PHYSICAL REVIEW A **71**, 064103 (2005)

"Superluminal" tunneling as a weak measurement effect

D. Sokolovski
School of Mathematics and Physics, Queen's University of Belfast, Belfast, BT7 1NN, United Kingdom

A. Z. Msezane
Department of Physics and Center for Theoretical Studies of Physical Systems, Clark Atlanta University, Atlanta, Georgia 30314, USA

V. R. Shaginyan
Petersburg Nuclear Physics Institute, Gatchina 188300, Russia
(Received 29 January 2004; published 21 June 2005)

We exploit the analogy between the transfer of a pulse across a scattering medium and Aharonov's weak measurements to resolve the long standing paradox between the impossibility to exceed the speed of light and the seemingly "superluminal" behavior of a tunneling particle in the barrier or a photon in a "fast-light" medium. We demonstrate that superluminality occurs when the value of the duration τ spent in the barrier is uncertain, whereas when τ is known accurately, no superluminal behavior is observed. In all cases only subluminal durations contribute to the transmission which precludes faster-than-light information transfer, as observed in a recent experiment.

DOI: 10.1103/PhysRevA.71.064103 PACS number(s): 03.65.Ta, 73.40.Gk

Recent experiments [1] on transmitting information-containing features of an optical pulse across the "fast-light" medium, in which the group velocity exceeds the vacuum speed of light c, have renewed the interest in the so-called "superluminal" propagation phenomenon. It is well known that a wave packet transmitted across the potential barrier, undersized wave guide, or fast-light medium may arrive in a detector ahead of the one that propagates freely, as if it has crossed the scatter infinitely fast. This phenomenon, often referred to as "apparent superluminality of quantum tunneling" was noticed more than 70 years ago [2] and often discussed since (for reviews see Refs. [3,4], a recent selection of different views on the subject can be found in Ref. [5]). It seems to raise the question about the possibility of faster-than-light travel, e.g., in classically forbidden regions, inaccessible to a classical particle. It is, however, broadly understood that the paradox results from an incorrect identification of the transmitted peak with the incident one, since the incident pulse undergoes severe reshaping in the barrier region. Although it has been suggested [6], that a barrier may, in some manner, carve the transmitted pulse from the forward tail of the incident one, the question of how exactly this is achieved remains open. The precise mechanism of this reshaping, its relation to Aharonov's "weak" measurements [7–9] of the time delay and the implications for the information transfer, studied in Ref. [1], are the subjects of this paper. Earlier work on the weak nature of the tunneling times can be found in Refs. [10–12], and a more recent approach relating superluminality to superoscillations is given in Ref. [13].

We start by revisiting [7,8] the analysis of a quantum system prepared at $t=0$ in the initial state $|I\rangle$ and then postselected (observed) at $t=T$ in the final state $|F\rangle$,

$$|I\rangle = \sum_{\nu} a_{\nu}|\nu\rangle, \quad |F\rangle = \sum_{\nu} b_{\nu}|\nu\rangle, \quad \hat{A}|\nu\rangle = A_{\nu}|\nu\rangle. \qquad (1)$$

If at some t, $0<t<T$, the system is subjected to a von Neumann-type measurement [14] of an operator $\hat{A}$, the state of the pointer with position τ after postselection is given by [7,8]

$$\langle\tau|M\rangle = \sum_{\nu} G(\tau - A_{\nu})\,\eta_{\nu}, \quad \eta_{\nu} \equiv b^{*}_{\nu}a_{\nu}, \qquad (2)$$

