Generation of entanglement and squeezing in the system of two ions trapped in a cavity

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The dynamical behavior of the entanglement and the squeezing in the system of two ions trapped in a lossy optical cavity is discussed, in which the vibrational mode of the first ion is coupled to the cavity field via a linear-mixing interaction and the vibrational mode of the second ion is coupled to the cavity field via an effective parametric interaction, respectively. It is found that when the effective strength r_1 of the coupling between the first ion and the cavity field is stronger than the strength r_2 of the coupling between the second ion and the cavity field, the motional state of the two ions develops into a robust two-mode mixed Gaussian entangled state and the two-mode squeezing of phonons appears. But the entanglement between the cavity mode and the vibrational mode of the second ion can only happen periodically in a small time range. When $r_1 < r_2$, only the entanglement between the two vibrational modes appears but the two-mode squeezing does not exist. And the entanglement between the cavity mode and the motional mode of the second ion can also take place in a small time range.

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I. INTRODUCTION

Entanglement in the continuous variable regime has attracted a lot of attention in the quantum optics and quantum information fields. Interest in continuous variable entanglement is being extensively excited due to successful experiments on quantum teleportation [1], quantum dense coding [2], and quantum swapping [3]. A lot of schemes for generating the continuous variable entangled states, such as utilizing the $\chi^{(2)}$ process in optical parametric amplifiers [4], the Kerr effect in optical fibers [5], and the interaction between a coherent linearly polarized field and cold atoms in a high finesse optical cavity [6], have been proposed theoretically and demonstrated experimentally.

Recent advances in ion cooling and trapping have opened interesting prospects in entangled state generation. By driving the ion appropriately with laser fields, its center-of-mass motion can be manipulated precisely. Recent experiments using linear traps have reached a very low rate of heating [7], so that the dissipation time associated with the vibration of the center of mass may be neglected. Various schemes of producing two-mode entangled states of the vibration modes, such as two-mode entangled coherent states [8], two-mode dark pair coherent states [9], two-mode SU(1,1) intelligent states [10], and the superposition of two-mode SU(2) coherent states [11], have been proposed in a two-dimensional ion trap. The experimental possibility of trapping an ion inside an optical cavity [12] has opened different options in conversion of quantum properties between the vibrational mode of the ion and the cavity field. The influences of the cavity field statistics on the ion dynamics [13], the transfer of coherence between motional states and the cavity field [14], the generation of photon-phonon Bell-type states [15], and the implementation of quantum phase gate operation when the cavity mode and the vibrational mode are used to separately represent a qubit [16], have been investigated and proposed. A scheme for the generation of an entangled coherent state for the photon-phonon modes in the system of a trapped ion inside a nonideal cavity was proposed by Rangel *et al.* [17]. On the other hand, both theoretical prediction and experimental progress show that trapped ions constitute one the most promising systems to implement scalable quantum computation (see, for example, Ref. [18]). Very recently a scheme for coherent-state quantum computation by exploiting the motional degrees of freedom of individually trapped ions in a cavity has been proposed [19]. Thus it is interesting to investigate how to prepare the vibrational entangled state for two individually trapped ions trapped in a cavity.

In this paper, the dynamical behavior of the entanglement and the squeezing in the system of two ions trapped in a lossy optical cavity is discussed, in which the motional mode of the first ion is coupled to the cavity field via a linear-mixing interaction and the motional mode of the second ion is coupled to the cavity field via an effective parametric interaction, respectively. It is found that when the effective strength r_1 of the coupling between the first ion and the cavity field is stronger than the strength r_2 of the coupling between the second ion and the cavity field, the motional state of the two ions develops into a robust two-mode mixed Gaussian entangled state and the two-mode squeezing of phonons appears. But the entanglement between the cavity mode and the motional mode of the second ion can only happen periodically in a small time range. When $r_1 < r_2$, only the entanglement between the two motional modes occurs but the two-mode squeezing does not appear. Different from the previous schemes for entanglement preparation between atoms or ions at separate nodes, such as entangling atoms or ions through the interaction of the quantum correlated field with separate atoms or ions [14,20] or through conditional measurements [21], the present scheme does not require nonclassical light source or projective measurements. The effective parametric interaction between the cavity mode and the second vibrational mode generates continuous variable entanglement between the second ion and the cavity field. The linear-mixing interaction between the cavity mode and the first vibrational mode can then transfer this entanglement to the first ion and thus the vibrational entanglement between the two ions takes place. The other advantage of the present

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scheme is that the entangled state can be prepared unconditionally and under steady-state conditions. This paper is structured as follows: in Sec. II, the Hamiltonian and the master equation to describe the coupling between the two ions trapped in the lossy optical cavity are presented. In Sec. III, we solve the master equation by use of the characteristic function method and get the time-dependent characteristic function analytically. Section IV is devoted to discussing the dynamical properties of the entanglement between the vibrational modes and the two-mode squeezing. The property of the entanglement between the cavity mode and one of the two vibrational modes is investigated in Sec. V. Finally we give a summary in Sec. VI.

II. MODEL

Consider that there are two far distant Paul traps inside a single-mode optical cavity, in each Paul trap an ion is confined to move in the x direction. Both of the ions are cooled down to very low temperature and may perform small oscillations with frequency ν_j $(j=1,2)$ around their equilibrium positions. Two of internal electronic levels for each ion, $|e_j\rangle$ and $|g_j\rangle$, separated by an energy ω_j, are coupled to the cavity field and an auxiliary laser with frequency ω_{Lj}. The trapping direction x is coincident with the axis of the cavity and the laser field is treated as a classical field and is assumed to propagate in the y-z plane. Therefore the coupling of the jth ion with the cavity and with the laser field is descried by the interaction Hamiltonian [13,14,16,17]

$$H_{\text{int}}^{j} = g_j \sin(kx_j)(a|e\rangle\langle g_j| + a^{\dagger}|g_j\rangle\langle e_j|) + \Omega_j(|e_j\rangle\langle g_j|e^{-i\omega_{Lj}t} + |g_j\rangle \times\langle e_j|e^{i\omega_{Lj}t}). \tag{1}$$

