\cdot\mathbf{v}_1\left(1-\frac{E_{1k}^2-E_{2k}^2}{E_{\alpha k}^2-E_{\beta k}^2}\right)+\frac{1}{2}\hbar\mathbf{k}\cdot\mathbf{v}_2\left(1+\frac{E_{1k}^2-E_{2k}^2}{E_{\alpha k}^2-E_{\beta k}^2}\right)-\frac{2\gamma_{12}^2n_1n_2\epsilon_{1k}\epsilon_{2k}(E_{\alpha k}^2+3E_{\beta k}^2)}{E_{\beta k}(E_{\alpha k}^2-E_{\beta k}^2)^3}\hbar^2(\mathbf{k}\cdot\mathbf{v}_1-\mathbf{k}\cdot\mathbf{v}_2)^2. \tag{24}$$

Note that at $\mathbf{v}_1=\mathbf{v}_2=\mathbf{v}$ the spectra (23) and (24) are reduced to common expressions for the energies of quasiparticles in a moving condensate: $\mathcal{E}_{\alpha(\beta)}(\mathbf{k})=E_{\alpha(\beta)k}+\hbar\mathbf{k}\cdot\mathbf{v}$.

Using Eqs. (21), (23), and (24) we obtain the following expression for the free energy:

$$F=F_0+\frac{V}{2}[(\rho_1-\rho_{n1})\mathbf{v}_1^2+(\rho_2-\rho_{n2})\mathbf{v}_2^2-\rho_{\mathrm{dr}}(\mathbf{v}_1-\mathbf{v}_2)^2], \tag{25}$$

where $F_0$ does not depend on $\mathbf{v}_i$. In Eq. (25) $\rho_i=m_in_i$ are the mass densities, the quantities

$$\rho_{n1}=-\frac{m_1}{3V}\sum_{\mathbf{k}}\epsilon_{1k}\left[\frac{dN_{\alpha k}}{dE_{\alpha k}}+\frac{dN_{\beta k}}{dE_{\beta k}}+\frac{E_{1k}^2-E_{2k}^2}{E_{\alpha k}^2-E_{\beta k}^2}\left(\frac{dN_{\alpha k}}{dE_{\alpha k}}-\frac{dN_{\beta k}}{dE_{\beta k}}\right)\right], \tag{26}$$

$$\rho_{n2}=-\frac{m_2}{3V}\sum_{\mathbf{k}}\epsilon_{2k}\left[\frac{dN_{\alpha k}}{dE_{\alpha k}}+\frac{dN_{\beta k}}{dE_{\beta k}}-\frac{E_{1k}^2-E_{2k}^2}{E_{\alpha k}^2-E_{\beta k}^2}\left(\frac{dN_{\alpha k}}{dE_{\alpha k}}-\frac{dN_{\beta k}}{dE_{\beta k}}\right)\right] \tag{27}$$

describe the thermal reduction of the superfluid densities, and the quantity

$$\rho_{dr} = \frac{4}{3V}\sqrt{m_1 m_2}\sum_{\mathbf{k}} \frac{\gamma_{12}^2 n_1 n_2 (\epsilon_{1k}\epsilon_{2k})^{3/2}}{E_{\alpha k}E_{\beta k}}\left[\frac{1+N_{\alpha k}+N_{\beta k}}{(E_{\alpha k}+E_{\beta k})^3} - \frac{N_{\alpha k}-N_{\beta k}}{(E_{\alpha k}-E_{\beta k})^3} + \frac{2E_{\alpha k}E_{\beta k}}{(E_{\alpha k}^2-E_{\beta k}^2)^2}\left(\frac{dN_{\alpha k}}{dE_{\alpha k}}+\frac{dN_{\beta k}}{dE_{\beta k}}\right)\right], \quad (28)$$

which we call the "drag density," yields the value of redistribution of the superfluid densities between the components. In Eqs. (26)–(28) $N_{\alpha(\beta)k}=[\exp(E_{\alpha(\beta)k}/T)-1]^{-1}$ is the Bose distribution function.

Using Eqs. (20) and (25) we arrive at the following expressions for the supercurrents:

$$\mathbf{j}_1 = (\rho_1 - \rho_{n1} - \rho_{dr})\mathbf{v}_1 + \rho_{dr}\mathbf{v}_2, \quad (29)$$

$$\mathbf{j}_2 = (\rho_2 - \rho_{n2} - \rho_{dr})\mathbf{v}_2 + \rho_{dr}\mathbf{v}_1. \quad (30)$$

One can see that at nonzero $\rho_{dr}$ the current of one component contains the term proportional to the superfluid velocity of the other component. It means that there is a transfer of motion between the components. In particular, at $v_1=0$ the current in the component 1 ($\mathbf{j}_1=\rho_{dr}\mathbf{v}_2$) is purely the drag current. Since $\rho_{dr}$ is the function of $\gamma_{12}^2$ [see Eqs. (28) and (22)] the drag current does not depend on the sign of the interaction between the components.