where $G(\tau)$ is the initial (e.g., Gaussian) state of the meter at $t=0$ and Σ_{ν} must be replaced by an integral $\int d\nu$ if $\hat{A}$ has a continuous spectrum. The meter is then read, i.e., the pointer position is accurately determined. For an accurate "strong" measurement, the width $\Delta\tau$ is small (compared to the separation between the eigenvalues or, if the spectrum is continuous, to the scale on which η_{ν} varies considerably) and the meter's readings occur close to the eigenvalues A_{ν}. For an inaccurate, or weak, measurement, $\Delta\tau$ is large, and the interference between overlapping Gaussians in Eq. (2) may, for a special choice of $|I\rangle$ and $|F\rangle$, produce anomalous readings in the regions where $\hat{A}$ has no eigenvalues [7,8]. Next we show that when using the coordinate of the tunneled particle to estimate the time delay it has experienced in the barrier, τ, one, in fact, performs a weak measurement of τ. The anomalously small value of τ that is obtained, results, just as an Aharonov's weak value, from the quantum uncertainty inherent to the procedure.

Consider a one-dimensional wave packet transmitted across a short-ranged potential barrier $V(x)$, contained inside the region $0<x<b$. At $t=0$ the particle is prepared as an incident wave packet with a mean momentum k_0, centered at $x=x_I<0$:

$$\Psi_0(x) \equiv \langle x|I\rangle = \int_{-\infty}^{\infty} C(k-k_0)\exp(ikx)dk, \qquad (3)$$

where the factor $C(k-k_0)$ insures that only positive momenta contribute to the integral. At some large $t=T$, it is postselected in the transmitted state ($\hbar=1$)

$$\Psi_F(x) \equiv \langle x|F\rangle = \int_{-\infty}^{\infty} T(k)C(k-k_0)\exp[ikx - iE(k)T]dk, \qquad (4)$$

where $T(k)$ is the barrier transmission amplitude. Using the convolution property of the Fourier integral, it is convenient to rewrite Eq. (4) as

$$\langle x|F\rangle = \int \Psi_T(x-x')\xi(x')dx', \qquad (5)$$

where

$$\Psi_T(x) = \int_{-\infty}^{\infty} C(k-k_0)\exp[ikx - iE(k)T]dk, \qquad (6)$$

is the state that would evolve from the initial one under free propagation, and $\xi(x')$ is the Fourier transform of $T(k)$. We can continue the discussion in terms of the "tunneling times" by identifying (v_0 is the velocity corresponding to the wave vector k_0)

$$\tau(x') \equiv -x'/v_0 \qquad (7)$$

with the delay experienced by the particle in the barrier. Changing the variables in Eq. (5) and separating the inessential phase associated with the free motion, we obtain

$$\Psi_F = \exp[ik_0x - iE(k_0)T]\int G(\tau(x)-\tau)\eta(\tau)d\tau, \qquad (8)$$

where G is the envelope of Ψ_T,

$$G(\tau) \equiv \exp[iE(k_0)T + ik_0v_0\tau]\Psi_T(-v_0\tau) \qquad (9)$$

and

$$\eta(\tau) \equiv -(2\pi)^{-1}v_0 \int T(k)\exp[i(k_0-k)v_0\tau]dk. \qquad (10)$$

Comparing Eq. (8) with Eq. (2) shows that the relation between the time delay τ and the particle's position x is that between the measured quantity, whose amplitude distribution is $\eta(\tau)$ and the position of the pointer, whose initial state is determined by the envelope of the initial pulse [15]. As in the original Aharonov's approach, the particle is postselected in its transmitted state. However, unlike in Refs. [7–9], no external pointer variable is employed and its role is played by the particle's own position. Equivalently, registering the transmitted particle at a location x amounts to measuring the time delay τ of a particle with the momentum k_0. Finding the particle roughly the width of the barrier b ahead of the free one, as it happens in tunneling [2], corresponds to a negative time delay of $\approx -b/v_0$. Importantly, the accuracy to which the time delayed is evaluated is limited (if the spreading of the wave packet is neglected [16]) by the uncertainty Δx of the particle's position in its initial state [17], $\Delta\tau \approx \Delta x/v_0$.

