Coherent destruction of tunneling in waveguide directional couplers

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It is theoretically shown that the phenomenon of coherent destruction of tunneling in bistable quantum systems induced by an external driving field can be realized in a periodically curved optical waveguide coupler, the periodic bending of the coupler simulating the effect of the external field in the quantum system.

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The issue of quantum tunneling control in a bistable (double-well) potential by the application of an external driving field has attracted considerable interest in the past few years in view of its potential applications in different physical, chemical, and biological systems (see, e.g., [1–4] and references therein). A paradigmatic and widely adopted model for quantum tunneling is the symmetric quartic double-well potential [5–8], which has allowed one to reach a deep understanding of both classical and quantum aspects involved in the tunneling dynamics and control. Depending on the amplitude and frequency of the driving field, either strong enhancement of the tunneling rate [5,6] or the coherent destruction of tunneling (CDT) [7] can be achieved. For a sinusoidally driven system, CDT generally occurs at frequencies of the external field comparable with—or larger than—the frequency splitting of the two lowest states of the double-well system, and the driving amplitude should be adjusted such that the periodically driven system admits of two degenerate Floquet states. In this case, a localized wave packet indefinitely trapped in one well can be built as a superposition of the two degenerate Floquet states. Enhancement of tunneling rate is instead obtained at driving frequencies close to the harmonic frequency at the bottom of each well, and it is usually referred to as chaos-assisted tunneling since it involves transition through an intermediate state which is chaotic for strong enough driving amplitudes [1]. Tunneling suppression can be observed as well in quantum systems with a periodic potential [1], where the phenomenon of dynamic localization [9] resembles that of CDT in a bistable potential [10]. An experimental observation of strong tunneling enhancement in a double-well system subjected to weak periodic perturbations has been recently reported using two optical waveguides [11], whereas waveguide coupling cancellation in a periodic array of waveguides with a zigzag geometry has been demonstrated in Ref. [12]. However, a clear correspondence between periodic perturbation of waveguide geometry and external driving field in the related quantum mechanical problems was not provided in these previous works.

In this Brief Report we show analytically and numerically that the tunneling process in a waveguide directional coupler [13] with a sinusoidally bent axis is fully analogous to the quantum tunneling problem in a bistable (double-well potential) system subjected to an external driving field. Though an optical realization of quantum tunneling enhancement using a directional waveguide coupler was previously reported in Ref. [11], the issue of tunneling control and CDT, achieved in quantum systems by an external driving field, was not addressed for that system. It is precisely the aim of this work to show that CDT can be realized in a sinusoidally curved waveguide coupler, the effect of periodic axis bending of the coupler playing precisely the role of the external driving field in the quantum mechanical system. Design parameters are also provided for an experimental observation of CDT in an optical waveguide coupler.

The starting point of our analysis is provided by a standard model of beam propagation at wavelength $\lambda=2\pi/k$ in an optical directional coupler, made of two identical waveguides separated by a distance a in the x direction, lying in the (x,z) plane. The axis of the coupler is assumed to be periodically bent along the propagation direction z, with a bending profile $x_0(z)$ which is assumed to vary slowly over a distance of the order of the waveguide spacing a [see Fig. 1(a)]. We further assume that the field is strongly localized in the vertical y direction by a planar waveguiding structure, so that beam dynamics can be effectively reduced to a two-dimensional problem (see, for instance, [14]). For a weak refractive index change of the waveguide channels from the substrate refractive index n_s, the electric field can be written as $E(x,z,t)=\psi(x,z)\exp(ikn_sz-i\omega t)+\text{c.c.}$, where $\omega=kc_0$ and the envelope $\psi(x,z)$ satisfies the scalar and paraxial wave equation (see, e.g., [11,15,16]):

$$i\lambda\!\!\!^{-}\frac{\partial\psi}{\partial z}=-\frac{\lambda\!\!\!^{-2}}{2n_s}\frac{\partial^2\psi}{\partial x^2}+V[x-x_0(z)]\psi. \qquad (1)$$