For simplicity we have taken a sine function as the cavity standing-wave mode and set the minimum of the two trapping potentials at two separate nodes of the cavity. The parameter Ω_j is the coupling constant between the jth ion and the jth laser, g_j is the single-photon coupling strength, and k is the wave number of the cavity field with frequency ω_c, which described by the operators a and $a^{\dagger}$. The position operator of the jth ion is related to the annihilation operator b_j and the creation operator $b_j^{\dagger}$ of the vibrational mode with frequency ν_j by $x_j=(1/2m_j\nu_j)^{1/2}(b_j+b_j^{\dagger})$, where m_j is the mass of the jth ion. In the Lamb-Dicke limit, the Hamiltonian (1), in the interaction picture, takes the form

$$H_I^{j} = \eta_j g_j(b_j e^{-i\nu_j t} + b^{\dagger}e^{i\nu_j t})(a|e_j\rangle\langle g_j|e^{i\Delta_j t} + \text{H.c.}) + \Omega_j(|e_j\rangle \times\langle g_j|e^{i(\Delta_j+\delta_j)t} + \text{H.c.}), \tag{2}$$

where $\Delta_j=\omega_j-\omega_c$ is the cavity-ion detuning, and $\delta_j=\omega_c-\omega_{L_j}$ is the frequency difference between the cavity field and the laser field incident to drive the jth ion. $\eta_j=k(1/2m\nu_j)^{1/2}\ll 1$ are the Lamb-Dicke parameters.

We now consider that the large cavity-ion detuning Δ_j is so large that $|\Delta_j \gg g_j, \Omega_j, \eta_j$, and ν_j. For $t\gg 1/|\Delta_j|$, and following the usual procedure for adiabatic elimination of the excited state $|e_j\rangle$ [14,16,17], we derive the following effective interaction Hamiltonian:

$$H_e^{j} \approx r_j(b_j a e^{-i(\delta_j+\nu_j)t} + b_j a^{\dagger} e^{-i(\nu_j-\delta_j)t} + \text{H.c.})\sigma_{zj}, \tag{3}$$

here $\sigma_{zj}=|e_j\rangle\langle e_j|-|g_j\rangle\langle g_j|$, $r_j=\eta_j\Omega_j g_j/\Delta$, and the assumption $\eta_j g_j \ll \Omega_j$ has been adopted. If the ion starts in one of its electronic levels, for example the ground state, the operator $|e_j\rangle\langle e_j|$ may be substituted by zero, since effectively no transition to the excited state occurs. For the first ion the frequency of the driving laser field is assumed as $\omega_{L1}=\omega_c-\nu_1$ and the trap frequency ν_1 is assumed to be large, such that $\nu_1 \gg |r_1|$, then utilizing the rotating-wave approximation gives the motional-light coupling in the first ion as

$$H_1 = r_1(ab_1^{\dagger} + a^{\dagger}b_1). \tag{4}$$

This Hamiltonian describes a linear-mixing process between the cavity photons and the vibrational mode of the first ion, in which the creation (absorption) of photons occurs simultaneously with the annihilation (creation) of the same number of vibrational quanta (phonons). This can lead to the possibility of exchanging the quantum information between the cavity field and the vibrational mode. For the second trapped ion, if we choose the frequency of the driving laser as $\omega_{L2}=\omega_c+\nu_2$ and employ the rotating-wave approximation once again under the condition that $\nu_2 \gg |r_2|$, then the effective Hamiltonian for the coupling between the field and the motion of the second ion is written as

$$H_2 = r_2(ab_2 + a^{\dagger}b_2^{\dagger}). \tag{5}$$

This is the Hamiltonian for a nondegenerate parametric amplification process and via this process continuous variable entanglement can be generated between the first motional mode and the cavity mode. Without loss of generality, in the following we assume $r_1, r_2 > 0$. From here we can see that via the nondegenerate parametric interaction, the cavity field may be entangled with the motional mode 2 belonging to the second ion. This cavity field is also coupled to the fist ion via the linear-mixing process, so entanglement of the cavity field with the vibrational mode 2 may be transferred to the vibrational mode 1, thus entanglement between the two motional modes may occur.

As pointed out in Ref. [7] in recent Paul trap experiments, the vibrational states are easier to protect from the environment than the cavity field. So here we only consider cavity losses and neglect the dissipation of the two vibrational modes of the two ions with the environment. Then the master equation for the total system density operator $\rho(t)$ is expressed as

$$\frac{d}{dt}\rho = -i[H_1 + H_2, \rho] + L_{\text{cav}}\rho, \tag{6}$$

with

$$L_{\text{cav}}\rho = \kappa(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a). \tag{7}$$

Here $L_{\text{cav}}\rho$ describes the cavity loss at zero temperature, κ is the cavity decay rate.

III. TIME-DEPENDENT SOLUTION OF THE CHARACTERISTIC FUNCTION FOR THE SYSTEM

In order to solve the master equation (6), we introduce two new Bosonic operators,

$$c_1 = \frac{1}{\Omega}(r_1 b_1 + r_2 b_2^\dagger), \tag{8}$$

$$c_2 = \frac{1}{\Omega r_2}[\Omega(r_2 b_1 + r_1 b_2^\dagger) + r_1(r_2 b_1^\dagger + r_1 b_2)],$$

with $\Omega^2 = r_1^2 - r_2^2$. Evidently, these new operators obey the commutation relations $[c_m, c_n^\dagger] = \delta_{mn}(m,n=1,2)$. Then the master equation (6) can be rewritten as

$$\frac{d}{dt}\rho = -i\Omega[a c_1^\dagger + a^\dagger c_1, \rho] + L_{\mathrm{cav}}\rho. \tag{9}$$

From the above equation we can see that the coupling system can be effectively treated as a linear mixer between the combined vibrational mode c_1 and the field mode a in a lossy cavity at zero temperature. Equation (9) can be solved analytically by use of the method of the characteristic function. The characteristic function for the system of the coupling between the motional and light modes in Wigner representation is defined as [22]

$$\chi(\xi_a, \xi_{c_1}, \xi_{c_2}, t) = \mathrm{Tr}[\rho(t)\exp(\xi_a a^\dagger + \xi_{c_1} c_1^\dagger + \xi_{c_2} c_2^\dagger + \mathrm{H.c.})]. \tag{10}$$

