Proposal for a planar Penning ion trap

J. R. Castrejón-Pita and R. C. Thompson
The Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, United Kingdom
(Received 12 April 2005; published 11 July 2005)

An alternative design for a planar Penning ion trap is presented. Although the analytical trapping potential is not exactly quadratic, approximate values for the characteristic frequencies around the equilibrium point can be found. An array of traps can be formed simply using long straight wires, and the open geometry of this design means that the structure can be miniaturized using well-known techniques. Consequently, these traps should be considered as excellent candidates for applications in quantum computation.

DOI: 10.1103/PhysRevA.72.013405 PACS number(s): 32.80.Pj, 39.10.+j, 42.50.−p, 03.67.−a

I. INTRODUCTION

The basic principles of ion traps are very simple: the motion of a charged particle is confined by electric and magnetic fields. Two configurations of three-dimensional ion traps are commonly used: the Penning and the Paul traps. Both traps consist of three electrodes, a ring electrode and two end-cap electrodes. These traps differ in the way in which the three-dimensional confinement is produced. The Penning trap utilizes an electrostatic potential between the ring and the end caps in order to confine the ion axially, and a magnetic field provides radial confinement. On the other hand, in the Paul trap a radio frequency (rf) voltage is applied between the end caps and the ring electrode. This rf component, combined with a constant electrostatic potential, produces axial and radial trapping (see [1]).

A different design of a planar trap has been proposed recently by Stahl *et al.* [2]. This trap consists of a planar conducting disk (inner electrode) surrounded by a ring electrode. When the electrodes are connected to their respective voltage supplies, a trapping region above the plane of the electrodes is produced. This trap is essentially a Penning trap because the final radial confinement is produced by a magnetic field perpendicular to the electrodes. From the point of view of the application to quantum computation, this concept has many advantages over the conventional designs of traps, in particular it presents better scalability and its open geometry would be useful to allow free optical access to the trapped ions. Based on the same principles and properties, an alternative design is presented in this work. Our proposal shares the same open geometry as the previous proposal by Stahl *et al.* [2] and the full equation of motion can be analytically treated.

II. THE PLANAR GUIDE

The planar trap [2] works because it is able to generate an axial potential minimum above the electrodes. At this point the forces acting on a charged particle generated by the two electrodes cancel out; the particle experiences an attractive force coming from the external ring which is compensated by a repulsive force coming from the inner disk [Fig. 1(a)]. In this paper we propose a trap made from thin wires or lines of charges that can be understood in the same way; the force coming from a central thin wire is compensated by the force coming from two outer parallel wires [Fig. 1(b)]. Although this configuration is only able to trap ions along a line above the central wire, a closed loop or the addition of another perpendicular set of wires would be suitable to confine ions in all directions (see Sec. II). The advantages of this trap are that it is scalable in a straightforward way, the expression for the electric potential can be analytically solved, and it should be easy to implement in the laboratory.

This paper is structured as follows. A solution is found for the electrostatic potential of a single charged line, and then the potential for a set of lines is presented. Afterward, the minimum points of the potential are calculated and quadratic expansions (Taylor series) around them are made. Then, using this potential and a magnetic field, the equation of motion of a charged particle can be established and solved. In order to make a more direct connection to a trap that would be realizable in the laboratory, the actual potentials required on thin wires to generate the same fields are also calculated. Finally, the same procedure is followed to calculate the equation of motion of a charged particle in a two-dimensional array of charged lines and the trapping conditions are explained.

In principle, any electrostatic potential can be obtained by solving Poisson's equation. In practice, charge distributions are usually too complicated and their associated Poisson equations cannot be solved analytically. However, for a few cases, the electrostatic field can be solved analytically through Gauss's law which can be expressed as

$$\int \vec{E} \cdot dS = \frac{q}{\epsilon_0} \tag{1}$$

where $\vec{E}$ is the electrostatic field, q is the enclosed charge, ϵ_0 is the permittivity of free space, and S is a surface which

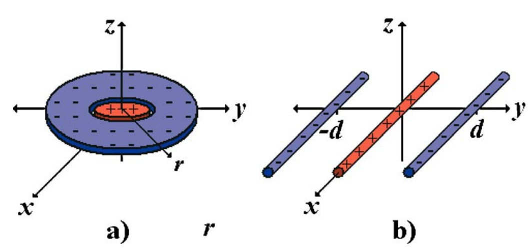

FIG. 1. (Color online) Two planar geometries for ion traps.