Equation (28) is the main result of the paper. This equation yields the value of the drag for the general case of the two-component Bose system with components of different densities, different masses of atoms, different interaction parameters, and for zero as well as for nonzero temperatures. Moreover, this equation is valid not only for the point interaction between the atoms, but for any central force interaction. In the latter case the interaction parameters $\gamma_{ik}$ in Eq. (28) and in the spectra (22) and (12) should be replaced with the Fourier components of the corresponding interaction potentials.

To estimate the absolute value of the drag we, for simplicity, specify the case $m_1=m_2=m$, that is realized when two components are two hyperfine states of the same atoms.

At $T=0$ Eq. (28) is reduced to

$$\rho_{dr} = \frac{4m}{3}\int_0^\infty d\epsilon \frac{\gamma_{12}^2 n_1 n_2 \nu(\epsilon)\epsilon^{1/2}}{\sqrt{(\epsilon+w_1)(\epsilon+w_2)}(\sqrt{\epsilon+w_1}+\sqrt{\epsilon+w_2})^3}, \quad (31)$$

where

$$\nu(\epsilon) = \frac{m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\sqrt{\epsilon}$$

is the density of states for free atoms, and

$$w_{1(2)} = \gamma_{11}n_1 + \gamma_{22}n_2 \pm \sqrt{(\gamma_{11}n_1 - \gamma_{22}n_2)^2 + 4\gamma_{12}^2 n_1 n_2}.$$

The integral in Eq. (31) can be evaluated analytically. To present the answer in a compact form it is convenient to introduce the dimensionless parameters

$$\eta = \frac{a_{12}^2}{a_{11}a_{22}} \text{ and } \kappa = \sqrt{\frac{n_1 a_{11}}{n_2 a_{22}}} + \sqrt{\frac{n_2 a_{22}}{n_1 a_{11}}}$$

($0 \leqslant \eta \leqslant 1$ and $\kappa \geqslant 2$).

Using these notations we have

$$\rho_{dr} = \sqrt{\rho_1\rho_2}\sqrt[4]{n_1 a_{11}^3 n_2 a_{22}^3}\frac{\eta}{\sqrt{\kappa}}F(\kappa,\eta), \quad (32)$$

where

$$F(\kappa,\eta) = \frac{256}{45\sqrt{2\pi}}\frac{(\kappa+3\sqrt{1-\eta})\sqrt{\kappa}}{(\sqrt{\kappa+\sqrt{\kappa^2-4+4\eta}}+\sqrt{\kappa-\sqrt{\kappa^2-4+4\eta}})^3}. \quad (33)$$

Direct evaluation of Eq. (33) shows that at allowed $\eta$ and $\kappa$ ($0 \leqslant \eta \leqslant 1$ and $\kappa \geqslant 2$) the factor $F(\kappa,\eta)$ is almost the constant (the range of variation of $F$ is [0.7–0.8]) and one can neglect the dependence of $F$ on the parameter of the system.

At $a_{11}n_1=a_{22}n_2$ we obtain from Eq. (32) the following approximate relation:

$$\rho_{dr} \approx \frac{1}{2}\rho_1\frac{a_{12}^2}{a_{11}a_{22}}\sqrt{n_1 a_{11}^3} = \frac{1}{2}\rho_2\frac{a_{12}^2}{a_{11}a_{22}}\sqrt{n_2 a_{22}^3}. \quad (34)$$

If the density of one component is much larger than that of the other and $a_{11} \sim a_{22}$, the "drag density" is approximated as

$$\rho_{dr} \approx 0.8\rho_1\frac{a_{12}^2}{a_{22}^2}\sqrt{n_2 a_{22}^3} \text{ at } n_1 \ll n_2,$$

$$\rho_{dr} \approx 0.8\rho_2\frac{a_{12}^2}{a_{11}^2}\sqrt{n_1 a_{11}^3} \text{ at } n_2 \ll n_1. \quad (35)$$

One can see that the "drag density" is proportional to the square root of the gas parameter. It means that the drag effect is larger in "less ideal" Bose gases.

The temperature dependence of the "drag density" at small $T$ can be evaluated analytically from Eq. (28) using the linear approximation for the spectra of the excitations. It yields $\rho_{dr}(T)=\rho_{dr}(0)(1-\alpha_T T^4/T_0^4)$, where $T_0=\sqrt{\gamma_{11}n_1\gamma_{22}n_2}$ and the factor $\alpha_T$ is positive. Numerical evaluation of the sum over $\mathbf{k}$ in Eq. (28) shows that the analytical approximation is valid only at $T \ll T_0$. At $T \gtrsim T_0$ the "drag density" decreases much slower under increase of the temperature. As an example, the dependence of $\rho_{dr}(T)$ at $n_1=n_2=n$, $\gamma_{11}=\gamma_{22}=\gamma$, and $\eta=0.5$ is shown in Fig. 1.