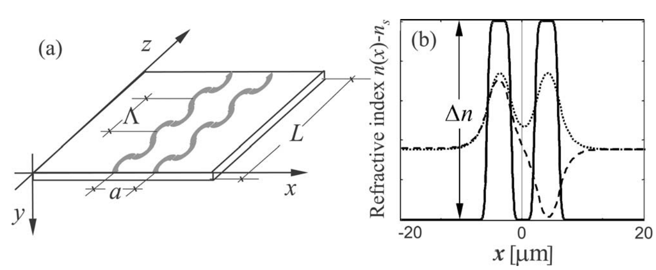

FIG. 1. (a) Schematic of a waveguide coupler with a periodically bent axis. (b) Refractive index profile $n(x)-n_s$ of the waveguide directional coupler used in the numerical simulations (solid curve), and behavior of odd (dashed curve) and even (dotted curve) supermodes $\xi_{1,2}(x)$ supported by the coupler. Parameter values are $n_s=2.138$, $\lambda=1.55$ μm, $w=2$ μm, $D_x=0.5$ μm, $\Delta n=0.01$, and $a=8$ μm.

In Eq. (1) we have set $\lambda\!\!\!^{-} \equiv \lambda/(2\pi) = 1/k$ and $V(x) \equiv [n_s^2 - n^2(x)]/(2n_s) \simeq n_s - n(x)$, where $n(x)$ is the refractive index profile of the coupler. Since we assume that the directional coupler is made of two identical waveguides, $V(x)$ turns out to be a symmetric function of x, i.e., $V(-x) = V(x)$, with a typical two-well shape [see e.g., Fig. 1(b)]. We want now to show that the periodic modulation of the waveguide axis $x_0(z)$ is fully equivalent, in the quantum mechanical analogy, to the application of an external ac driving field. To this aim, let us introduce the new variables:

$$x' = x - x_0(z), \quad z' = z, \tag{2}$$

and the phase transformation for the envelope:

$$\phi(x',z') = \psi(x',z')\exp\left[-i\frac{n_s}{\lambda\!\!\!^{-}}\dot{x}_0 x' - i\frac{n_s}{2\lambda\!\!\!^{-}}\int_0^{z'} d\xi \dot{x}_0^2(\xi)\right], \tag{3}$$

where the dot indicates the derivative with respect to z'. Substitution of Eqs. (2) and (3) into Eq. (1) yields the following wave equation for the envelope $\phi(x',z')$:

$$i\lambda\!\!\!^{-}\frac{\partial\phi}{\partial z'} = -\frac{\lambda\!\!\!^{-2}}{2n_s}\frac{\partial^2\phi}{\partial x'^2} + V(x')\phi + S(z')x'\phi, \tag{4}$$

where we have set

$$S(z') \equiv n_s \ddot{x}_0(z'). \tag{5}$$

It is worth observing that, after the formal substitution $z' \to t$, $n_s \to m$, and $\lambda \to h$, Eq. (4) describes the one-dimensional quantum dynamics of a particle of mass m, in the bistable double-well potential $V(x')$, subjected to an external and periodic driving field $S(z')$ [1,5,7], which is related to the waveguide axis bending through Eq. (5). To capture the beam dynamics along the waveguide coupler and to determine the conditions for CDT, let us assume that each waveguide of the coupler be single-mode, so that a set of two coupled-mode equations can be derived using a standard mode projection technique (see, for instance, [13]). In the quantum mechanical analogy, coupled-mode equation analysis corresponds to the two-level model of quantum tunneling [1,17]. Let us indicate by $\xi_1(x')$ and $\xi_2(x')$ the two supermodes of the straight waveguide coupler [13], with nearly degenerate propagation constants β_1 and β_2, i.e., $\mathcal{H}\xi_{1,2} = \beta_{1,2}\xi_{1,2}$ with $\mathcal{H} \equiv -\lambda\!\!\!^{-2}/(2n_s)(\partial^2/\partial x'^2) + V(x')$; the supermodes are normalized such that $\langle\xi_1|\xi_1\rangle = \langle\xi_2|\xi_2\rangle = 1$, $\langle\xi_1|\xi_2\rangle = 0$, where the bracket denotes the scalar product. Note that, owing to the symmetry of $V(x')$, $\xi_{1,2}(x')$ are real-valued and have opposite parity [see Fig. 1(b)]. The combinations of waveguide supermodes $u_{1,2}(x') = [\xi_1(x') \pm \xi_2(x')]/\sqrt{2}$ correspond to beam localization in each one of the two waveguides of the coupler, and the shift $\Delta\beta \equiv \beta_1 - \beta_2$ of propagation constants is responsible for optical tunneling of the straight waveguide coupler [13]. To study beam dynamics in the periodically curved waveguide coupler within a coupled-mode equation analysis, we neglect excitation of radiation modes, so that $\psi(x',z')$ can be taken as a superposition of supermodes ξ_1 and ξ_2, with coefficients which depend on z', or, equivalently, of their linear combinations $u_1(x')$ and $u_2(x')$. After setting $\psi(x',z') = [c_1(z')u_1(x') + c_2(z')u_2(x')]\exp(-i\beta_{av}z')$, where $\beta_{av} \equiv (\beta_1 + \beta_2)/2$, using the orthonormal properties $\langle\xi_i|\xi_k\rangle = \delta_{i,k}$ and the property $\langle\xi_1|x'|\xi_1\rangle = \langle\xi_2|x'|\xi_2\rangle = 0$, Eq. (4) yields