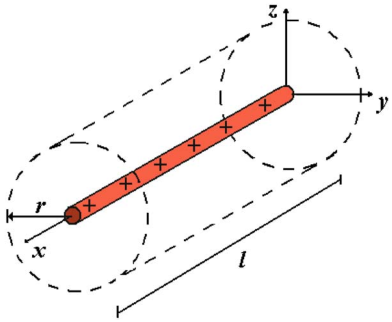

FIG. 2. (Color online) Geometry of the line of charges.

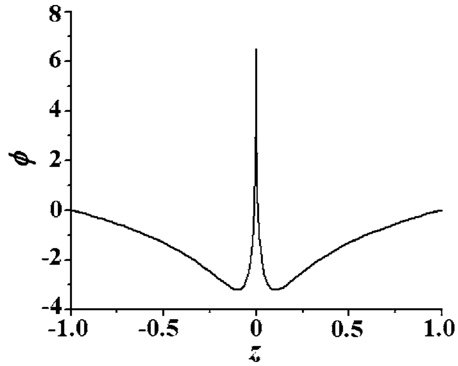

FIG. 3. Electrostatic potential along the z axis, at $y=0$ mm, $d=0.1$ mm, and $R=1$ mm.

surrounds the charge. For example, Eq. (1) can be easily used to find the electrostatic potential of an infinite line of charges. Using a cylindrical surface surrounding a line of charges parallel to x (Fig. 2), the solution of this problem is

$$E = \frac{\sigma}{2\pi\epsilon_0 r} \tag{2}$$

where σ is the linear charge density [3]. From Eq. (2), the electric potential can be easily calculated by direct integration, giving in Cartesian coordinates

$$\phi(y,z) = \frac{\sigma}{4\pi\epsilon_0} \ln \frac{R^2}{(y-d)^2 + (z-z_0)^2} \tag{3}$$

which is the well-known potential for an isolated line charge at $(y=d, z=z_0)$, where R is an arbitrary distance at which the potential is set to zero, $R \gg d$ [3]. Following this result, by means of the superposition principle, the potential for a set of three equally spaced charged lines placed along x, in $z=0$, with line charges $-\sigma$, σ, and $-\sigma$, centered at $y=0$ and each separated by a distance d [see Fig. 1(b)], is

$$\phi = -\frac{\sigma}{4\pi\epsilon_0}\left(\ln \frac{R^2}{(y+d)^2 + z^2} - \ln \frac{R^2}{y^2 + z^2} + \ln \frac{R^2}{(y-d)^2 + z^2}\right). \tag{4}$$

This arrangement is able to generate a trapping potential along z for positively charged ions; if $\sigma < 0$ the trapping potential would be suitable for negatively charged ions. The form of the axial normalized potential, for $d=0.1$ mm and $R=1$ mm, is presented in Fig. 3. Although this potential is not accurately quadratic, as in the Penning trap, such a potential can be approximated by a Taylor expansion to second order around the stationary points $\xi(y=0, z=\pm d)$, i.e.,

$$\phi(y,z) = \phi(\xi) + \left.\frac{\partial^2 \phi}{\partial y^2}\right|_{\xi} y^2 + \left.\frac{\partial^2 \phi}{\partial z^2}\right|_{\xi} (z \mp d)^2.$$

The fact that two real minima are found $(y=0, z=\pm d)$ implies that charged particles can be trapped above and below the lines of charge. The second order Taylor expansion around the stationary points becomes

$$\phi(y,z) \approx \frac{\sigma}{2\pi\epsilon_0}\left(\ln \frac{2d}{R} - \frac{y^2}{d^2} + \frac{(z \mp d)^2}{d^2}\right). \tag{5}$$

Using this potential, the components of the electrostatic field are

$$E_x = -\frac{\partial \phi(y,z)}{\partial x} = 0, \tag{6}$$

$$E_y = -\frac{\partial \phi(y,z)}{\partial y} = \frac{\sigma}{\pi\epsilon_0 d^2} y, \tag{7}$$

$$E_z = -\frac{\partial \phi(y,z)}{\partial z} = -\frac{\sigma}{\pi\epsilon_0 d^2}(z \mp d), \tag{8}$$

which are the formulas needed to obtain the equation of motion for a charged particle around the potential minimum along z. If a magnetic field (B) along z is added (as in the Penning trap), the equation of motion is found from the Lorentz force equation. Under the conditions described above, this equation can be easily solved for the component along z because this motion is independent of the magnetic field: a similar deduction can be found in [1] for the Penning trap. Then, the equation of motion in z is

$$\frac{d^2 z'}{dt^2} = -\frac{q\sigma}{\pi m \epsilon_0 d^2} z' \tag{9}$$

where $z' = z \mp d$, q is the charge of the particle, and m is the mass of the particle. The axial frequency associated with this harmonic motion is $\omega_z = \sqrt{q\sigma/\pi m \epsilon_0 d^2}$.