In the case of tunneling, the momentum spread of the initial wave packet must be at least [18] small enough for all its components to tunnel, rather than to pass over the barrier. Such wave packets, broad in the coordinate space, correspond to inaccurate weak measurements which may produce anomalous superluminal readings, even when the amplitude distribution $\eta(\tau)$ contains only non-negative time delays. Indeed, in the complex k plane, $T(k)$ may only have poles on the positive imaginary axis and in the lower half-plane [19]. The poles of the first kind (I) correspond to the bound states supported by $V(x)$, while those of the second kind (II) correspond to scattering resonances. Closing the contour of integration in Eq. (10) as appropriate, we obtain

$$\eta(\tau) = 2\pi i \sum_{\mathrm{I}} \mathrm{Res}_n T \exp(ik_nv_0\tau), \quad \tau < 0 \qquad (11)$$

$$= -2\pi i \sum_{\mathrm{II}} \mathrm{Res}_n T \exp(-ik_nv_0\tau), \quad \tau > 0, \qquad (12)$$

where $\mathrm{Res}_n T$ denotes the residue of $T(k)$ at the nth pole. Thus, the poles (I), if present, produce negative time delays and are responsible, in the classical limit, for the speed up of a particle passing above a potential well. It is interesting to note that a potential well too shallow to support a bound state would not speed up a passing wave packet. If, on the other hand, no bound states are present, then

$$\eta(\tau) \equiv 0 \quad \text{for } \tau < 0 \qquad (13)$$

and the "spectrum" of the time delays in Eq. (8) is confined in the $0 \geqslant \tau < \infty$ semiaxis. Note that Eq. (13) demonstrates the causal nature of the scattering process, since the condition $\mathrm{Im}k_n < 0$, used in its derivation, also ensures that $\mathrm{Im}E(k_n) < 0$ for $\mathrm{Re}k_n > 0$ and the resonance states containing outgoing waves, $\mathrm{Re}k_n > 0$, are emptied, rather than filled up, as the time increases [19].

It is clear now that with a careful choice of $T(k)$, and, therefore, $\eta(\tau)$ the superluminal pulse can be produced, in an explicitly causal manner, from the front tails of $\Psi_T(x-x')$, all delayed relative to free propagation. One such system is a particle tunneling across a potential barrier. However, our approach only relies on the analyticity of the transmission amplitude $T(k)$ and can also be applied to evanescent propagation in waveguides [5] and optical propagation through fast-light media (see Refs. [1,3]), which have additional advantage of dealing with wave packets not subject to spreading in vacuum.

Information transfer is often associated with propagation of nonanalytic features, such as cutoffs [1] and next we will show that Eq. (13) ensures that it cannot be transferred faster than light. For a simple example, consider the propagation of an electromagnetic pulse across two narrow semitransparent mirrors, broadly similar to the setup studied in [20]. If the mirrors are modeled by δ functions of magnitude Ω located at $x=0$ and $x=b$, respectively, $T(k)$ is given by the multiple scattering expansion