$$i\dot{c}_1 = \frac{\Delta\beta}{2}c_2 + \kappa S(z')c_1, \tag{6}$$

$$i\dot{c}_2 = \frac{\Delta\beta}{2}c_1 - \kappa S(z')c_2, \tag{7}$$

where we have set $\kappa \equiv \langle\xi_1|x'|\xi_2\rangle/\lambda\!\!\!^{-}$. Note that if a is the distance between the two waveguides of the coupler [Fig. 1(a)] and the two wells in the potential $V(x')$ are well separated, one can approximately set $\langle\xi_1|x'|\xi_2\rangle \simeq a/2$, i.e., $\kappa \simeq a/(2\lambda\!\!\!^{-}) = a\pi/\lambda$. Equations (6) and (7) capture the basic dynamics underlying optical tunneling within a coupled-mode equation approach and are similar to the two-level model of quantum tunneling [17]. For a straight coupler ($S=0$), a periodic change of power between the two channels occurs over a spatial periodicity $\pi/\Delta\beta$, which is a well-known result [13]. For a periodically bent waveguide axis, Eqs. (6) and (7) are nonauthonomous and a Floquet analysis should be performed. In order to achieve CDT, let us assume that the modulation period Λ of the waveguide axis bending is sufficiently smaller than the beating length $\pi/\Delta\beta$ of the straight coupler. CDT occurs whenever the Floquet exponents of Eqs. (6) and (7) become degenerate, so that coupling between channels u_1 and u_2 is inhibited. In the limit $\epsilon \equiv \Lambda\Delta\beta \ll 1$, an approximate solution to Eqs. (6) and (7) can be searched by using a multiple scale asymptotic technique. At leading order in ϵ, one can write $c_1(z') = a_1(z')\exp[-i\kappa\int_0^{z'} S(y)dy]$ and $c_2(z') = a_2(z')\exp[i\kappa\int_0^{z'} S(y)dy]$, where the amplitudes $a_{1,2}(z')$ vary slowly over one period Λ according to the coupled mode equations:

$$i\dot{a}_1 = \frac{\Delta\beta_{eff}}{2}a_2, \tag{8}$$

$$i\dot{a}_2 = \frac{\Delta\beta_{eff}^*}{2}a_1, \tag{9}$$

where we have set

$$\Delta\beta_{eff} \equiv \Delta\beta\frac{1}{\Lambda}\int_0^{\Lambda} dz' \exp\left[2i\kappa\int_0^{z'} S(y)dy\right]. \tag{10}$$

CDT is thus obtained whenever $\Delta\beta_{eff} = 0$, which yields [using Eqs. (5) and (10)]:

$$\int_0^{\Lambda} dz' \exp[2i\kappa n_s \dot{x}_0(z')] = 0. \tag{11}$$

A particularly simple and relevant case, which corresponds to CDT by a sinusoidal driving field [7], is that of a coupler with a sinusoidally bent axis, i.e., $x_0(z) = A\sin(2\pi z/\Lambda)$, where A is the modulation depth of axis bending. In this case, from Eqs. (5) and (10) one obtains:

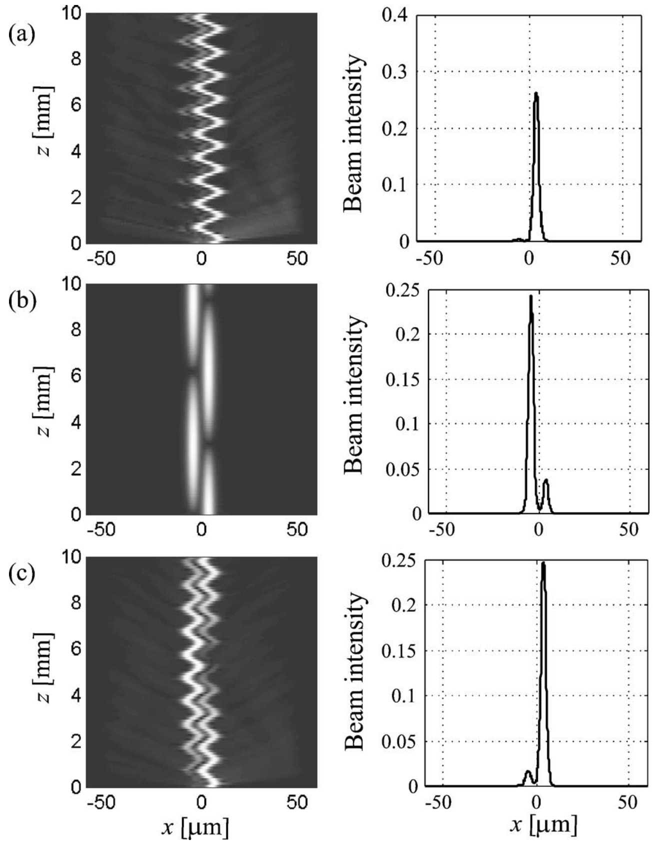

FIG. 2. Snapshots of beam amplitude evolution $|\psi|$ in a $L=1$ cm-long sinusoidally curved waveguide coupler (left pictures) and corresponding output beam intensity $|\psi|^2$ (right pictures) for a few values of modulation depth A. In (a) $A=5.51$ μm, corresponding to the condition for CDT; in (b) $A=0$, corresponding to the straight coupler, and in (c) $A=3$ μm. The period of sinusoidal axis bending is $\Lambda=1$ mm. The other parameter values are the same as in Fig. 1(b).

$$\Delta\beta_{eff}=\Delta\beta J_0\left(\frac{4\pi\kappa n_s A}{\Lambda}\right), \tag{12}$$

and the condition (11) for CDT yields:

$$J_0\left(\frac{4\pi\kappa n_s A}{\Lambda}\right)=0. \tag{13}$$

Note that Eq. (13) is analogous to the CDT condition derived in a two-level quantum system [17]. In particular, assuming $\kappa\simeq\pi a/\lambda$ and taking the first root of Bessel J_0 function, from Eq. (13) one can derive an approximate expression for the waveguide axis modulation depth A at which CDT occurs:

$$A\simeq 2.405\frac{\Lambda\lambda}{4\pi^2 n_s a}. \tag{14}$$

We checked the validity of the previous analysis and the occurrence of CDT in a periodically curved directional coupler, beyond the coupled-mode equation approximation, by a direct numerical integration of the scalar wave equation (1) using a pseudospectral split-step beam propagation technique. In order to account for radiation losses due to

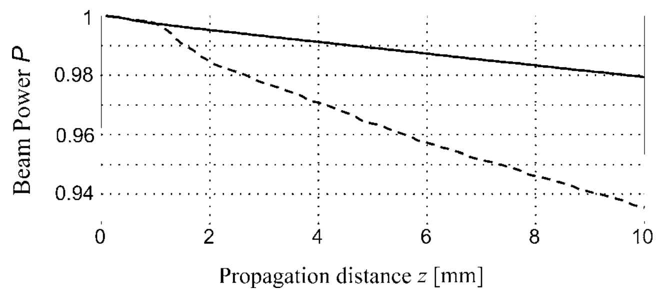

FIG. 3. Behavior of beam power $\mathcal{P}(z)=\int|\psi(x,z)|^2dx$ vs propagation distance z for the curved waveguide coupler in the CDT condition of Fig. 2(a) (dashed curve) and for the straight waveguide coupler of Fig. 2(b) (solid curve).