On the other hand, the movement in the xy plane is more complicated because the equations of motion for these dimensions are coupled. This is

$$\ddot{x} = \omega_c \dot{y}, \tag{10}$$

$$\ddot{y} = -\omega_c \dot{x} + \omega_z^2 y, \tag{11}$$

where the cyclotron frequency is defined by $\omega_c = qB/m$. For the arrangement described above, the equations of motion are not symmetric as they are in the Penning trap, but they still have an analytical solution. The solution for this system is given by the expressions

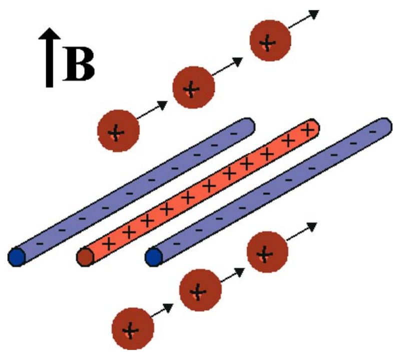

FIG. 4. (Color online) Schematic view of the *ion guide*.

$$x(t) = c_1 + c_2 t + c_3 \sin \omega_1 t + c_4 \cos \omega_1 t,$$

$$y(t) = c_2 \frac{\omega_c}{\omega_z^2} + \frac{\omega_1}{\omega_c}(c_3 \cos \omega_1 t - c_4 \sin \omega_1 t),$$

where $\omega_1 = \sqrt{\omega_c^2 - \omega_z^2}$. These equations represent a circular motion plus a linear drift along x. The oscillating solutions require the condition $\omega_c > \omega_z$. From these results it is easy to see that the potential produced by the three lines is not able to trap in three dimensions. The electrostatic potential traps axially and the magnetic field confines in one other dimension; the free dimension is the one along the charged lines. Consequently, this can be used as an ion guide because the ion would follow the "path" of the lines (see Fig. 4).

In fact, the lines can have different charge density and by changing this, the position of the potential minimum along z can be modified. The new potential can be written as

$$\phi = -\frac{1}{4\pi\epsilon_0}\left(\sigma_- \ln \frac{R^2}{(y+d)^2 + z^2} - \sigma_+ \ln \frac{R^2}{y^2 + z^2} + \sigma_- \ln \frac{R^2}{(y-d)^2 + z^2}\right)$$

where $-\sigma_-$ is the linear charge density of the outer lines and σ_+ is the charge density of the central line. Then, the expression for the position of the potential minima is

$$z_{min} = \pm \sqrt{\frac{\gamma}{2-\gamma}} d \tag{12}$$

where γ is the ratio of charge densities between the lines ($\gamma = \sigma_+/\sigma_-$). The variation of the position is presented graphically in Fig. 5. The position of the minimum can be adjusted with this method until the total charge is zero ($\gamma = 2$); after this point the charged particle cannot be trapped (this potential corresponds to a linear charge quadrupole).

So far, the motion of the trapped ions has been solved assuming lines of charges. This would, however, be very hard to realize in practice. Fortunately, thin conductors connected to some potential are a good approximation for a line of charges and are easy to implement in the laboratory. A procedure to find the equivalence between linear charge densities and voltages, when the lines have the same magnitude of charge density, is through the following equations. The voltage at the surface of the outer wires can be approximated as

$$V_- = -\frac{\sigma}{2\pi\epsilon_0}\left(\ln \frac{R}{a} - \ln \frac{R}{d} + \ln \frac{R}{2d}\right) = -\frac{\sigma}{2\pi\epsilon_0} \ln \frac{R}{2a} \tag{13}$$

where a is the radius of the wire, as long as $a \leqslant d$. On the other hand, the voltage at the surface of the middle wire is

$$V_+ = -\frac{\sigma}{2\pi\epsilon_0}\left(\ln \frac{R}{d} - \ln \frac{R}{a} + \ln \frac{R}{d}\right) = -\frac{\sigma}{2\pi\epsilon_0} \ln \frac{Ra}{d^2}. \tag{14}$$

Using these equations the following relationship is obtained:

$$\frac{V_-}{V_+} = \frac{\ln(R/2a)}{\ln(Ra/d^2)}. \tag{15}$$

Therefore, if these voltages are applied to the three wires, the potential generated is equivalent to that created by the three lines of charges. To illustrate this, the equipotential lines generated by a charge density ($|\sigma| = 1.16 \times 10^{-10}$ C/m) along the lines and the simulated equipotential lines generated by finite wires connected to the equivalent voltages ($V_+ = 2.00$ V and $V_- = -9.28$ V) are shown in Fig. 6 for $R = 50$ mm, $a = 1$ mm, and $d = 10$ mm.