Using the standard operator correspondence we find that the above characteristic function obeys

$$\frac{\partial}{\partial t}\chi(t) = -[i\Omega(A_1 + A_2) + \kappa A_3 + \kappa B_1]\chi(t), \tag{11}$$

in which

$$A_1 = \xi_a^* \frac{\partial}{\partial \xi_{c_1}^*} - \xi_a^* \frac{\partial}{\partial \xi_{c_1}},$$

$$A_2 = \xi_{c_1}^* \frac{\partial}{\partial \xi_a^*} - \xi_{c_1}^* \frac{\partial}{\partial \xi_a},$$

$$A_3 = \xi_a \frac{\partial}{\partial \xi_a} + \xi_a^* \frac{\partial}{\partial \xi_a^*},$$

$$A_4 = \xi_{c_1} \frac{\partial}{\partial \xi_{c_1}} + \xi_{c_1}^* \frac{\partial}{\partial \xi_{c_1}^*},$$

$$B_1 = |\xi_a|^2, \quad B_2 = |\xi_{c_1}|^2,$$

$$B_3 = -\xi_{c_1}\xi_a^* + \xi_{c_1}^*\xi_a. \tag{12}$$

Using the commutation relations among the operators $A_j(j=1,2,3,4)$ and $B_k(k=1,2,3)$, we have

$$\chi(\xi_a, \xi_{c_1}, \xi_{c_2}, t) = \exp(g_1 B_1 + g_2 B_2 + g_3 B_3) e^{f_2 A_2} e^{f_1 A_1} \times e^{f_3 A_3 + f_4 A_4} \chi(\xi_a, \xi_{c_1}, \xi_{c_2}, 0), \tag{13}$$

in which

$$f_1 = -\frac{2i\Omega \sinh\frac{\Omega' t}{2}}{\Omega'^2}\left(\Omega' \cosh\frac{\Omega' t}{2} - \kappa \sinh\frac{\Omega' t}{2}\right),$$

$$f_2 = -\frac{2i\Omega \sinh\frac{\Omega' t}{2}}{\Omega' \cosh\frac{\Omega' t}{2} - \kappa \sinh\frac{\Omega' t}{2}},$$

$$f_3 = -\kappa t + \ln\frac{\cosh\frac{\Omega' t}{2} - \kappa \sinh\frac{\Omega' t}{2}}{\Omega'},$$

$$f_4 = -\kappa t - \ln\frac{\cosh\frac{\Omega' t}{2} - \kappa \sinh\frac{\Omega' t}{2}}{\Omega'},$$

$$g_1 = \frac{e^{-\kappa t}}{2\Omega'^2}(-4\Omega^2 + \kappa^2 \cosh \Omega' t - \kappa\Omega' \sinh \Omega' t) - \frac{1}{2},$$

$$g_2 = \frac{e^{-\kappa t}}{2\Omega'^2}(-4\Omega^2 + \kappa^2 \cosh \Omega' t + \kappa\Omega' \sinh \Omega' t) - \frac{1}{2},$$

$$g_3 = \frac{i\Omega\kappa e^{-\kappa t}}{\Omega'^2}(\cosh \Omega' t - 1),$$

where $\Omega' = \sqrt{\kappa^2 - 4\Omega^2}$.

Assuming initially that the cavity mode and both of the motional modes are in the vacuum state $|0_a, 0_{b_1}, 0_{b_2}\rangle$, we can obtain the time-dependent characteristic function $\chi(\xi_a, \xi_{c_1}, \xi_{c_2}, t)$ for the coupling system explicitly. The Weyl-Wigner characteristic function for the phonon-photon coupling system in the representation of Bosonic operators a, b_1, b_2 is defined as

$$\chi(\xi_a, \xi_{b_1}, \xi_{b_2}, t) = \mathrm{Tr}[\rho(t)\exp(\xi_a a^\dagger + \xi_{b_1} b_1^\dagger + \xi_{b_2} b_2^\dagger + \mathrm{H.c.})]. \tag{14}$$

By use of the relations among the operators b_1, b_2, c_1, and c_2, we can get the time-dependent characteristic function $\chi(\xi_a, \xi_{b_1}, \xi_{b_2}, t)$ in the three-mode photon-phonon coupling system as

$$\begin{aligned}\chi(\xi_a, \xi_{b_1}, \xi_{b_2}, t) = \exp\Bigg[& -\left(e_1^2 + \frac{1}{2}\right)|\xi_{b_1}|^2 - \left(e_2^2 - \frac{1}{2}\right) \\ & \times |\xi_{b_2}|^2 - \left(\frac{1}{2} + e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t}\sinh^2\frac{\Omega' t}{2}\right) \\ & \times |\xi_a|^2 - e_1 e_2(\xi_{b_1}\xi_{b_2} + \xi_{b_1}^*\xi_{b_2}^*) \\ & + i e_1 e_3(\xi_a \xi_{b_1}^* - \xi_a^* \xi_{b_1}) \\ & + i e_2 e_3(\xi_a \xi_{b_2} - \xi_a^* \xi_{b_2}^*)\Bigg],\end{aligned} \tag{15}$$

where

$$g(t) = e^{-\kappa t/2}\frac{\Omega' \cosh\frac{\Omega' t}{2} + \kappa \sinh\frac{\Omega' t}{2}}{\Omega'},$$

$$e_1 = \frac{r_1 r_2}{\Omega^2}[g(t) - 1],$$

$$e_2 = \frac{1}{\Omega^2}[r_2^2 g(t) - r_1^2],$$

$$e_3 = \frac{2r_2}{\Omega'}e^{-\kappa t/2}\sinh\frac{\Omega' t}{2}. \quad (16)$$

After having the characteristic function $\chi(\xi_a,\xi_{b_1},\xi_{b_2},t)$, we can discuss the nonclassical properties such as entanglement and squeezing in the three-mode photon-phonon coupling system.

IV. SQUEEZING AND ENTANGLEMENT OF THE MOTION FOR THE TWO IONS

It is interesting to study the entanglement properties of the bipartite system which is obtained when one of the three modes is traced out. This study may be interesting for possible applications of the present scheme for quantum information processing. In the following we discuss the quantum properties of the motion for the two ions, i.e., to trace the cavity field out. The time-dependent characteristic function $\chi_b(\xi_{b_1},\xi_{b_2},t)$ for the two vibrational modes can be obtained through the identity relation [22,23] $\chi_b(\xi_{b_1},\xi_{b_2},t)=\chi(\xi_a=0,\xi_{b_1},\xi_{b_2},t)$. The expression of $\chi_b(\xi_{b_1},\xi_{b_2},t)$ is written as

$$\chi_b(\xi_{b_1},\xi_{b_2},t) = \exp\left\{-\left[\left(e_1^2+\frac{1}{2}\right)|\xi_{b_1}|^2 + \left(e_2^2-\frac{1}{2}\right)|\xi_{b_2}|^2 + e_1 e_2(\xi_{b_1}\xi_{b_2} + \xi_{b_1}^*\xi_{b_2}^*)\right]\right\} = \exp\left(-\frac{1}{2}\xi_b^\dagger V_{b_1 b_2}\xi_b\right). \quad (17)$$

Here $\xi_b^\dagger=(\xi_{b_1}^*,\xi_{b_1},\xi_{b_2}^*,\xi_{b_2})$ is a four-dimensional vector and the matrix $V_{b_1b_2}$ is a 4×4 covariance matrix for the two motional modes.