Now let us discuss how the drag effect can reveal itself in a real physical situation. If one deals with the stationary superflow one implies that it is the circulating superflow, e.g., the tangential superflow in a hole cylinder. In such a case the

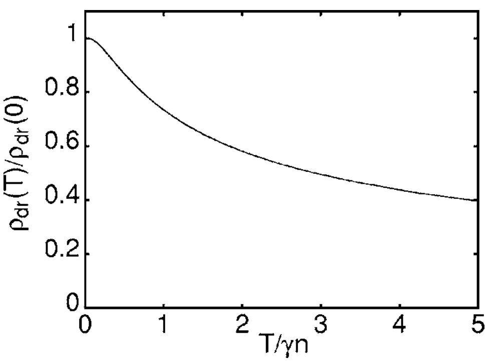


FIG. 1. Dependence of the "drag density" on the temperature.

superfluid velocities satisfy the Onsager-Feynman quantization condition

$$\oint \mathbf{v}_i \cdot d\mathbf{l} = \frac{2\pi\hbar N_i}{m_i}, \tag{36}$$

where the vorticity parameters $N_i$ are integer. Then, the drag effect can be understood as the appearance of the circulating current in the drive component (e.g., specie 1), when the circulation of the superfluid velocity of the drive component (e.g., specie 2) is fixed ($N_2$=const). The current of the specie 1 [Eq. (29)] depends on the superfluid velocities of both species and if the superfluid velocity of the drag component is directed antiparallel to the superfluid velocity of the drive component the current of the drag component might vanish. But since the velocities are quantized it may happen only under certain special conditions (see below). The superfluid velocity of the drag component is determined by that at fixed $N_2$ the free energy (25) has a minimum with respect to discrete values of $\mathbf{v}_1=\hbar N_1/(m_1 R)$ [where the $R$ is the radius of the contour in Eq. (36)]. Depending on the value of the parameter $\alpha=[\rho_{dr}/(\rho_2-\rho_{n2}-\rho_{dr})](m_1/m_2)N_2$ several possibilities can be realized. At $|\alpha|<1/2$ the minimum of the energy (25) corresponds to $N_1=0$ (and $\mathbf{v}_1=0$). In this case the current of the drag component is directed along the drive current and it is proportional to the drag density. At $|\alpha|=p$ ($p$ is natural) the value $N_1=-p$ minimizes the energy. In this case two terms in Eq. (29) compensate each other and the current in the drag component vanishes. At half-integer $\alpha$ the degenerate situation takes place: two states (with codirected currents, and counterdirected currents) have the same energy. At $p-1/2<|\alpha|<p$ the state with counterdirected currents gains the energy and at $p<|\alpha|<p+1/2$ the codirected currents are energetically preferable. In the latter two cases the nonzero vorticity of the drag component ($N_1\neq 0$) is also induced. This behavior is analogous to the behavior of a superconducting ring in a magnetic field. We note that since $\rho_{dr}\ll\rho_2$, the most realistic case is $|\alpha|<1/2$ when the simple picture of the transfer of part of the motion from the drive to the drag component takes place.

In this study we have concentrated on the analytical derivation of the drag effect in the uniform Bose gases. The consideration of the nonuniform case requires the solution of the eigenvalue problem for the elementary excitations in the two-component Bose gas in the external potential. But even for the simplest case of a spherically symmetric trap this problem can be solved analytically only in the long-wavelength limit and the Thomas-Fermi approximation [12] (the spectrum of elementary excitations in one-component Bose gases was obtained analytically for a number of potentials but also in the same limit [11,14–16]). Since the main contribution to the drag density comes from the excitations with the wave vectors of order of the healing length [see Eq. (28)], the rigorous analysis of the drag effect can be done only numerically. Nevertheless, in the Tomas-Fermi situation the drag effect can be evaluated basing on the following arguments. When the linear size of the Bose cloud is much larger than the healing length, the spectrum of the excitations at the wave vectors of order or higher than the inverse healing length is well-described by the quasiuniform approximation. Therefore the drag effect can be described by the same equations, as in the uniform case with the only modification that the quantities $n_1$ and $n_2$, and, correspondingly, $\rho_i$, $\rho_{ni}$, $\rho_{dr}$, and $j_i$ in Eqs. (26)–(30) are understood as functions of coordinates.