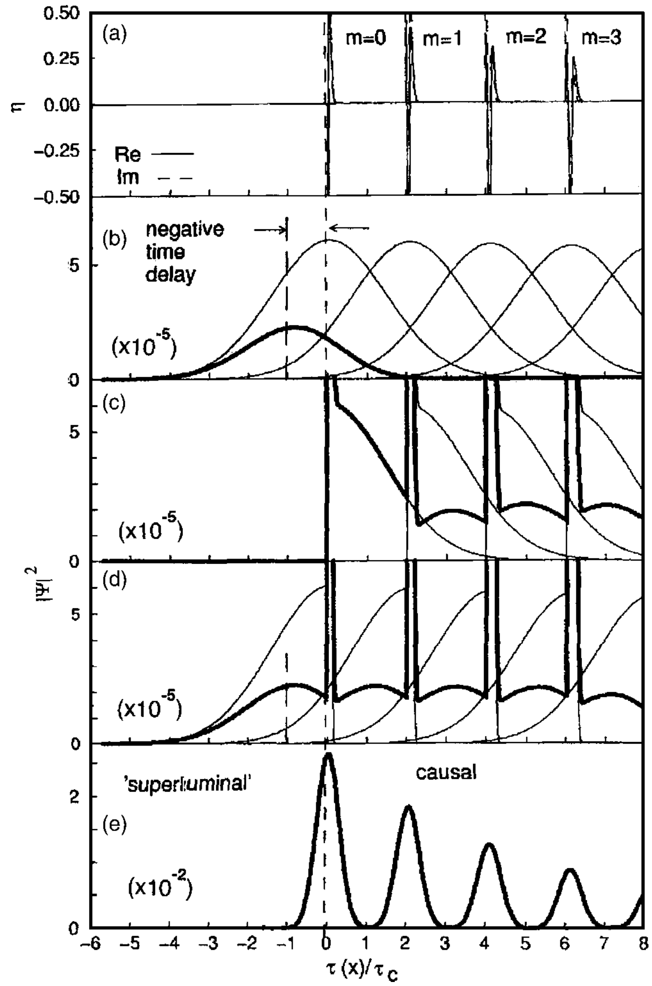

FIG. 1. (a) Real (solid) and imaginary (dashed) part of the time delay amplitude distribution, $\eta(\tau)$, for $\Omega b=100$. (b) Transmitted field $\Psi_F(x)$ (thick solid) and its components Ψ_m, corresponding to different terms in Eq. (14), for $\Omega b=100$, and a Gaussian incident pulse $k_0b=1.4\pi$ and $\Delta x/b=2.85$. (c) Same as (b) but for an incident Gaussian pulse truncated at front. (d) Same as (b) but for an incident Gaussian pulse truncated at rear. (e) Same as (b) but for $k_0b=7\pi$ and $\Delta x=0.57$.

$$T(k)=\sum_{m=0}^{\infty}T^{(m)}(k)\equiv[1+R(k)]\sum_{m=0}^{\infty}R(k)^{2m}\exp(2imkb), \tag{14}$$

where

$$R(k)=-i\Omega/(2k+i\Omega) \tag{15}$$

is the reflection amplitude for a single δ function placed at the origin $x=0$. Accordingly, the distribution $\eta(\tau)$ is decomposed into subamplitudes $\eta_m(\tau)$, $\eta_m(\tau)\equiv 0$ for $\tau<2mb/c$, each peaked near $\tau_m=2mb/c$ [Fig. 1(a)]. For a large $\Omega,\Omega b \gg 1$, the widths can be neglected and the incident pulse is split into a number of discrete path modes corresponding to $2m$, $m=0,1,2\ldots$ additional reflections experienced by the ray between $x=0$ and $x=b$ [20], and Eq. (4) reduces to Eq. (2) for a variable with a discrete spectrum $\{\tau_m\}$,

$$\Psi_F(x)\approx\sum_m G[\tau(x)-\tau_m]T^{(n)}(k_0). \tag{16}$$

A numerical evaluation of $\Psi_F(x)$, for $\Omega b=100$, corresponding to a weak Gaussian measurement

$$G(x)=\exp(-x^2/\Delta x^2), \quad \Delta x>b,$$

shows [see Fig. 1(b)] how a set of nearly Gaussian shapes, each delayed by $\tau_m>0$, interfere to produce a *negative* time delay $\approx -b/c$. Note that this is the best speed up which can be achieved with the model Eq. (14) [21].

