Fermionic atoms in optical superlattices

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Fermionic atoms in an optical *superlattice* can realize a very peculiar Anderson lattice model in which impurities interact with each other through a *discretized* set of delocalized levels. We show that under these finite-size features strongly correlated phases appear. By tuning the parameters of the superlattice two different phases could be observed: a Kondo-singlet phase in which each impurity forms a singlet with the Fermi level of its neighboring conducting islands, and a magnetic phase in which long range magnetic order is established between the impurities mediated by the intermediate islands. The interplay between these two phenomena depends on the parity of the number of particles per discretized set. We show how Kondo-induced resonances of measurable size can be observed through the atomic interference pattern.

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Cold atoms in optical lattices are capturing a lot of attention from both experimental [1,2] and theoretical sides [3–9]. This interest is highly motivated by the possibility of investigating the domain of strongly correlated phenomena, the interaction effects (typically small in free space) being enhanced due to the periodic confinement. As a unique feature of these atomic systems, the full control of the system's parameters allows one to explore several fascinating directions. On the one hand, atoms in optical lattices can be used to provide illuminating and critical insight into models describing strongly correlated systems. For example, quantum phase transitions in both bosonic [3] and fermionic [5] Hubbard models, as well as in spin Hamiltonians [9], can be explored with unprecedented control. On the other hand, in what perhaps is even more challenging, exotic scenarios can be created in which strongly correlated phenomena may occur under novel conditions. A variety of possibilities already accessible experimentally (different lattice topologies created with superpositions of multiple laser beams [10], independent periodic potentials for different internal atomic states [2,4], interactions controlled by Feshbach resonances [11], etc.) can be combined, promising new ways to strongly entangle atomic ensembles.

In this paper we study the physics of fermionic atoms in an optical *superlattice* [10]. We will show that the system realizes an Anderson lattice Hamiltonian (ALH) [12]. The ALH has been extensively studied in the context of strongly correlated electrons [13,14], and is known to capture the physics of a variety of strongly correlated phenomena, from Kondo effect [13] to Ruderman-Kittel-Kasuya-Yosida magnetism [13,14]. Typical condensed matter systems described by Anderson models are metallic or intermetallic compounds with a low concentration of magnetic impurities. The usual scenario is then that of impurities located far from each other, each of them coupled to a *continuum* of delocalized electrons. In an interesting volte-face, atoms in a superlattice naturally realize a quite different situation, allowing us to investigate a very peculiar regime. For realistic experimental situations supersites (that will play the role of impurities) will be separated typically by a small number of lattice sites. Therefore (if, for instance, tunneling is only allowed along one direction) the system realizes an array of impurities connected through small "islands" with a *discretized* set of levels. The situation resembles that of an array of the Kondo boxes theoretically studied in [15], where impurities can now interact with each other through the intermediate "conducting islands." We will show that, in such a situation, finite size effects give rise to strongly correlated phenomena. We explain how to induce and observe the strongly correlated effects we predict by combining several different techniques.

We consider a gas of fermionic atoms embedded in a superlattice of period L with potential depth V_0 for "normal" sites and V_0' for "supersites" (see Fig. 1). We assume that two kinds of atoms are present (generalized spin $\sigma = \uparrow, \downarrow$).

For sufficiently low temperatures atoms will be confined to the lowest Bloch band of the superlattice and the system can be described by an ALH of the form

$$H_{\mathrm{ALH}} = -t \sum_{\langle \ell,\ell' \rangle \sigma} c^{\dagger}_{\ell\sigma} c_{\ell'\sigma} + U \sum_{\ell} n_{\ell\uparrow} n_{\ell\downarrow} - \Delta\epsilon \sum_{s\sigma} n^{f}_{s\sigma} + U_f \sum_{s} n^{f}_{s\uparrow} n^{f}_{s\downarrow} + V \sum_{\langle \ell,s \rangle \sigma} (f^{\dagger}_{s\sigma} c_{\ell\sigma} + \mathrm{H.c.}), \quad (1)$$

where $c_{\ell\sigma}$, $f_{s\sigma}$ are fermionic operators that annihilate an atom