At an arbitrary symmetry of the trap potential the superfluid velocity of the drag component cannot be equal to zero in each point. Indeed, in the general case of space dependent $\rho_i$, $\rho_{ni}$, and $\rho_{dr}$ the velocity field $\mathbf{v}_2(\mathbf{r})$ cannot satisfy two independent continuity conditions $\boldsymbol{\nabla}\cdot[(\rho_2-\rho_{n2}-\rho_{dr})\mathbf{v}_2]=0$ and $\boldsymbol{\nabla}\cdot(\rho_{dr}\mathbf{v}_2)=0$. To analyze this case one should find the velocity fields $\mathbf{v}_1(\mathbf{r})$ and $\mathbf{v}_2(\mathbf{r})$ that satisfy the continuity conditions and the quantization conditions. To illustrate this point let us consider a simple example of a trap having the shape of a hollow cylinder with the densities that depend only on the polar angle $\phi$. We will seek the velocity fields that do not have radial components. Then, Eqs. (29) and (30), written for the tangential components of the currents and the velocities, can be presented in the matrix form

$$\begin{pmatrix} j_1 \\ j_2 \end{pmatrix} = \hat{R}\begin{pmatrix} v_1(r,\phi) \\ v_2(r,\phi) \end{pmatrix}, \tag{37}$$

where

$$\hat{R} = \begin{pmatrix} \rho_{s1}(\phi)-\rho_{dr}(\phi) & \rho_{dr}(\phi) \\ \rho_{dr}(\phi) & \rho_{s2}(\phi)-\rho_{dr}(\phi) \end{pmatrix} \tag{38}$$

with $\rho_{si}(\phi)=\rho_i(\phi)-\rho_{ni}(\phi)$. Due to the continuity conditions the currents $j_1$ and $j_2$ in Eq. (37) do not depend on $\phi$. According to Eq. (37) the velocities $v_1(r,\phi)$ and $v_2(r,\phi)$ are connected with the currents by the equation

$$\begin{pmatrix} v_1(r,\phi) \\ v_2(r,\phi) \end{pmatrix} = \hat{R}^{-1}\begin{pmatrix} j_1(r) \\ j_2(r) \end{pmatrix}. \tag{39}$$

Integrating Eq. (39) over $\phi$ and taking into account the quantization conditions (36) we obtain the equation for the currents

$$\hat{T}\begin{pmatrix} j_1(r) \\ j_2(r) \end{pmatrix} = \frac{2\pi\hbar}{r}\begin{pmatrix} N_1/m_1 \\ N_2/m_2 \end{pmatrix}, \tag{40}$$

where

$$\hat{T} = \begin{pmatrix} \int_0^{2\pi} d\phi \frac{\rho_{s2}-\rho_{dr}}{\rho_{s1}\rho_{s2}-\rho_{dr}(\rho_{s1}+\rho_{s2})} & -\int_0^{2\pi} d\phi \frac{\rho_{dr}}{\rho_{s1}\rho_{s2}-\rho_{dr}(\rho_{s1}+\rho_{s2})} \\ -\int_0^{2\pi} d\phi \frac{\rho_{dr}}{\rho_{s1}\rho_{s2}-\rho_{dr}(\rho_{s1}+\rho_{s2})} & \int_0^{2\pi} d\phi \frac{\rho_{s1}-\rho_{dr}}{\rho_{s1}\rho_{s2}-\rho_{dr}(\rho_{s1}+\rho_{s2})} \end{pmatrix}. \tag{41}$$

If a given vorticity of the drive component $N_2$ is not very large the minimum of energy is reached at $N_1=0$. In the latter case the solution of Eq. (40) in the leading order in $\rho_{dr}$ yields the following expression for the current of the drag component:

$$j_1(r) \approx \frac{2\pi\hbar N_2}{m_2 r} \frac{\int_0^{2\pi} d\phi \frac{\rho_{dr}(\phi)}{\rho_{s1}(\phi)\rho_{s2}(\phi)}}{\int_0^{2\pi} d\phi \frac{1}{\rho_{s1}(\phi)} \int_0^{2\pi} d\phi \frac{1}{\rho_{s2}(\phi)}}. \tag{42}$$

One can see that if at some $\phi$ the density $\rho_{s1}$ has a sharp minimum the first factor in the denominator in Eq. (42) becomes large. On the other hand, the integral in the numerator is not very sensitive to lowering of $\rho_{s1}$ [see Eqs. (35)]. Thus in a system with a "bottle neck" in the drag component the drag current decreases strongly and the main consequence of the drag effect is the emergence of the gradient of the phase of the order parameter of the drag component. A similar situation takes place in a system with a weak link. The latter case is analyzed in the next section. In the uniform case Eq. (42) is reduced to $j_1=\rho_{dr}v_2$.

To complete the discussion we emphasize that the crossed term ($\rho_{dr}\mathbf{v}_1\cdot\mathbf{v}_2$) in the free energy (25) [and, consequently, the drag terms in the currents (29) and (30)] comes only from the second and third terms in Eq. (21). Consequently, the drag effect considered in this paper is solely by the excitations. At the mean field level of approximation (which can be also formulated in terms of the Gross-Pitaevsky equation) the effect does not appear, while the coupling between the components is also present at that level of approximation. We would note that at the mean field level the drag effect of another type may emerge. That effect takes place in the case when one of the species is subjected by an asymmetric rotating external potential (see, for instance, [17], where such an effect has been studied with reference to the system of two coupled traps).