bending-induced coupling with radiation modes, Eq. (1) has been solved on a domain with a finite extension in the transverse x direction and assuming absorbing boundary conditions. The refractive index profile of each waveguide forming the coupler has been assumed to be given by $n_w(x)=n_s+\Delta n[\mathrm{erf}((x+w)/D_x)-\mathrm{erf}((x-w)/D_x)]/[2\,\mathrm{erf}(w/D_x)]$; such a profile typically describes a channel waveguide fabricated by diffusion processes [18], $2w$ and D_x being the channel width and the diffusion length, respectively. The index profile of the coupler used in our numerical simulations is shown in Fig. 1(b). The occurrence of CDT is clearly shown in Fig. 2, where beam propagation along the coupler is shown for a sinusoidally curved waveguide axis [Fig. 2(a)] and compared with the propagation in a straight waveguide coupler [Fig. 2(b)]. One of the two waveguides of the coupler was excited, at the input plane, in its fundamental mode, and propagation length L was chosen to be a few times longer than the coupling length for the straight coupler, which is given by $\pi/\Delta\beta\simeq 3$ mm. The modulation amplitude of the periodically curved coupler in Fig. 2(a) was chosen according to the approximate value given by Eq. (14) for CDT, as obtained by

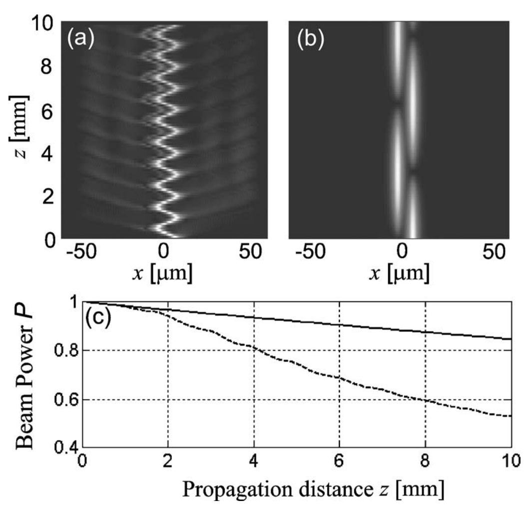

FIG. 4. (a) and (b): Same as Figs. 2(a) and 2(b) but for a waveguide coupler with $w=1$ μm and $a=8.4$ μm. In (a) the modulation depth, satisfying the CDT condition, is $A\simeq 5.45$ μm. (c) shows the behavior of the beam power $\mathcal{P}$ vs propagation distance corresponding to the straight coupler (solid curve) and to the sinusoidally curved coupler (dashed curve).

the coupled-mode equation analysis. For a modulation amplitude A such that $J_0(4\pi\kappa n_s A/\Lambda)\neq 0$, tunneling is not completely suppressed, however, the effective coupling length increases according to Eq. (12); this result is shown in Fig. 2(c). The total beam power $\mathcal{P}(z)=\int dx|\psi(x,z)|^2$ contained in the transverse integration domain as a function of propagation distance z, for both straight and curved waveguide couplers, is shown in Fig. 3. Note that in the CDT regime radiation losses–though maintaining at a relatively low level–are non-negligible owing to bending-induced coupling with radiation modes. Note also that radiation losses are absent in quartic bistable potentials [5,7], which support solely bound states, and cannot be accounted for by a simple two-mode analysis as given by Eqs. (6) and (7). For a double-well potential with finite barriers, such as that shown in Fig. 1(b), one expects radiation losses to sensitively depend on the waveguide coupler geometry. In particular, for a fixed value of the coupling length, i.e., of the splitting $\Delta\beta$, radiation losses are expected to increase as the bound modes of the double-well move toward the continuum, i.e., when the bound mode of each waveguide becomes less guided by the structure. This can be achieved, in practice, by reducing the channel width w and simultaneously increasing–though slightly–the distance a between the two waveguides to keep the same value of mode splitting $\Delta\beta$. As an example, in Fig. 4 it is shown the increase of radiation losses in a sinusoidally bent waveguide coupler, with the same coupling length as that shown in Fig. 2, obtained when the channel width is halved.

In conclusion, we have theoretically shown that the phenomenon of CDT predicted more than one decade ago for bistable quantum systems subjected to an external harmonic field [7] can be elegantly realized in a periodically curved optical waveguide coupler, the periodic bending of the coupler simulating the effect of the external field in quantum mechanical systems. The analytical predictions based on an extended coupled-mode equation analysis, similar to the two-level theory of CDT for quantum systems [17], have been confirmed by a direct numerical integration of the beam propagation wave equation. It is envisaged that the present analysis may stimulate an experimental observation of the phenomenon of CDT in a waveguide-based optical system.

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