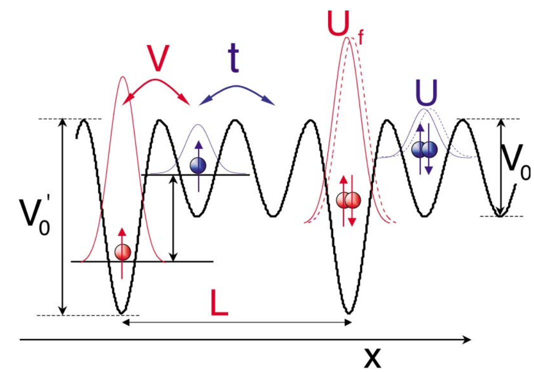

FIG. 1. (Color online) AHM for atoms in a superlattice (see text).

with spin state σ on normal site ℓ and supersite s, respectively, and $n_{\ell\sigma}=c^{\dagger}_{\ell\sigma}c_{\ell\sigma}$, $n^{f}_{s\sigma}=f^{\dagger}_{s\sigma}f_{s\sigma}$.[1]

The Hamiltonian (1) has been extensively studied in the context of strongly correlated electrons. It is well known that in the regime in which U_f, $\Delta\epsilon \gg t$, $V \gg U$, the so-called Kondo regime [16], strongly correlated effects appear. Within this regime the low-energy physics of Hamiltonian (1) can be described by an effective model in which the f-atoms degrees of freedom are represented by localized spins, the well-known Kondo lattice model (KLM) [13,16],

$$H_{\mathrm{KLM}} = -t\sum_{\langle \ell,\ell'\rangle\sigma} c^{\dagger}_{\ell\sigma}c_{\ell'\sigma} + J\sum_{s} \mathbf{S}^{f}_{s}\cdot\mathbf{S}^{c}_{s}, \qquad (2)$$

where $\mathbf{S}^{f}_{s}=\frac{1}{2}\sum_{\sigma,\sigma'}\tau_{\sigma,\sigma'}f^{\dagger}_{s\sigma}f_{s\sigma'}$ are localized spins and $\mathbf{S}^{c}_{s}=\frac{1}{2}\sum_{\sigma,\sigma'}\tau_{\sigma,\sigma'}d^{\dagger}_{s\sigma}d_{s\sigma'}$ with $d_{s\sigma}=\sum_{\langle s,\ell\rangle}c_{\ell\sigma}$, τ being the vector of Pauli matrices. The exchange interaction $J=2V^2/\Delta\epsilon$ is antiferromagnetic, and though typically very small ($J \ll t$) in condensed matter systems, is the source of interesting many-body effects. The KLM has been studied typically in two different situations [16]. (1) A single localized spin weakly coupled to a continuum, which is the usual Kondo problem [13,14]; and (2) a Kondo lattice with typically one impurity per conduction electron [16].

The atomic system that we have under consideration realizes a situation which is essentially different from these two typical condensed matter cases, the finite size effects playing here a dominant role. Let us assume that tunneling is only allowed along one direction (potential barriers have been made very high in the other directions) and that the parameters in Eq. (1) fulfill the conditions to be in the Kondo regime. The system realizes then a one-dimensional (1D) Kondo lattice in which impurities interact with each other through a discretized set of levels. A new characteristic energy scale appears, namely the separation between levels of an island, Δ. For $\nu=1/2$ ($\nu=M/N$, N being the number of particles and M the total number of sites), the separation of the Fermi level and the next excitation is $\Delta=2t\sin(\pi/L)$, which is finite in our case. In addition, the ratio t/J (as we show later) can be varied, so that discussion of both the strong and weak coupling limits is experimentally relevant in this case.