## III. MODEL OF BOSE-EINSTEIN QUBIT WITH EXTERNAL DRAG FORCE

It is known that Bose systems in the Bose-Einstein condensed state may demonstrate Josephson phenomenon [9]. In this paper we consider the external Josephson effect that takes place in two-well Bose systems. It was shown in [18] that in such systems one can realize the situation, when two states, that differ in the expectation value of the relative number operator, can be used as qubit states.

To include the drag force into the play we consider the following geometry. Let our two-component system be confined in a toroidal trap and the Bose cloud of the component 1 (the drag component) is situated inside and overlaps with the Bose clouds of the component 2 (the drive component). Such a situation can be realized if $|\gamma_{12}|<\min(\gamma_{11},\gamma_{22})$.

Deforming the confining potential one can cut the drag component into two clouds of a half-torus shape (separated by two Josephson links) leaving the Bose cloud of the drive component uncut (Fig. 2). In what follows we use the following notations: $R_t$ is the large radius of the toroidal trap, and $r_{t1}$ and $r_{t2}$ are the small radii of the toroidal Bose clouds of the drag and the drive components, correspondingly.

Rotating this trap one can excite a superflow in the drive component. After the rotation is switched off there will is a circulating superflow in the drive component and no superflow in the drag component (at negligible small Josephson coupling). The superfluid velocity of the drive component is

$$v_2 = \frac{N_2\hbar}{m_2 R_t}. \tag{43}$$

In Eq. (43), we imply that $R_t \gg r_{t1}, r_{t2}$ and neglect, for simplicity, the effect caused by a dependence of $r_{t2}$ on the polar angle.

Since $j_1=0$, the phase gradient $\nabla\varphi_1$ should be nonzero to compensate for the drag effect. In the polar coordinates the $\phi$ component of the phase gradient is given by the relation

$$(\nabla\varphi_1)_\phi = -\frac{N_2}{R_t} f_{dr} = -f_{dr}(\nabla\varphi_2)_\phi, \tag{44}$$

where

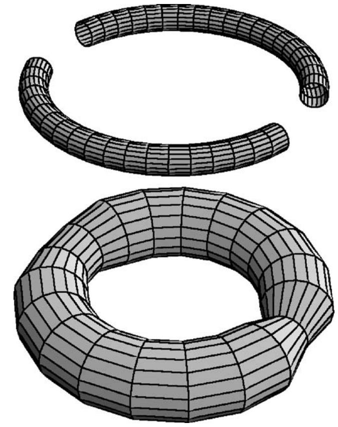

FIG. 2. Schematic shapes of Bose clouds for the drag (top figure) and drive (bottom figure) components. The drag component is situated inside and overlaps with the drive component.

$$f_{dr} = \frac{m_1}{m_2} \frac{\rho_{dr}}{\rho_{s1} - \rho_{dr}}. \quad (45)$$

The quantity $f_{dr}$ yields the ratio between the phase gradients in the drag and the drive components in the situation when the drag component is in the open circuit (i.e., the current cannot flow in the circuit). We call this quantity the drag factor.

We imply that $r_{t1}$ and $r_{t2}$ are much larger than the healing lengths that allows one to describe the drag effect in quasi-uniform approximation. For definiteness, we specify the case of $\rho_1 \ll \rho_2$ and $\rho_2 \approx$const in the overlapping region. In this case one can neglect the space dependence the drag factor [see Eqs. (35)].

At nonzero Josephson coupling the current $j_1$ can be nonzero, but it cannot exceed the maximum Josephson current $j_m$. Relation (44) remains approximately correct at nonzero Josephson coupling if an inequality $j_m \ll \hbar\rho_1/(m_1 R_t)$ is satisfied. Here we specify just such a case. It is important to emphasize that we consider the situation, when there is only the external Josephson effect between two half-torus traps, and there is no internal Josephson effect between the drag and the drive species.

The drag force can be considered as an effective vector potential $\mathbf{A}_{dr} = -\hbar f_{dr} \nabla \varphi_2$ (in units of $e=c=1$) that corresponds to an effective magnetic flux $\Phi_{dr} = -2\pi\hbar f_{dr} N_2$. Thus our Bose system is similar to the Cooper pair box system that implements the Josephson charge qubit with the Josephson coupling controlled by an external magnetic flux [19]. To extend this analogy we formulate the model of the Bose-Einstein qubit subjected by the drag force. In what follows we use the approach of Ref. [18].