From Eq. (17) we can see under the interaction with the cavity field that the two ions initially in the vacuum state $|0_{b_1},0_{b_2}\rangle$ evolve into a two-mode Gaussian state. The purity of the two-mode Gaussian state for the two motional modes can be measured by the linear entropy as follows:

$$S(t) = 1 - \mathrm{Tr}_{b_1,b_2}(\rho_b^2(b_1,b_2,t)) = 1 - \frac{1}{\pi^2}\int d^2\xi_{b_1}d^2\xi_{b_2}\chi_b(\xi_{b_1},\xi_{b_2},t)\chi_b(-\xi_{b_1},-\xi_{b_2},t) = \frac{2(e_2^2-e_1^2-1)}{2(e_2^2-e_1^2)-1} = \frac{2(\langle b_2^\dagger b_2\rangle - \langle b_1^\dagger b_1\rangle)}{2(\langle b_2^\dagger b_2\rangle - \langle b_1^\dagger b_1\rangle)+1}, \quad (18)$$

where $\langle b_1^\dagger b_1\rangle=e_1^2$ and $\langle b_2^\dagger b_2\rangle=e_2^2-1$ are the expectation values of the phonon number operators $b_1^\dagger b_1$ and $b_2^\dagger b_2$. Here $S(t)=0$ corresponds to the phonons in the two ions trapped in the cavity being in a pure two-mode Gaussian state and $S(t)\neq 0$ means the motional phonons in a mixed two-mode Gaussian state. If the cavity dissipation can be neglected, i.e., $\kappa=0$, then $\langle a^\dagger a\rangle=\langle b_2^\dagger b_2\rangle-\langle b_1^\dagger b_1\rangle$ because the operator $a^\dagger a+b_1^\dagger b_1-b_2^\dagger b_2$ commutates with the Hamiltonian H_1+H_2 of the system and all the three modes are initially assumed in their ground states. So the purity of the two-mode Gaussian state for the two motional modes is decided by the average photon number in the cavity. For the case of the coupling constants r_1 and r_2 satisfying $r_1>r_2$, i.e., the linear-mixing process is stronger than the parametric amplification process, the whole system can be regarded as an effective linear mixer with the coupling strength $\Omega=\sqrt{r_1^2-r_2^2}$. The average photon number is obtained as $\langle a^\dagger a\rangle=r_2^2\sin^2\Omega t/\Omega^2$. We can see that when $\Omega t=2n\pi$ $(n=1,2,\ldots)$, $S(t)=0$ corresponding to that the two vibrational modes evolve back to their initial ground state, a pure state. And for $\Omega t_n=(2n+1)\pi$ $(n=1,2,\ldots)$, $S(t_n)=0$ indicating that the two vibrational modes evolve to another pure state. This is because at these moments, the cavity field is in its vacuum state, the strong linear-mixing process between the cavity mode and the first motional mode transfers completely the cavity property to the motional mode 1 so that the cavity mode is decoupled with both of the two motional modes. In fact, at these moments, the two vibrational modes are in a two-mode squeezed vacuum state $|\Psi_{b_1,b_2}\rangle=(1/\cosh r)\Sigma_{n=0}^{\infty}(-\tanh r)^n|n,n\rangle$ with $\tanh 2r=4r_1r_2(r_1^2+r_2^2)/[(r_1^2-r_2^2)^2+8r_1^2r_2^2]$. If $r_1<r_2$, which means that parametric amplification process is stronger than the linear-mixing process, we find that $\langle a^\dagger a\rangle=r_2^2\sinh^2\sqrt{r_2^2-r_1^2}t/(r_2^2-r_1^2)$. Evidently, the behavior of the whole system looks like an effective parametric amplifier since the parametric amplification process is dominant. In this case, $\langle a^\dagger a\rangle>0$ so that $S(t)\neq 0$, that is, the two motional modes evolve into a mixed state although there is no cavity dissipation. This is because the linear-mixing process is weaker than the parametric amplification process; the linear mixer cannot transfer completely the cavity field into the first ion during the parametric amplification between the cavity field and the second ion. So except at $t=0$, the two motional modes are in a mixed state. However, if the cavity dissipation is taken into account, i.e., $\kappa\neq 0$, it is easy to find that $S(t)\neq 0$ except $t=0$ no matter when $r_1>r_2$ or $r_1<r_2$, that is to say, the two motional modes initially in their ground state evolve into a mixed Gaussian state due to the cavity dissipation.

The expectation values of two phonon operators b_1b_2 and $b_1^\dagger b_2^\dagger$ can be easily obtained from Eq. (17) as $\langle b_1^\dagger b_2^\dagger\rangle=\langle b_1b_2\rangle=-e_1e_2$. It is easy to check that $|\langle b_1^\dagger b_2^\dagger\rangle|>\sqrt{\langle b_1^\dagger b_1\rangle\langle b_2^\dagger b_2\rangle}$, which indicates that nonclassical correlation between the two motional modes can be established through the interaction with the cavity field. This nonclassical correlation may lead to the appearance of the squeezing and the entanglement for the two motional modes. The position and momentum operators for each motional mode can be defined as

$$X_j = b_j + b_j^\dagger, \quad P_j = -i(b_j - b_j^\dagger)(j=1,2). \quad (19)$$

The variances in the sum and difference operators are derived as

$$\Delta X_{\pm}^2 = \langle (X_1 \pm X_2)^2\rangle - (\langle X_1 \pm X_2\rangle)^2$$
$$= \langle (P_1 \mp P_2)^2\rangle - (\langle P_1 \mp P_2\rangle)^2 = 2\left(\frac{r_2 g \pm r_1}{r_1 \pm r_2}\right)^2. \tag{20}$$

For the ions that are both in their ground motional states, the variances equal 2. Here we can see that the variances of the sum position operator X_1+X_2 and the difference momentum operator P_1-P_2 may be smaller than 2, indicating that a two-mode squeezing motional state can be generated.