The immediate application of the ion guide presented here is in the transport of charged particles; however, an ion trap can also be made using this configuration. If the guiding wires are given a slight curvature, a closed track would then be able to produce the extra confinement required to trap the ions. If the radius of curvature (r_c) of the lines is much greater than d, the equations presented above would be a good approximation. The angular velocity around the centre of the ring would come from the initial tangential velocity of the ion (related to the linear drift term presented above).

Using wires connected to fixed voltages and with a grounded plate at some large distance, simulations were carried out confirming trapped ion trajectories. In Fig. 7 a simulated flight of an ion with $m = 60$ amu, $d = 10$ mm, $a = 1$ mm,

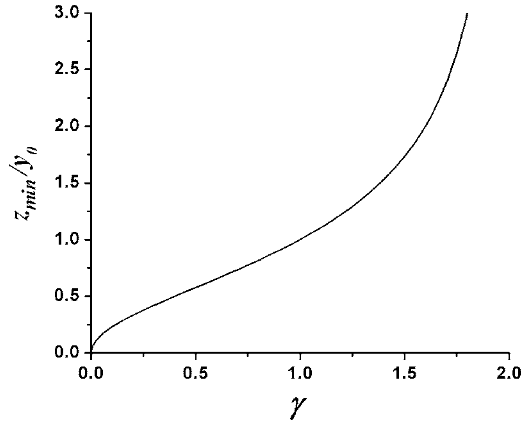

FIG. 5. Relationship between the relative position of the minima along z and the ratio of the relative charge in the wires.

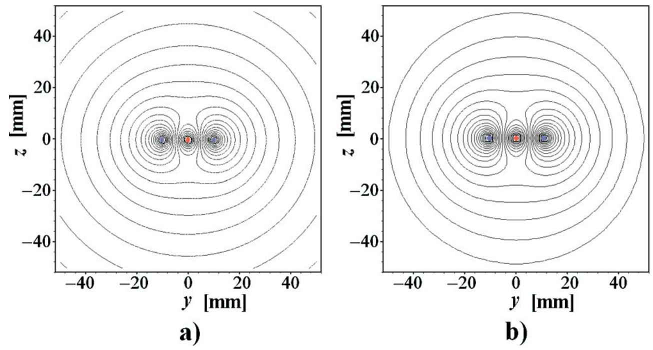

FIG. 6. (Color online) Equipotential lines for (a) three lines of charge and (b) three conductors at fixed voltages (simulations using SIMION). The potential lines are evaluated at the same voltages in both cases. In the case of (b), the equipotential lines are limited by the position of the ground at $R=50$ mm.

$r_c=70$ mm, $B=0.50$ T, initial kinetic energy of 1 eV, $V_-=-9.28$ V, and $V_+=2.00$ V (equivalent to $|\sigma|=1.16\times10^{-10}$ C/m) is presented. These values were chosen to show a clear motion in the simulated trap.

III. THE TRAP

The natural next step to generate a three-dimensional trapping potential is to try to symmetrize Eq. (4). This is done by adding to the existing two-dimensional potential (in the plane yz) the potential generated in a perpendicular plane (xz) by another set of charged lines parallel to the y axis. The geometry presented in Fig. 8 produces such an effect. The potential generated by this geometry can be solved by the superposition of two perpendicular potentials of the form discussed above. Consequently, the full three-dimensional potential is

$$\phi(x,y,z) = -\frac{\sigma}{4\pi\epsilon_0}\left(\ln\frac{R^2}{(x+d)^2+(z+z_0)^2} - \ln\frac{R^2}{x^2+(z+z_0)^2} + \ln\frac{R^2}{(x-d)^2+(z+z_0)^2}\right) - \frac{\sigma}{4\pi\epsilon_0} \times\left(\ln\frac{R^2}{(y+d)^2+(z-z_0)^2} - \ln\frac{R^2}{y^2+(z-z_0)^2} + \ln\frac{R^2}{(y-d)^2+(z-z_0)^2}\right) \quad (16)$$

which represents the potential due to two perpendicular sets of three charged lines, separated by $2z_0$. The separation between charged lines in a single set is d. Now, following the same procedure used before, it is possible to find the stationary points along the z direction. These points are at $z=0$ and