In the two mode approximation the Bose field operators for the drag component can be presented in the form:

$$\hat{\Psi}_1(\mathbf{r},t) = \sum_{l=L,R} a_l(t)\Psi_l(\mathbf{r}-\mathbf{r}_l),$$

$$\hat{\Psi}_1^\dagger(\mathbf{r},t) = \sum_{l=L,R} a_l^\dagger(t)\Psi_l^*(\mathbf{r}-\mathbf{r}_l), \quad (46)$$

where $a_{L(R)}^\dagger$ and $a_{L(R)}$ are the operators of creation and annihilation of bosons in the condensates confined in the left(right) half-torus, and $\Psi_L$, $\Psi_R$ are two almost orthogonal local mode functions

$$\int d^3r \Psi_l^*(\mathbf{r})\Psi_{l'}(\mathbf{r}) \approx \delta_{ll'}, \quad l,l' = L,R$$

that describe the condensate in the left and right traps [20].

Substituting Eq. (46) into Hamiltonian (1), we obtain the following expression for the parts of the Hamiltonian that depend on the operators $a_l^\dagger$ and $a_l$:

$$H_a = \sum_{l=L,R} (K_l a_l^\dagger a_l + \lambda_l a_l^\dagger a_l^\dagger a_l a_l) + (J a_L^\dagger a_R + J^* a_R^\dagger a_L), \quad (47)$$

with

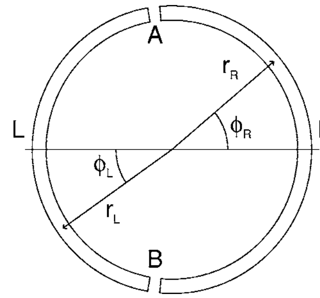


FIG. 3. Left (L) and right (R) half-torus of the drag component, separated by Josephson links $A$ and $B$.

$$K_l = \int d^3r \Psi_l^* \left[ -\frac{\hbar^2}{2m}\nabla^2 + V_{tr} + \gamma_{12}\Psi_2^*\Psi_2 \right]\Psi_l, \quad (48)$$

$$\lambda_l = \frac{\gamma_{11}}{2} \int d^3r |\Psi_l|^4, \quad (49)$$

$$J = \int d^3r \left[ \frac{\hbar^2}{2m} \nabla \Psi_L^* \cdot \nabla \Psi_R + V_{tr}\Psi_L^*\Psi_R \right]. \quad (50)$$

The functions $\Psi_L$ and $\Psi_R$ contain the phase factors $e^{i\varphi_L(\mathbf{r})}$ and $e^{i\varphi_R(\mathbf{r})}$, where the phases satisfy Eq. (44). Taking these factors into account, one can choose the following basis for the one mode functions

$$\Psi_{L(R)}(\mathbf{r}) = |\Psi_{L(R)}(\mathbf{r})| \exp[-iN_2 f_{dr}\phi_{L(R)}(\mathbf{r})], \quad (51)$$

where $\phi_L, \phi_R$ are the polar angles counted from the centers of $L$ and $R$ half-torus, correspondingly (see Fig. 3). The angles $\phi_{L(R)}(\mathbf{r})$, defined as shown in Fig. 3, satisfy the relation

$$\phi_R(\mathbf{r}_A) - \phi_L(\mathbf{r}_A) = \phi_L(\mathbf{r}_B) - \phi_R(\mathbf{r}_B) = \pi, \quad (52)$$

where $\mathbf{r}_A$ and $\mathbf{r}_B$ are the radius-vectors of Josephson links.

Substituting Eq. (51) into Eq. (50), using Eq. (52), and taking into account that the functions $\Psi_L$ and $\Psi_R$ overlap in a small vicinity of $A$ and $B$ links, we obtain the following expression for the Josephson coupling parameter:

$$J = (J_A + J_B)\cos\left(\pi\frac{\Phi_{dr}}{\Phi_0}\right) + i(J_A - J_B)\sin\left(\pi\frac{\Phi_{dr}}{\Phi_0}\right), \quad (53)$$

where $\Phi_0 = 2\pi\hbar$ is the "flux quantum" and

$$J_{A(B)} \approx \int_{V_{A(B)}} d^3r \left[ \frac{\hbar^2}{2m} \nabla |\Psi_L| \cdot \nabla |\Psi_R| + V_{tr}|\Psi_L||\Psi_R| \right]. \quad (54)$$

Here $V_A$ and $V_B$ are the areas of overlapping of two one mode functions at links $A$ and $B$, correspondingly.

Considering the Hilbert space in which the total number operator

$$\hat{N} = a_L^\dagger a_L + a_R^\dagger a_R \quad (55)$$

is a conservative quantity ($\hat{N} = N$) we present the Hamiltonian (47) in the following form:

$$H_a = E_c(\hat{n}_{RL} - n_g)^2 + (J a_L^\dagger a_R + \text{H.c.}) + \text{const}, \quad (56)$$

where

$$\hat{n}_{RL} = \frac{a_R^\dagger a_R - a_L^\dagger a_L}{2} \quad (57)$$

is the number difference operator,

$$E_c = \lambda_R + \lambda_L \quad (58)$$

is the interaction energy, and the quantity

$$n_g = \frac{1}{2E_c}[K_L - K_R + (N-1)(\lambda_L - \lambda_R)] \quad (59)$$

describes an asymmetry of L and R half-tore.