First we discuss the case in which the coupling parameters r_1 and r_2 obey $r_1>r_2$. In this case the linear-mixing process is stronger than the parametric amplification process, the dynamic behavior of the system is dominated by the linear-mixing process, and the system can be effectively treated as a new linear mixer. If the cavity dissipation is neglected, i.e., $\kappa=0$, then $\Delta X_+^2=2[(r_2\cos\Omega t+r_1)/(r_1+r_2)]^2 \leqslant 2$. We find at $\Omega t_n=(2n+1)\pi$, the variance ΔX_+^2 reaches its minimum value

$$\Delta X_+^2(t_n) = 2\left(\frac{r_1 - r_2}{r_1 + r_2}\right)^2. \tag{21}$$

This may be understood as follows: the nondegenerate parametric process between the cavity mode and the second motional mode produces the strong correlation for the cavity mode and the second motional mode, via the linear-mixing process between the cavity mode and the first motional mode, the quantum property of the cavity field is exchanged to the first motional mode, so that nonclassical correlation between the two vibrational modes is established. At time points t_n, the cavity field evolves into its vacuum and $\langle b_1^\dagger b_1\rangle=\langle b_2^\dagger b_2\rangle$, the cavity field completely transfers its nonclassical correlation with the second motional mode to the first motional mode, so that a pure two-mode squeezed vacuum state for the motional modes arises. But for $t\neq t_n$ and $\Omega t\neq 2n\pi$, $\langle a^\dagger a\rangle\neq 0$ and $\langle b_1^\dagger b_1\rangle\neq\langle b_2^\dagger b_2\rangle$, which means that the cavity field only transfers partially its nonclassical correlation with the second vibrational mode to the first vibrational one, so the resulted vibrational state of the two ions is a two-mode mixed Gaussian state and its squeezing becomes weaker. The larger $\langle a^\dagger a\rangle$ is, i.e., the difference between the two vibrational modes is, the weaker the squeezing is. Evidently, if r_2 tends to r_1, $\Delta X_+^2(t_n)\to 0$, which indicates that the pure two-mode motional state becomes a perfectly squeezed vacuum state. Unfortunately, it cannot be achievable in the present scheme. Because at these time points, the phonon numbers are expressed as $\langle b_1^\dagger b_1\rangle=\langle b_2^\dagger b_2\rangle=4r_1^2r_2^2/(r_1^2-r_2^2)^2$. Evidently when $r_2\to r_1$, the phonon numbers in the two ions tend to infinity and the state becomes the original Einstein-Podolsky-Rosen (EPR) state [24]. This is invalid in the present scheme, because the Lamb-Dicke limit assumption breaks down when the mean phonon numbers become large enough such that the extent of the atomic wave packet is no longer much smaller than the wavelength of the light. Therefore, when we choose the parameters r_1 and r_2, the case of r_2 tends to r_1 should be avoided. Because the system can be effectively regarded as a linear mixer when $r_1>r_2$, the system can reach a steady state when the cavity dissipation is taken into account (i.e., $\kappa\neq 0$). In the limit $t\to\infty$, the variance ΔX_+^2 reduces as

$$\Delta X_+(\infty)^2 = \frac{2r_1^2}{(r_1 + r_2)^2}. \tag{22}$$

We can check that when r_2 tends to r_1, $\Delta X_+^2(\infty)$ reaches its minimum value $1/2$ and the two-mode vibrational squeezing achieves its maximum value. This can also not be achieved, because in the long time limit, the phonon numbers are expressed as $\langle b_1^\dagger b_1(\infty)\rangle=r_1^2r_2^2/(r_1^2-r_2^2)^2$ and $\langle b_2^\dagger b_2(\infty)\rangle=r_2^2(2r_1^2-r_2^2)/(r_1^2-r_2^2)^2$. Evidently when $r_2\to r_1$, the phonon numbers in the two ions tend also to infinity, so the present model is not valid. Comparing Eq. (21) with Eq. (22), we can find that the cavity dissipation is harmful for the two-mode squeezing for the motional modes, for the same values of the parameters r_1 and r_2, the fluctuation of the X_+ component when $\kappa=0$ is squeezed deeper than that when $\kappa\neq 0$. But from Eqs. (21) and (22) we see that the strongest squeezing for the two motional modes happens at $r_2\to r_1$, although this cannot be reached in the present scheme. This is because for $r_2\to r_1$, the difference between the two vibrational modes becomes much smaller so the two-mode squeezing for the motional modes becomes stronger.

Second we consider the situation of $r_1<r_2$, in which the parametric interaction process is stronger than the linear-mixing process; the behavior of the system is similar to a parametric amplifier. In this case the analytical expressions of the characteristic functions $\chi(\xi_a,\xi_{b_1},\xi_{b_2},t)$ [Eq. (15)] and $\chi(\xi_{b_1},\xi_{b_2},t)$ [Eq. (17)] are still valid, but the total system could not reach a steady state since it gains more energy from the external driving lasers than it dissipates through the cavity dissipation. The average phonon numbers exhibit an exponential growth so that the Lamb-Dicke assumption can be also violated as mentioned in the above. That is to say, the present model described by Hamiltonian Eqs. (4) and (5) may be valid in a small time regime. However, the value of the function $g(t)>1$ for $t>0$, since $\Omega'=\sqrt{\kappa^2+4(r_2^2-r_1^2)}>\kappa$. The variances $\Delta X_\pm^2$ are larger than 2 for all times. Therefore no two-mode squeezing happens when $r_1<r_2$. The reason why there is no two-mode squeezing when $r_1<r_2$ is that the linear-mixing process is weaker than the parametric interaction process; the behavior of the system is dominated by an effective parametric amplification. This leads to the photon number of the cavity growing exponentially with time. Therefore the linear-mixing process could only transfer partially the nonclassical correlation of the cavity mode with the second vibrational mode generated by the parametric interaction process to the first vibrational mode. Although the nonclassical correlation between the two motional modes can also be established, the difference between the two vibrational modes becomes large because of the dominant behavior of the system being an effective parametric interaction process. So no two-mode squeezing appears. Although there is no two-mode squeezing for the phonons, we can show that there still exists the entanglement between the phonons in the two ions due to the nonclassical correlation between the two vibrational modes.