It is now straightforward to show that this speed up effect cannot be used to send information faster than light. Aharonov and co-workers have already demonstrated [8] that a weak von Neumann pointer cannot be used for this purpose. Rather, they argued, the meter acts as a filter, extracting, in a nontrivial manner, the signal, otherwise hidden by a noise. The same argument applies to superluminal propagation. If the incident wave packet is chosen to be the rear half of a Gaussian with a sharp front, $G_-(x)=[1-\theta(x)]\exp(-x^2/\Delta x^2)$, Eqs. (8) and (13) show that the transmitted field will vanish outside the causal boundary $x_B=cT+x_I$, or for $\tau<0$, as shown in Fig. 1(c). Note that in Fig. 1(c) the narrow spikes near $\tau=\tau_m$ result from large oscillations of $\eta_m(\tau)$ clearly visible in Fig. 1(a). Equally unsuccessful would be an attempt at superluminal transfer of information encoded in a sharp cutoff at the rear of the incident pulse, $G_+(x)=\theta(x)\exp\times(-x^2/\Delta x^2)$. Figure 1(d) shows that by inspecting the field at $x>cT$, an observer cannot decide whether the whole Gaussian, or only its front half was incident on the barrier, and must await the arrival of the information-carrying part of the signal.

Recent experiments, in which a detector was to distinguish between a cut (G_+) and an uncut (G) signals, [1] have shown that the information detection time for pulses propagating through the fast-light medium is somewhat longer than that in vacuum, even though the group velocity in the medium is in the highly superluminal regime. As in Fig. 1(d), the absence of superluminal information transfer has a simple explanation. Since the medium (in this case, the potassium vapor) does not bind photons, the transmission amplitude cannot have poles in the upper half of the k plane and $\xi(x')$ in Eq. (5) must vanish for $x'>0$. The most advanced term in Eq. (5), $\Psi_T(x)$ contains a cutoff at $x=x_B\equiv cT+x_I$, and the superluminal ($x>x_B$) part of the transmitted field in Fig. 1(d) builds up from the front tails of $\Psi_T(x-x')$, $x'\leqslant 0$, unaffected by the cutoff, and for this reason is the same as the advanced uncut field {cf. Fig. 2(a) of Ref. [1]}. For $x<x_B$ the field has a complicated shape and the actual delay in detecting the backface of the pulse is likely to be caused by its deformation while propagating through the medium. To advance the cutoff beyond x_B, and achieve a truly superluminal information transfer, one requires a one-dimensional system capable of supporting both bound and scattering states of a photon, in the way a finite-depth potential well supports bound states of an electron. At present, we are not aware of the existence of such systems.

In summary, the notion of superluminality in wave packet propagation is based on relating the final position x of the transmitted particle to the time τ it is supposed to have spent in the scatterer. Quantally, τ and x are related as the measured quantity and the pointer position in a measurement, whose accuracy is determined by the coordinate width of the pulse. In cases where apparent superluminality is observed, e.g., in tunneling or optical propagation through fast-light media, the measurement is inevitably weak. Even if no negative time delays contribute to the transmission [$\eta(\tau)\equiv 0$ for $\tau<0$], it may, therefore, produce an anomalous reading by constructing a superluminal pulse from the front tails of the components of the transmitted pulse, all delayed relative to free propagation. Just as an improvement in the accuracy destroys anomalous weak values [9], a choice of a narrow incident pulse destroys superluminal propagation by making higher incident momenta pass over the barrier, or spill outside the anomalous dispersion region. As a result, a "strong" measurement registers only the subluminal time delays, as illustrated in Fig. 1(e) for the simple model described above. The contradiction between the impossibility of faster-than-light travel and observing an apparently superluminal pulse is, therefore, resolved in a typically quantum mechanical fashion: when superluminality is present, one does not know the delay, and cannot claim that the duration spent in the scatterer is shorter than b/c. Conversely, when the delay is known, no superluminal transmission is observed. The absence of negative virtual delays in optical propagation does, however, limit the speed of information transfer to c and below, as a nonanalytic feature (e.g., a cutoff) of the initial pulse may only travel as far as the most advanced component in Eq. (5), i.e., at most by cT. Beyond this point the field will either vanish if the front part of the pulse was discarded, or remain identical to the uncut field if its rear tail has been removed as the recent experiments by Stenner *et al.* [1] show.