$$z_{min} = \pm\sqrt{z_0^2 \pm \sqrt{4z_0^2 d^2 + d^4}}. \quad (17)$$

On the other hand, if the lines have different relative charge, the positions of the minima are given by

$$z_{min} = \pm\frac{\sqrt{d_0^2(\gamma-1) + z_0^2(2-\gamma) \pm \sqrt{4\gamma(2-\gamma)z_0^2 d^2 + d^4}}}{2-\gamma}, \quad (18)$$

where γ is the charge ratio between the lines. The main point is that in both cases (same charge or not), four minimum points are found. The first point is above the wires, the second one is below the wires, and there are two more points situated symmetrically between the wires [see Fig. 9(b)]. Therefore, in principle, this trap would be able to trap ions over, below, and between the lines. The latter points are valid only when Eq. (17) has real values. This gives the condition

$$(z_0/d)^4 \geqslant 1 + 4(z_0/d)^2 \quad (19)$$

which implies that $(z_0/d)^2 > 2+\sqrt{5}$. On the other hand, Eq. (16) has another stationary point at $z=0$. The concavity of this point changes with the relative values of d and z_0 and is related to the sign of the second derivative of the potential evaluated at $z=0$. If this value is positive, the potential around this point will be concave and it will be able to trap positive ions [Fig. 9(a)]. In contrast, if the value is negative,

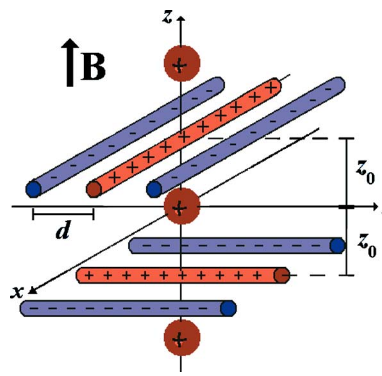

FIG. 8. (Color online) Schematic view of the planar trap; the scale is not preserved.

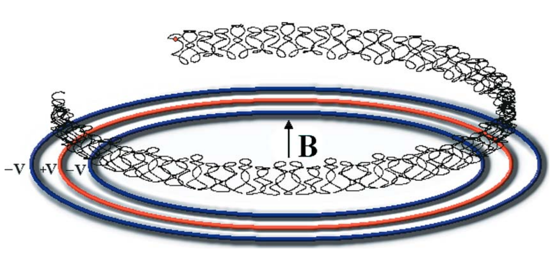

FIG. 7. (Color online) Simulation of the motion of an ion above the ion guide, $B=0.5$ T, $V_-=-9.28$ V, and $V_+=2.00$ V.

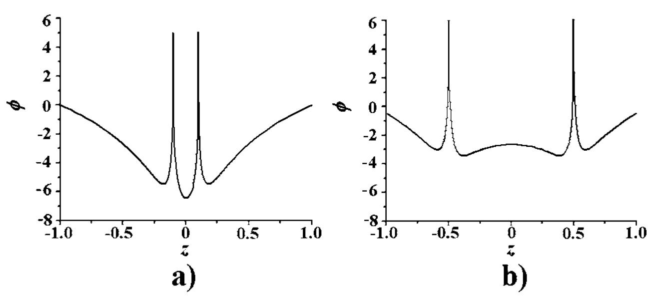

FIG. 9. Potential along the z direction. $z_0/d=$(a) 1 and (b) 5.

the potential between the wires will be convex and it will be able to trap negative ions [Fig. 9(b)]. The condition to store positive ions at this point is then

$$\left.\frac{\partial^2\phi}{\partial z^2}\right|_{0,0,0} = \frac{\sigma}{4\pi\epsilon_0}\left(-16\frac{z_0^2}{(d^2+z_0^2)^2} + \frac{8}{d^2+z_0^2} + \frac{4}{z_0^2}\right) > 0 \tag{20}$$

which implies that $(z_0/d)^2 < 2+\sqrt{5}$. In other words, depending on the geometry, Eqs. (19) and (20), the potential has either one or two trapping regions for positive ions between the sets of lines. Continuing with the solution for the motion of the trapped ions and following the same procedure as before, the potential around the minimum $\xi=(x=0,y=0,z=z_{min})$ is approximately

$$\phi(x,y,z) = \phi(\xi) + \left.\frac{\partial^2\phi}{\partial x^2}\right|_{\xi} x^2 + \left.\frac{\partial^2\phi}{\partial y^2}\right|_{\xi} y^2 + \left.\frac{\partial^2\phi}{\partial z^2}\right|_{\xi} (z-z_{min})^2$$