The entanglement property of the state characterized by Eq. (17) can be analyzed by use of the separability criteria for two-mode Gaussian states established by Simon [25]. The separability criterion established by Simon [25] can be understood as a valid Wigner-class-conservative quantum map under local time reversal [25,26]. A necessary and sufficient condition for a two-mode Gaussian quantum state to be separable is that its covariance matrix must satisfy $\tilde{V}+(1/2)\mathbf{E}\geqslant 0$, under a partial phase space mirror reflection $\tilde{V}=TVT$ [26], with

$$T=\begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}\end{pmatrix},\quad X=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

$$E=\begin{pmatrix}\mathbf{Z} & \mathbf{0}\\ \mathbf{0} & \mathbf{Z}\end{pmatrix},\quad Z=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{23}$$

Otherwise the state is entangled. There are four eigenvalues of $\tilde{V}_{b_1b_2}+(1/2)\mathbf{E}$ for the two-mode motional state described by Eq. (17), which are listed as

$$\lambda_{1,2,3,4}=\frac{1}{2}(e_1^2+e_2^2\pm 1)\pm\frac{1}{2}\sqrt{(e_1^2-e_2^2+1)^2+4e_1^2e_2^2}. \tag{24}$$

It is easily verified that only the eigenvalue $\lambda_4=1/2(e_1^2+e_2^2-1)-1/2\sqrt{(e_1^2-e_2^2+1)^2+4e_1^2e_2^2}$ is negative and the negativity of λ_4 is important to determine the nonseparability of the two-mode vibrational state. In fact, from the expression of $\lambda_4(t)$, we find that $\lambda_4(t)<0$ is equivalent to $\sqrt{e_1^2(e_2^2-1)}<|e_1e_2|$, which can be re-expressed as $\sqrt{\langle b_1^\dagger b_1\rangle\langle b_2^\dagger b_2\rangle}<|\langle b_1^\dagger b_2^\dagger\rangle|$. That is to say, the nonclassical correlation between the two motional modes established through the interaction with the cavity field results in the entanglement between the two vibrational modes. For the pure two-mode squeezed motional state produced at t_n when $r_1>r_2$ and $\kappa=0$, we have $\lambda_4(t_n)=-2r_1r_2/(r_1+r_2)^2$. As pointed out above, when $r_2\to r_1$, this two-mode squeezed state reaches the perfect EPR state [24], the maximally entangled state with continuous variable, we find that $\lambda_4(t_n)=-1/2$. So for the two-mode Gaussian entangled state described by Eq. (17), the value of the negative eigenvalue $\lambda_4(t)$ varies from $-1/2$ to 0. The deviation of $\lambda_4(t)$ from $-1/2$ means that as the entanglement decreases, the larger $\lambda_4(t)$ is, the weaker the entanglement. Thus the negative eigenvalue $\lambda_4(t)$ is a good measurement of the entanglement for the mixed state described by Eq. (17).

Figure 1 displays the time evolution of the negativity $\lambda_4(t)$ and the variance $\Delta X_+^2(t)$ for coupling constants obeying $r_1>r_2$. In this case, the system can be effectively treated as a linear mixer with coupling constant $\sqrt{r_1^2-r_2^2}$. We can see that when $r_1^2>r_2^2+\kappa^2/4$, both the entanglement between the two motional modes and the two-mode squeezing of phonons in two ions happen simultaneously. The functions $\Delta X_+^2(t)$ and $\lambda_4(t)$ do a damped periodic oscillation with period $2\pi/\sqrt{(r_1^2-r_2^2)-\kappa^2/4}$. With the increase of the ratio r_2/r_1, both the entanglement between the two motional modes and the two-mode squeezing of the phonons increase. This is because with the increase of the ratio $r_2/r_1<1$, the cavity field has a strong tendency to transfer its nonclassical correlation with the second vibrational mode to the first vibrational one; the difference between the two vibrational modes decreases, so that both the entanglement and the two-mode squeezing increases. The long time behavior of $\lambda_4(t)$ and $\Delta X_+^2(t)$ reaches two different constants dependent on the ratio r_2/r_1. This means that the two ions evolve into a robust two-mode Gaussian entangled state. As proved in Eq. (18), the linear entropy of this state is smaller than 1, so this state is a mixed state. If $r_2^2<r_1^2<r_2^2+\kappa^2/4$, the periodic oscillation disappears in the time behavior of $\lambda_4(t)$ and $\Delta X_+^2(t)$ because of the strength of the effective linear mixer being weaker than the dissipation rate of the cavity mode. The entanglement and the two-mode squeezing occur even in the long time limit. And in the long time limit, the two-mode motional state of the two ions evolves into a robust mixed entangled state. Comparing Fig. 1(a) with Fig. 1(b), we see that for the case of $r_1>r_2$, when $t\to\infty$, the values of $\lambda_4(t)$ and $\Delta X_+^2(t)$ are only dependent on the ratio r_2/r_1 but independent of the values of r_1 and r_2.

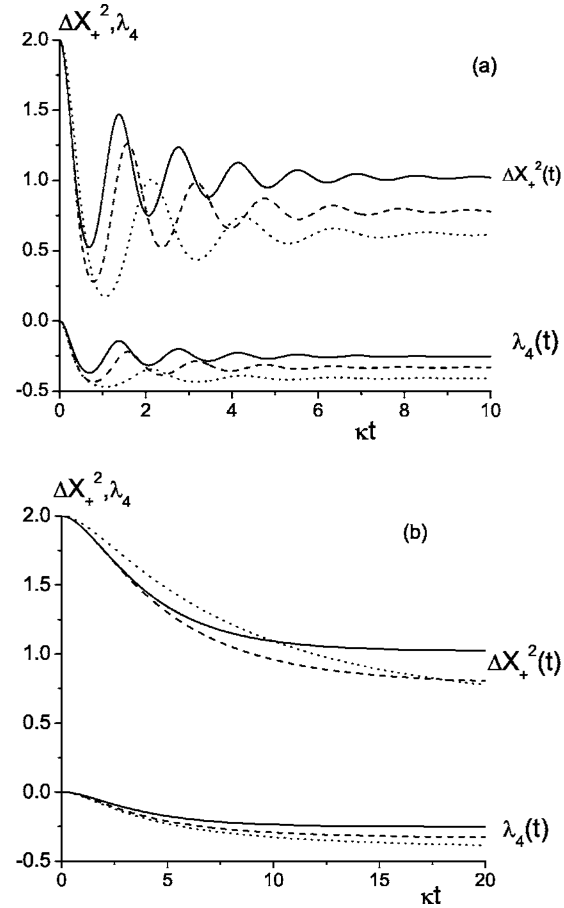

FIG. 1. The dynamic behavior of the negativity $\lambda_4(t)$ and the variance $\Delta X_+^2(t)$ for the two-mode vibrational modes when $r_1>r_2$, where (a) $r_1=5\kappa$, $r_2/r_1=0.4$ (solid), 0.6 (dashed), and 0.8 (dotted); (b) $r_1=0.5\kappa$, $r_2/r_1=0.4$ (solid), 0.6 (dashed), and 0.8 (dotted).