Two of us (D.S. and V.S.) gratefully acknowledge the support of the CTSPS, Clark Atlanta University. A.Z.M. was supported by the U.S. DOE, Division of Chemical Sciences, Office of Basic Research.

[1] M. D. Stenner, D. J. Gauthier, and M. A. Neifeld, Nature (London) **425**, 695 (2003).

[2] L. A. MacColl, Phys. Rev. **40**, 621 (1932).

[3] R. Y. Chiao and A. M. Steinberg, Prog. Opt. **37**, 345 (1997).

[4] *Time in Quantum Mechanics*, edited by J. G. Muga, R. Sala-Mayato, and I. L. Eguisquiza (Springer, New York, 2002).

[5] Special Issue, Ann. Phys. **7**, 662 (1998).

[6] J. M. Deutch and F. E. Low, Ann. Phys. (N.Y.) **228**, 184 (1993).

[7] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).

[8] Y. Aharonov and L. Vaidman, Phys. Rev. A **41**, 11 (1990).

[9] Y. Aharonov and L. Vaidman, in Ref. [4], pp. 369–412.

[10] A. M. Steinberg, Phys. Rev. Lett. **74**, 2405 (1995).

[11] A. M. Steinberg, Phys. Rev. A **52**, 32 (1995).

[12] G. Iannaccone, in *Time and Its Implications*, edited by D. Mugnai, A. Ranfagni, and L. S. Schulman (World Scientific, Singapore, 1997), pp. 292–309.

[13] Y. Aharonov, N. Erez, and B. Reznik, Phys. Rev. A **65**, 052124 (2002); J. Mod. Opt. **50**, 1139 (2003).

[14] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, NJ, 1955).

[15] For a slightly different way to relate superluminal propagation to weak measurements of the tunneling time see: D. Sokolovski and Y. Liu, Phys. Rev. A **63**, 012109 (2000); D. Sokolovski, Proc. R. Soc. London **460**, 499 (2004).

[16] For a massive particle, the wave packet would spread and the accuracy of the measurement would deteriorate further, as the time progresses. We, however, assume that the transmitted particle is detected before significant spreading has occurred. No such restriction is necessary for a photon, whose wave packet retains its shape in free propagation.

[17] Consider, for example, free motion, in which case $\eta(\tau)\approx v_0^{-1}\delta(\tau)$ and different values of $\tau(x)$ axise solely from the spread in the particle's initial position.

[18] A more stringent condition, $\partial_k \mathrm{Re}\{\ln T(k_0)\}/\Delta x \ll 1$ is needed if one also wants to avoid distortion.

[19] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed. (Pergamon, Oxford, 1977).

[20] Wang Yun-ping and Zhang Dian-lin, Phys. Rev. A **52**, 2597 (1995); Y. Japha and G. Kurizki, *ibid.* **53**, 586 (1996).

[21] For $\Omega b \gg 1$ and $k \ll \Omega$, the transmission probability is small, $|T(k)|\ll 1$, except near equally spaced resonances at $k_j=\pi j/b$, $j=1,2,\ldots$. Between the resonances we have $(\phi' \equiv \partial_k \phi)\phi'(k)\approx -b, \phi''(k)\ll 1$. Thus for the transmitted pulse with $k_0\approx\pi(j+1/2)$ we obtain $\langle x|F\rangle \approx \exp[ik_0x - iE(k_0)T]T(k_0)\int[iqx + iq\phi'(k_0)] \approx T(k_0)G_T(x+b)$ so that $\tau=-b/c$ is the best speed up achieved with this model. In addition, we must ensure that the momentum width of the incident pulse is small compared to the spacings between the resonances, $\Delta k \ll \pi$. If this is not the case, the wings of the momentum distribution are caught in the neighboring resonances, and the barrier loses its "filtering" property, transmitting, instead of a single pulse, an extended wave, sustained by the decay of the resonances. The minimal coordinate width of a Gaussian pulse is, therefore, limited by the condition $4\pi\Delta x/b \gg 1$.