where

$$\left.\frac{\partial^2\phi}{\partial x^2}\right|_{\xi} = \frac{\sigma}{2\pi\epsilon_0}\left(\frac{2}{d^2+(z_{\min}+z_0)^2} - \frac{1}{(z_{min}+z_0)^2} - \frac{4d^2}{[d^2+(z_{min}+z_0)^2]^2}\right), \tag{21}$$

$$\left.\frac{\partial^2\phi}{\partial y^2}\right|_{\xi} = \frac{\sigma}{2\pi\epsilon_0}\left(\frac{2}{d^2+(z_{min}-z_0)^2} - \frac{1}{(z_{min}-z_0)^2} - \frac{4d^2}{[d^2+(z_{min}-z_0)^2]^2}\right), \tag{22}$$

$$\left.\frac{\partial^2\phi}{\partial z^2}\right|_{\xi} = \frac{\sigma}{2\pi\epsilon_0}\left(\frac{2}{d^2+(z_{min}+z_0)^2} + \frac{1}{(z_{min}+z_0)^2} - \frac{4(z_{min}+z_0)^2}{[d^2+(z_{min}+z_0)^2]^2}\right) + \frac{\sigma}{2\pi\epsilon_0}\left(\frac{2}{d^2+(z_{min}-z_0)^2} + \frac{1}{(z_{min}-z_0)^2} - \frac{4(z_{min}-z_0)^2}{[d^2+(z_{min}-z_0)^2]^2}\right) \tag{23}$$

where the following relation is preserved (as expected from Maxwell's equations)

$$\left.\frac{\partial^2\phi}{\partial x^2}\right|_{\xi} + \left.\frac{\partial^2\phi}{\partial y^2}\right|_{\xi} + \left.\frac{\partial^2\phi}{\partial z^2}\right|_{\xi} = 0. \tag{24}$$

Then, the final equations of motion for trapped ions, assuming the existence of a magnetic field parallel to the z axis, are

$$\ddot{x} = \omega_c\dot{y} + \omega_x^2 x, \tag{25}$$

$$\ddot{y} = -\omega_c\dot{x} + \omega_y^2 y, \tag{26}$$

$$\ddot{z}' = -\omega_z^2 z', \tag{27}$$

where

$$\omega_x^2 = -\frac{2q}{m}\left.\frac{\partial^2\phi}{\partial x^2}\right|_{\xi}, \quad \omega_y^2 = -\frac{2q}{m}\left.\frac{\partial^2\phi}{\partial y^2}\right|_{\xi}, \quad \omega_z^2 = \frac{2q}{m}\left.\frac{\partial^2\phi}{\partial z^2}\right|_{\xi},$$

and $z'=z-z_{min}$. Again, the equation of the axial motion is uncoupled and easily recognized as the harmonic oscillator equation. For a stable trap we require ω_x^2, ω_y^2, and ω_z^2 to be positive.

In the case of the motion around ($z_{min}=0$), Eqs. (25) and (26) can be reduced because

$$\omega_x^2 = \omega_y^2 = \frac{\omega_z^2}{2} = \frac{q\sigma}{m\pi\epsilon_0}\left(\frac{2}{d^2+z_0^2} + \frac{1}{z_0^2} - \frac{4z_0^2}{(d^2+z_0^2)^2}\right). \tag{28}$$

Using these relations, the equations of motion are

$$\ddot{x} = \omega_c\dot{y} + \frac{\omega_z^2}{2}x, \tag{29}$$

$$\ddot{y} = -\omega_c\dot{x} + \frac{\omega_z^2}{2}y, \tag{30}$$

$$\ddot{z} = -\omega_z^2 z, \tag{31}$$

where the motion in the xy plane is symmetric and its solution is the same as in the Penning trap [1]. However, ions can also be trapped in the other minima where $\omega_x \neq \omega_y$. There,