However, for the case of $r_2>r_1$, the behavior of the system is dominated by the parametric amplification; with the

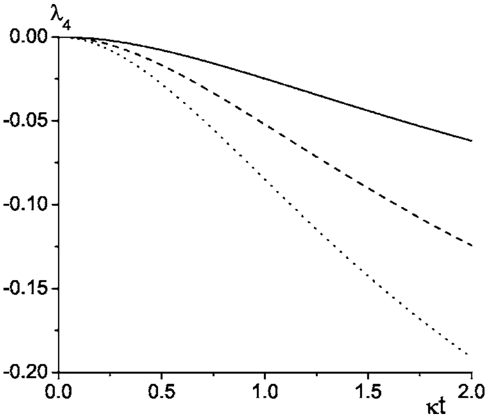

FIG. 2. The time evolution of $\lambda_4(t)$ for the two-mode vibrational modes when $r_1 < r_2$, in which $r_2 = \kappa$, $r_1/r_2 = 0.4$ (solid), 0.6 (dashed), and 0.8 (dotted).

time development, the cavity field grows exponentially, so the ability of the cavity field transferring its nonclassical correlation with the second vibrational mode to the first one becomes weak. The difference between the two vibrational modes becomes large with the time development. So the two-mode squeezing cannot take place. But the nonclassical correlation between the cavity field with the second ion can still partially transfer to the first ion, so the entanglement between the two motional modes still happens as shown in Fig. 2; the entanglement degree increases with the time and with the increase of the ratio r_1/r_2, and the two ions cannot evolve into a steady state. These results are only valid in the short time range. As mentioned previously, since the mean phonon numbers increase exponentially, the assumption of the Lamb-Dicke limit will be violated at some certain time.

V. ENTANGLEMENT BETWEEN THE CAVITY MODE AND ONE OF THE TWO MOTIONAL MODES

In this section we discuss the properties of entanglement between the cavity mode and one of the two motional modes. Setting $\xi_{b_2} = 0$ in Eq. (15), we get the characteristic function for the coupling system between the cavity mode and the first motional mode as

$$\chi(\xi_a, \xi_{b_1}, t) = \exp\left[-\left(\frac{1}{2} + e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t} \sinh^2 \frac{\Omega' t}{2}\right)|\xi_a|^2 - \left(e_1^2 + \frac{1}{2}\right)|\xi_{b_1}|^2 + i e_1 e_3 (\xi_a \xi_{b_1}^* - \xi_a^* \xi_{b_1})\right] = \exp\left(-\frac{1}{2}\xi_{ab_1}^\dagger V_{ab_1} \xi_{ab_1}\right). \tag{25}$$

Here $\xi_{ab_1}^\dagger = (\xi_a^*, \xi_a, \xi_{b_1}^*, \xi_{b_1})$ is a four-dimensional vector and the matrix V_{ab_1} is a 4×4 covariance matrix for the cavity mode and the first motional mode. The four eigenvalues of the operator $TV_{ab_1}T + (1/2)\mathbf{E}$ are listed as follows:

$$\lambda_{1,2,3,4}^{ab_1} = \pm \frac{\sqrt{\left(e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t} \sinh^2 \frac{\Omega' t}{2} - e_1^2 \pm 1\right)^2 + 4 e_1^2 e_3^2}}{2} + \frac{e_1^2 + e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t} \sinh^2 \frac{\Omega' t}{2} + 1}{2}. \tag{26}$$

It is easily to prove that $\lambda_{1,2,3,4}^{ab_1} \geq 0$ when $t \geq 0$ for arbitrary values of r_1 and r_2, so there is no entanglement between the cavity mode a and the motional mode b_1 of the first ion. This is because when the interaction between the cavity mode and the first motional mode is via linearly mixing processes, the entanglement between these two modes under this interaction requires that the input state be nonclassical [23,27], but here the chosen initial state of these two modes is classical, so no entanglement happens between the cavity mode and the motional mode of the first ion.

If we trace out the motional mode of the first ion, then the characteristic function for the coupling system between the cavity field and the motional mode of the second ion is expressed as

$$\chi(\xi_a, \xi_{b_2}, t) = \exp\left[-\left(\frac{1}{2} + e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t} \sinh^2 \frac{\Omega' t}{2}\right)|\xi_a|^2 - \left(e_2^2 - \frac{1}{2}\right)|\xi_{b_1}|^2 + i e_2 e_3 (\xi_a \xi_{b_2} - \xi_a^* \xi_{b_2}^*)\right] = \exp\left(-\frac{1}{2}\xi_{ab_2}^\dagger V_{ab_1} \xi_{ab_2}\right), \tag{27}$$

where $\xi_{ab_2}^\dagger = (\xi_a^*, \xi_a, \xi_{b_2}^*, \xi_{b_2})$ is a four-dimensional vector and the matrix V_{ab_2} is a 4×4 covariance matrix for the cavity mode and the second motional mode. The possible negative eigenvalue of the matrix $TV_{ab_2}T + (1/2)\mathbf{E}$, which is important to determine the entanglement between the cavity mode and the second motional mode, is written as

$$\lambda_- = -\frac{\sqrt{\left(e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t} \sinh^2 \frac{\Omega' t}{2} - e_2^2 + 1\right)^2 + 4 e_2^2 e_3^2}}{2} + \frac{1}{2}\left(e_2^2 + e_3^2 + \frac{\kappa^2}{2\Omega'^2} e^{-\kappa t} \sinh^2 \frac{\Omega' t}{2} - 1\right) = -\frac{\sqrt{(\langle b_2^\dagger b_2\rangle - \langle a^\dagger a\rangle)^2 + 4|\langle ab_2\rangle|}}{2} + \frac{1}{2}(\langle b_2^\dagger b_2\rangle - \langle a^\dagger a\rangle), \tag{28}$$

where $\langle a^\dagger a\rangle = e_2^2 + (\kappa^2/2\Omega'^2) e^{-\kappa t} \sinh^2(\Omega' t/2)$, $\langle b_2^\dagger b_2\rangle = e_2^2 - 1$, and $|\langle ab_2\rangle| = |e_2 e_3|$. Similar to the case for the entanglement between the two motional modes, here the entanglement between the cavity mode and the second motional mode requires the existence of the nonclassical correlation between the cavity mode and the second vibrational mode, i.e., $|\langle ab_2\rangle| > \sqrt{\langle a^\dagger a\rangle\langle b_2^\dagger b_2\rangle}$. However, the nonclassical correlation established by the parametric interaction process is damaged by the direct dissipation of the cavity, therefore this inequality cannot be held at all times and the entanglement between the cavity mode and the second vibrational mode may appear only in some time period.