In what follows we imply that the system is in the Fock regime [9] ($|J|N \ll E_c$) and use the number representation

$$|n_{RL}\rangle \equiv |n_R, n_L\rangle \equiv \left| \frac{N}{2} + n_{RL}, \frac{N}{2} - n_{RL} \right\rangle.$$

In this representation the first term in Eq. (56) is diagonal. The second term in Eq. (56) can be considered as a small nondiagonal correction. But if $n_g$ is biased near one of the degeneracy points

$$n_{deg} = \begin{cases} M + \frac{1}{2} & \text{for even } N \\ M & \text{for odd } N \end{cases} \quad (60)$$

(where $M$ is an integer and $|M| < N/2$), the second term in Eq. (56) results in a strong mixing of two lowest states ($|\uparrow\rangle = |n_{deg}+1/2\rangle$ and $|\downarrow\rangle = |n_{deg}-1/2\rangle$) and the low energy dynamics of the system can be described by a pseudospin Hamiltonian

$$H_{eff} = -\frac{\Omega_x}{2}\hat{\sigma}_x - \frac{\Omega_y}{2}\hat{\sigma}_y - \frac{\Omega_z}{2}\hat{\sigma}_z, \quad (61)$$

where $\hat{\sigma}_i$ are the Pauli operators, and

$$\Omega_x = -(J_A + J_B)\sqrt{(N+1)^2 - 4n_{deg}^2}\cos\left(\pi\frac{\Phi_{dr}}{\Phi_0}\right),$$

$$\Omega_y = -(J_A - J_B)\sqrt{(N+1)^2 - 4n_{deg}^2}\sin\left(\pi\frac{\Phi_{dr}}{\Phi_0}\right),$$

$$\Omega_z = 2E_c(n_g - n_{deg}) \quad (62)$$

are the components of the pseudomagnetic filed. In experiments one can control the parameters $n_g$, $J_A$, and $J_B$ independently and, consequently, the pseudomagnetic field $\mathbf{\Omega}(t)$ can be switched arbitrarily. It means that Eq. (61) represents the standard Hamiltonian of the qubit system. The parameters of the qubit (61) depend on the "drag flux" $\Phi_{dr}$. Therefore one can determine its value from the measurement of the state of the system after a controlled evolution of a certain reproducible initial state.

Let us consider two possibilities. For definiteness, we specify the case of odd $N$ and the degeneracy point $n_{deg}=0$. If the Josephson coupling is switched off and $n_g$ is switched on to some positive value (much less than unity) the system is relaxed to the state $|\psi_{in}\rangle = |\uparrow\rangle$. This state can be used as the reproducible initial state. The quantity that should be measured is the expectation value of the number difference operator. In the initial state the expectation value of this operator is $n_{RL}=1/2$.

When the system is switched suddenly to the degeneracy point $n_g=0$ and the Josephson couplings are switched on for some time $\tau$ the initial state evolves to another state with another $n_{RL}$.

If one sets $J_A = J_B = J$ the result of evolution ($|\psi_f\rangle = U|\psi_{in}\rangle$ is described by the unitary operator

$$U_1(\tau) = \begin{pmatrix} \cos(\alpha_1\tau) & -i\sin(\alpha_1\tau) \\ -i\sin(\alpha_1\tau) & \cos(\alpha_1\tau) \end{pmatrix}$$

where $\alpha_1 = (J/\hbar)(N+1)\cos(\pi\Phi_{dr}/\Phi_0)$. One can see that at the time of evolution $\tau = \tau_1 = \pi/(4|\alpha_1|)$ the expectation value of the number difference operation will be equal to zero.

For the case $J_A = J$ and $J_B = 0$ the operator of evolution reads as

$$U_2(\tau) = \begin{pmatrix} \cos(\alpha_2\tau) & -ie^{-i\pi\Phi_{dr}/\Phi_0}\sin(\alpha_2\tau) \\ -ie^{i\pi\Phi_{dr}/\Phi_0}\sin(\alpha_2\tau) & \cos(\alpha_2\tau) \end{pmatrix}$$

with $\alpha_2 = (J/2\hbar)(N+1)$. Respectively, the expectation value $n_{RL}$ will be equal to zero at $\tau = \tau_2 = \pi/(4\alpha_2)$.

The ratio $\tau_2/\tau_1 = |\cos(\pi\Phi_{dr}/\Phi_0)|/2$ depends only on $\Phi_{dr}$ and the quantity $\Phi_{dr}$ can be extracted from the measurements of $\tau_1$ and $\tau_2$. It is important to note that to provide this scheme one should control only the ratio of $J_A$ and $J_B$, but not their absolute values.