Strong coupling regime: $J \gg \Delta$. Kondo screening dominates the physics of the problem. Since tunneling is very small, impurities are basically disconnected from each other and singlet formation occurs independently for each of them. In this limit we can use a generalization of the variational wave function of Varma and Yafet [17] for the ground state:

$$|\Psi\rangle = \prod_{s}\left(\beta + \sum_{k}\beta_k(f^{\dagger}_{s\uparrow}A_{sk\uparrow} + f^{\dagger}_{s\downarrow}A_{sk\downarrow})|FS\rangle_s\right), \qquad (3)$$

where $A_{sk\sigma}=\sqrt{(2/L)}\sum_{\langle s,\ell\sigma\rangle}\sin(k\ell)c_{\ell\sigma}$, $k=\pi n/L$, $n=1,\ldots,L-1$, and $|FS\rangle_s=\Pi^{k_F}_{k,\sigma}A^{\dagger}_{sk\sigma}$ with $k_F=\pi N/2L$. The variational coefficients β and β_k satisfy $|\beta|^2+2\sum_k|\beta_k|^2=1$. For each f-site, Eq. (3) describes singlet formation with a delocalized state of momentum k with an amplitude given by β_k. Minimization of $E=\langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle$ with respect to the β's yields: $\beta_k/\beta=-(2/\sqrt{L})V\sin k/(T+\Delta_k)$, where T is the gain in energy due to the formation of the singlet, given by

[1]Considering Gaussian wave packets we have: $t=3\eta^2 e^{-(\pi\eta/2)^2}E_R$, $U=a_s k\sqrt{8/\pi}\eta^3$, $U_f=U(V'_0/V_0)^{3/4}$, $\Delta\epsilon=[\eta^4(\gamma^4-1)-3\eta^2(\gamma^2-1)]E_R$, $V=\sqrt{2}\beta e^{-(\pi\eta\beta)^2/4}t$, with $E_R=\hbar^2k^2_{\mathrm{laser}}/2m$, $\gamma=(V'_0/V_0)^{1/4}$, $\beta=\gamma/(1+\gamma^2)^{1/2}$.

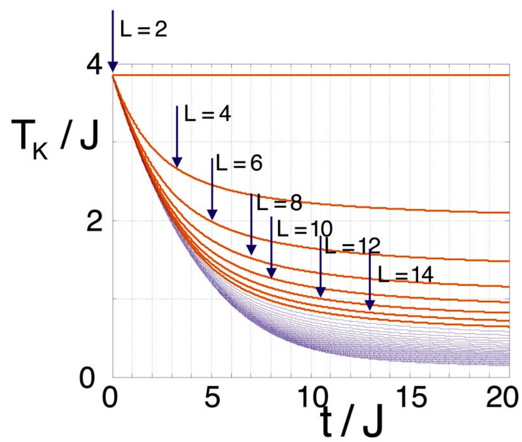

FIG. 2. (Color online) Energy gain, T, as a function of J/t. Different curves correspond to increasing values of $L(N=L/2)$. For realistic cases (in orange), the arrows mark points at which $T=2\Delta$, to the right of which expression (4) is not valid.

$$T = \Delta\epsilon - \epsilon(k_F) + \frac{16V^2}{L}\sum_{k}^{k_F}\frac{\sin^2 k}{T+\Delta_k}. \qquad (4)$$

Here, $\Delta_k=\epsilon(k_F)-\epsilon(k)$, $\epsilon(k)=-2t\cos k$. This gain in energy defines a characteristic temperature below which a singlet is formed and the system is strongly correlated. Note that under realistic conditions we will have $L<10$ and the sum in Eq. (4) will be a discrete sum with a few ($\sim L/2$ for $\nu=1/2$) terms, so that T is always analytic in J. In particular, T does not go to zero with t/J (Fig. 2), but remains $\sim J$ due to the finite size of the conducting island. When Δ becomes of the order of J the size of the singlets becomes comparable to the separation L between supersites, so that the screening cloud of one impurity starts affecting the next supersite. An interplay between singlet formation (localized in the vicinity of the Fermi level of an island) and magnetism takes place.