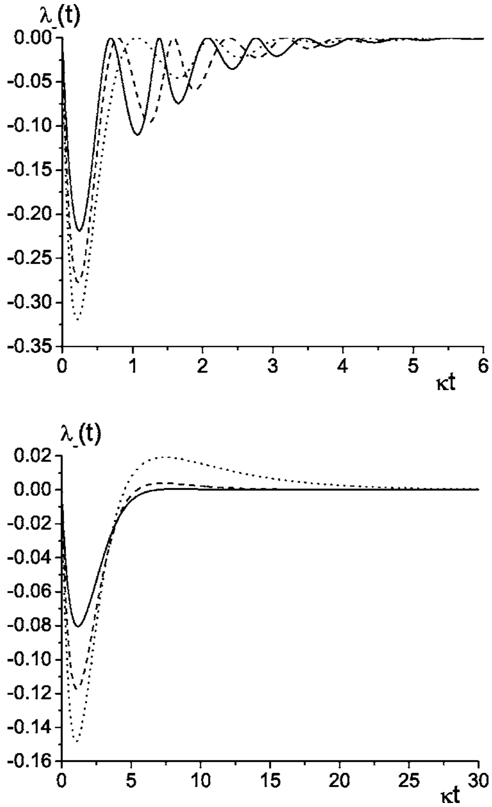

FIG. 3. The dynamic behavior of the negativity $\lambda_{-}(t)$ for the cavity mode and the second vibrational mode in the case of $r_1 > r_2$, where (a) $r_1 = 5\kappa$, $r_2/r_1 = 0.4$ (solid), 0.6 (dashed), and 0.8 (dotted); (b) $r_1 = 0.5\kappa$, $r_2/r_1 = 0.4$ (solid), 0.6 (dashed), and 0.8 (dotted).

For the case of $r_1 > r_2$, the behavior of the system is dominated by the linear-mixing process; the system can reach a steady state at the long time limit because of the cavity dissipation. That is to say, the cavity field is in a vacuum state at $t \to \infty$ so the cavity field is decoupled with the two motional modes. Therefore $\lambda_{-}(\infty)$ equals zero and no entanglement happens between the cavity mode and the motional mode of the second ion. so the entanglement can only appear in the short time range. When $r_1^2 > r_2^2 + \kappa^2/4$, we can see from Fig. 3(a) that $\lambda_{-}(t)$ displays a damped periodic oscillation. At time points $t_m = 2m\pi/\sqrt{4(r_1^2 - r_2^2) - \kappa^2}(m = 0, 1, 2, \ldots)$, we can find that $\langle a^{\dagger}a \rangle = 0$ which means that the cavity field returns back to its initial ground state, so no entanglement exists between the cavity field and the second ion. But if $t \neq t_m$, $\lambda_{-}(t) < 0$ holds in a certain time range, which reflects that the entanglement between the cavity mode and the second motional mode appears. With the increase of the ratio r_2/r_1, at the short time range, the entanglement increases. When $r_2^2 < r_1^2 < r_2^2 + \kappa^2/4$, the degree of the entanglement increases with the ratio r_2/r_1, but the time range for the appearance of the entanglement decreases. When $r_2 > r_1$, that is, the strength of the parametric interaction between the cavity and the second ion is stronger that of the linear-mixing interaction between the cavity and the first ion, the dynamics of the system is dominated by the parametric interaction. However, due to the damage of the cavity dissipation, the nonclassical correlation between the cavity field and the second ion established by the parametric interaction may disappear with the time development. So the entanglement can only happen within the short time range. We can see from Fig. 4 that the entanglement decreases with the increase of the ratio r_1/r_2.

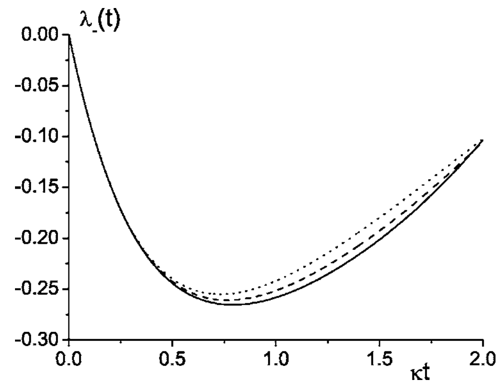

FIG. 4. The time evolution of $\lambda_{-}(t)$ for $r_1 < r_2$, in which $r_2 = \kappa$, $r_1/r_2 = 0.4$ (solid), 0.6 (dashed), and 0.8 (dotted).

VI. CONCLUSION

In conclusion, the dynamical behavior of the entanglement and the squeezing in the system of two ions trapped in a lossy optical cavity is discussed, in which the motional mode of the first ion is coupled to the cavity field via a linear-mixing interaction and the motional mode of the second ion is coupled to the cavity field via an effective parametric interaction, respectively. The time-dependent characteristic function for the two vibrational modes and the cavity mode are solved analytically. It is found that the motional state of the two ions develop in a robust two-mode mixed Gaussian entangled state and the two-mode squeezing of phonons appears when the two coupling constants between the cavity mode and the motional modes obey $r_1 > r_2$. When $r_1 < r_2$, only the entanglement between the two motional modes appears but the two-mode squeezing does not exist. There is no entanglement between the cavity mode and the motional mode of the first ion because the interaction between the first ion and the cavity field is via linear-mixing processes. For $r_1 > r_2$, due to the cavity decay, the cavity field evolves into a vacuum state in the long time limit so that the entanglement between the cavity mode and the motional mode of the second ion can only happen periodically in the small time range. And for $r_1 < r_2$, the entanglement can also take place in the small time range.

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