Another possibility can be based on detection of the Berry phase [21]. Equation (61) contains all three components of the field $\mathbf{\Omega}$ and they can be controlled independently. The general scheme of detection of the Berry phase in such a situation was proposed [22]. A concrete realization of this scheme in the Josephson charge qubit was described in [23]. Here we extend the ideas of [22,23] to the case of the "dragged" Bose-Einstein qubit.

We start from the same initial state and switch to $J_A = J_B = J$ and $n_g = 0$. The initial state $|\uparrow\rangle$ can be presented as the superposition of two instantaneous eigenstates $|e_a\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ and $|e_b\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$:

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}}(|e_a\rangle + |e_b\rangle). \quad (63)$$

An adiabatic cyclic evolution of the parameters of the Hamiltonian (61) results in the appearance of the Berry phase in the $|e_a\rangle$ and $|e_b\rangle$ eigenstates if the vector $\mathbf{\Omega}$ subtends a nonzero solid angle at the origin.

Let us consider the following four stage cyclic adiabatic evolution starting from the point $J_A = J_B = J$ and $n_g = 0$: 1—$J_B$ is switched off; 2—$J_A$ is switched off and simultaneously $n_g$ is switched to $n_{g1} > 0$; 3—$n_g$ is returned to the same degeneracy point ($n_g = 0$) and $J_B$ is switched to $J_B = J$; and 4—$J_A$ is switched to $J_A = J$ (all switches should be done slowly: $\hbar|d\mathbf{\Omega}/dt| \ll \Omega^2$).

After such an evolution the system arrives at the state

$$|\psi_m\rangle = \frac{1}{\sqrt{2}}(e^{i\delta_a+i\gamma}|e_a\rangle + e^{i\delta_b-i\gamma}|e_b\rangle), \tag{64}$$

where $\gamma=\pi\Phi_{dr}/\Phi_0$ is the Berry phase (equal to half of the solid angle subtended by $\mathbf{\Omega}$) and $\delta_a$, $\delta_b$ are the dynamical phases. Elimination of the dynamical phases can performed by swapping the eigenstates ($\pi$-transformation) and repeating the same cycle of evolution in a reverse direction (see [22]).

The $\pi$-transformation can be done by fast switching off the Josephson coupling and switching on $n_g=n_{g2}>0$ during the time interval $t_\pi=\hbar\pi/(2E_c n_{g2})$. After the $\pi$-transformation the state becomes

$$|\psi_{m\pi}\rangle = -\frac{i}{\sqrt{2}}(e^{i\delta_a+i\gamma}|e_b\rangle + e^{i\delta_b-i\gamma}|e_a\rangle). \tag{65}$$

After the cyclic evolution in the reverse direction we arrive at the state

$$|\psi_f\rangle = -\frac{i}{\sqrt{2}}e^{i(\delta_a+\delta_b)}(e^{2i\gamma}|e_b\rangle + e^{-2i\gamma}|e_a\rangle). \tag{66}$$

One can see that the expectation value of the number difference operator in the final state (66) $n_{RL}=\cos(4\gamma)/2=\cos(4\pi\Phi_{dr}/\Phi_0)/2$ depends only on $\Phi_{dr}$ and the measurement of this difference allows one to determine the value of the "drag flux."

Thus the measurements of a relative number of atoms in left and right condensates under controlled evolution of the state of the system allows one to observe the nondissipative drag and determine the drag factor (if the vorticity of the drive component is known).

## IV. CONCLUSIONS

We have investigated the nondissipative drag effect in three-dimensional weakly interacting two-component superfluid Bose gases. The expression for the drag current is derived microscopically for the general case of two species of different densities, different masses, and different interaction parameters. It is shown that the drag current is proportional to the square root of the gas parameter. The drag effect is maximal at zero temperatures and it decreases when the temperature increases, but at temperatures of order of the interaction energy the drag current remains of the same order as at zero temperature.

We have considered the toroidal double-well geometry, where the nondissipative drag influences significantly the Josephson coupling between the wells. In the system considered the drag force can be interpreted as an effective vector potential applied to the drag component. The effective vector potential is equal to $\mathbf{A}_{dr}=-\hbar f_{dr}\nabla\varphi_{drv}$ (in units of $e=c=1$), where $\varphi_{drv}$ is the phase of the drive component, and $f_{dr}$ is the drag factor. In the toroidal geometry the effective vector potential can be associated with an effective flux of external field $\Phi_{dr}=2\pi\hbar f_{dr}N_v$, where $N_v$ is the vorticity of the drive component. In the Fock regime the system can be considered as a Bose-Einstein counterpart of the Josephson charge qubit in an external magnetic field. The measurement of the state of such a qubit allows one to observe the drag effect and determine the drag factor.

## ACKNOWLEDGMENT

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