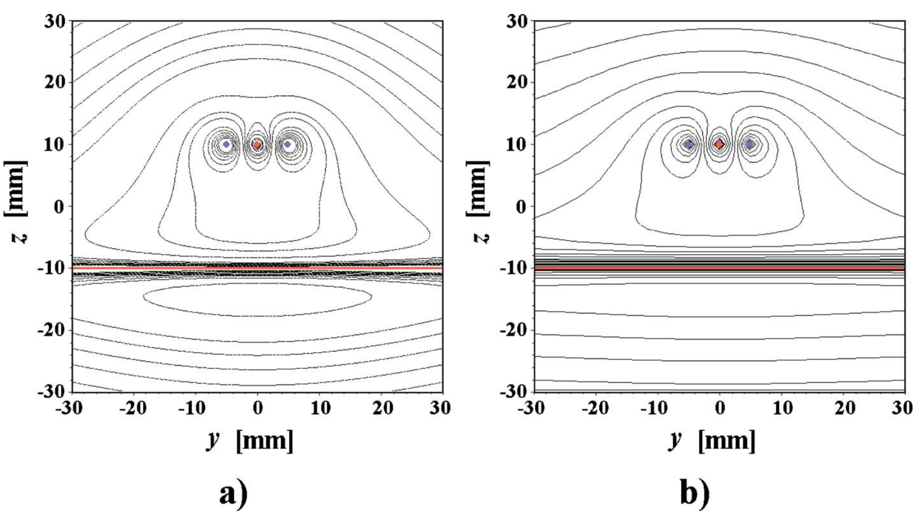

FIG. 10. (Color online) Equipotential lines for (a) two perpendicular sets of linear charges and (b) two perpendicular sets of three conductors at fixed voltages (simulations using SIMION).

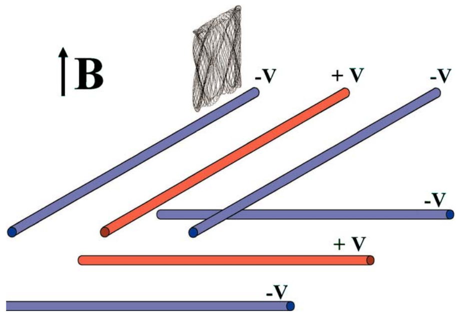

FIG. 11. (Color online) Simulation of the motion of a $^{40}Ca^{+}$ ion in the trap.

their movement in the plane xy can be understood as an elliptical motion [5]. A solution to Eqs. (29) and (30) can be proposed as

$$x = x_0 e^{-i\omega_{xy} t},$$

$$y = y_0 e^{-i(\omega_{xy} t + \pi/2)},$$

where ω_{xy} is the frequency associated with the plane xy. Substituting the last expressions in the equation of motion, the following relationships are obtained:

$$(\omega_{xy}^2 + \omega_x^2)x_0 - i\omega_{xy}\omega_c y_0 e^{-i\pi/2} = 0,$$

$$i\omega_{xy}\omega_c x_0 + (\omega_{xy}^2 + \omega_y^2)y_0 e^{-i\pi/2} = 0,$$

which has solutions

$$\omega_{xy}^2 = \frac{(\omega_c^2 - \omega_x^2 - \omega_y^2) \pm \sqrt{(\omega_c^2 - \omega_x^2 - \omega_y^2)^2 - 4(\omega_x^2\omega_y^2)}}{2} \tag{32}$$

and

$$\frac{y_0^2}{x_0^2} = \frac{\omega_{xy}^2 + \omega_x^2}{\omega_{xy}^2 + \omega_y^2}. \tag{33}$$

Finally, the full radial motion in the trap would consist of the linear superposition of the two solutions for ω_{xy}.

Although the latter equations are applied to lines of charges, the expression presented in Eq. (15) can be used to find the equivalent voltages on conducting wires. In Fig. 10 equipotential lines are presented for a trap with the following characteristics: $|\sigma| = 1.2\times 10^{-10}$ C/m (equivalent to $V_+ = 0.50$ V and $V_- = -8.26$), $R = 40$ mm, $a = 0.5$ mm, $d = 5$ mm, and $z_0 = 10$ mm. The main differences in such graphs are because the simulations are limited in size, which restricts the size of R. In addition, simulations were carried out using $^{40}Ca^{+}$ with the geometry $z_0 = 5$ mm, $a = 1$ mm, $d = 8$ mm, $R \approx 50$ mm, $B = 0.60$ T, and initial kinetic energy of 1 eV (typical experimental values [4]). Wires connected to $V_- = -13.04$ V and $V_+ = 1.00$ V were used (equivalent to $|\sigma| = 2.25\times 10^{-10}$ C/m). The motion of such an ion in the upper position of the trap is shown in Fig. 11. Here $\omega_{xy}^{+} = 856.95$ kHz and $y_0^{+}/x_0 = 0.88$, $\omega_{xy}^{-} = 9.83$ kHz and $y_0^{-}/x_0 = 0.05$ for the two motions.

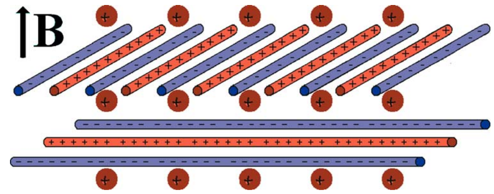

FIG. 12. (Color online) Schematic view of the planar multiple trap.

A. Scalability

One of the great advantages of this trap is its high scalability (see Fig. 12). The equation to solve, when $2n+1$ wires are used in each plane, to give n^2 traps (where n is an odd number), is

$$\begin{aligned}\phi(x,y,z) = {} & \frac{\sigma}{4\pi\epsilon_0}\sum_{i=1}^{n}(-1)^i\left(\ln\frac{R^2}{(x+id)^2+(z+z_0)^2}\right. \\ & \left. + \ln\frac{R^2}{(x-id)^2+(z+z_0)^2}\right) + \frac{\sigma}{4\pi\epsilon_0}\ln\frac{R^2}{x^2+(z+z_0)^2} \\ & + \frac{\sigma}{4\pi\epsilon_0}\sum_{i=1}^{n}(-1)^i\left(\ln\frac{R^2}{(y+id)^2+(z-z_0)^2}\right. \\ & \left. + \ln\frac{R^2}{(y-id)^2+(z-z_0)^2}\right) + \frac{\sigma}{4\pi\epsilon_0}\ln\frac{R^2}{y^2+(z-z_0)^2}.\end{aligned} \tag{34}$$

Following the same procedure as before, the following relationships are found for the potential minima:

$$x_{min} \approx \pm 2m_x d, \tag{35}$$

$$y_{min} \approx \pm 2m_y d, \tag{36}$$

where m is a finite integer and $-n/2 < m_x$, $m_y < -n/2$. The relationship to find all the minima in z is too complicated to be shown here; instead of this, the relationship at $m=0$ (at the center) is presented below.

$$\left.\frac{\partial\phi}{\partial z}\right|_{0,0,0} = \sum_{i=1}^{n}(-1)^{i+1}\left(\frac{z_{min}+z_0}{(id)^2+(z_{min}+z_0)^2} + \frac{z_{min}-z_0}{(id)^2+(z_{min}-z_0)^2}\right) - \frac{z_{min}}{z_{min}^2 - z_0^2} = 0 \tag{37}$$

where the minimum at $z_{min}=0$ is obvious. This equation is an alternating series which converges because of the Leibniz criterion, $a_i > a_{i+1}$ and $\lim_{i\to\infty}(a_i)=0$ where a_i is the factor inside the sum $[\Sigma(-1)^{i+1}a_i]$. The convergence of this series indicates that eventually the minimum in z is mainly influ-

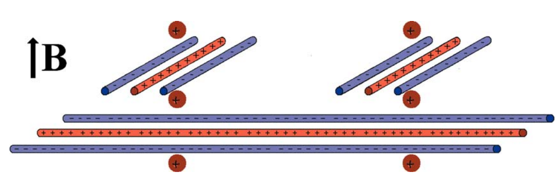

FIG. 13. (Color online) Alternative scalable geometry.

enced by the nearest wires. As before, voltages can be found for each line charge to give the same potential distribution as the analytical calculations.

There are other modifications that can be made to the proposals in this paper, for example, the addition of extra wires to decrease the higher order terms $O(x^4, y^4, z^4, \ldots)$ in the Taylor expansion (improving the quadratic behavior of the potential). Another interesting case is when the two sets of wires do not have the same relative values of voltages, when it can be shown that even in the external trapping zones a circular motion can be found (as in the Penning trap). Finally, another scalable design where the interaction between the traps can be neglected is shown in Fig. 13.

IV. CONCLUSIONS

A different design for a planar trap has been presented. The theory behind its function has been developed. Different points of trapping have been demonstrated; two of them share the open geometry presented in a previous work [2]. The great advantage of this trap is that it can be easily constructed and its three-dimensional potential is analytical. Simulations have been carried out demonstrating the validity of the model.

ACKNOWLEDGMENTS

This project was supported by the European Commission within the RTD program FP5 (Grant No. HPRI-CT-2001-50036 HITRAP). J.R.C.P. acknowledges support by CONACyT, SEP, and the ORS. We also acknowledge support from the EPSRC.

[1] P. K. Ghosh, *Ion Traps* (Oxford University Press, Oxford, 1995).
[2] S. Stahl *et al.*, Eur. Phys. J. D **32**, 139 (2005).
[3] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1998).
[4] H. F. Powell, D. M. Segal, and R. C. Thompson, Phys. Rev. Lett. **89**, 093003 (2002).
[5] G. Z. K. Horvath, Ph.D. thesis, Blackett Laboratory, Imperial College, London, 1995 (unpublished).