Weak coupling regime $\Delta \gg J$. A very different situation corresponds to the regime in which the spacing between energy levels in the conducting islands [$\sim 2t\sin(\pi/L)$] becomes much larger than the temperature $T(\sim J)$. Within this limit atomic orbital degrees of freedom are completely frozen, with excitations above the Fermi level in each of the islands taking part of the problem only as virtual states. Performing adiabatic elimination of these excitations in Hamiltonian (2), we obtain an effective Hamiltonian for the spin degrees of freedom. As an interesting feature the resulting Hamiltonian depends on the parity of the number of particles per conducting island, N_c. This even-odd effect is a clear manifestation of the finite size of the conducting islands.

N_c=even. The Fermi level of each island is occupied by two atoms. In this case the only spin degrees of freedom correspond to atoms localized in supersites. An effective spin-spin interaction between neighboring supersites appears, mediated by the Fermi sea in between them. To second order perturbation theory in J,

$$H_{SS} = J_{eff}^{M} \sum_{\langle s,s' \rangle} \mathbf{S}_s^f \cdot \mathbf{S}_{s'}^f, \qquad (5)$$

where $J_{eff}^{M}=(J^2/tL)\sin(k_F)\sin[k_F(L-1)]\sin(k_F+\Delta k)\sin[(k_F+\Delta k)(L-1)]$, and $\Delta k=\pi/L$. We see that magnetism is induced for localized atoms, the ground state being antiferromagnetic or ferromagnetic depending on both L and k_F. We note that due to the characteristic topology imprinted by the superlattice, only neighboring impurities interact with each other.

N_c=odd. The Fermi level of each island is occupied by one atom, whose spin comes into play. The effective Hamiltonian is in this case $H=J_{eff}^{K}\Sigma_s \mathbf{S}_s^f \cdot \mathbf{S}_s^{k_F}$, where $\mathbf{S}_s^{k_F}=\frac{1}{2}\Sigma_{\sigma,\sigma'}\tau_{\sigma,\sigma'}A_{sk_F\sigma}^{\dagger}A_{sk_F\sigma'}$, and $J_{eff}^{K}=4J/L$. Singlet formation at the Fermi level takes place in this case and magnetism is not induced. The ground state consists in this case of singlets formed by each localized atom and the atoms at the Fermi level in neighboring islands. The characteristic temperature is $T=2J_{eff}^{K}$.

Numerical results. To illustrate the predictions above we have numerically diagonalized Hamiltonian (1) for a small 1D superlattice. In Figs. 3 and 4 we plot the spin-spin correlation functions $\langle \mathbf{S}_f \cdot \mathbf{S}_\ell \rangle$ (spatial correlation of a fixed f-spin with the rest of sites in the chain, ℓ), and $\langle \mathbf{S}_f \cdot \mathbf{S}_k \rangle$ (correlation of a fixed f-spin with a delocalized spin with momentum k) for the exact ground state. We consider different cases. (a) $L=4$, $N_c=4$ (Fig. 3). Figure 3(a) shows a clear smooth transition from local Kondo singlet formation to magnetism of localized spins. For small values of t/J each localized f-spin is antiferromagnetically correlated with its next neighboring sites (forming a singlet with them). As t/J increases correlations of each impurity with its neighboring islands disappear at the same time that correlations between next supersites are induced. As stated by Hamiltonian (5) impurities are antiferromagnetically coupled ($J_{eff}^{M}=-J^2/16t$). (b) $L=4$, $N_c=3$ (Fig. 4). Localization of the singlet at the Fermi level of the island appears in this case as t/J increases. In real space Fig. 4(a) shows how singlet-type correlations become more and more extended along the conducting islands next to each impurity, whereas neighboring impurities remain uncorrelated. Delocalization of the singlet becomes more evident in momentum space [Fig. 4(b)], where a resonance, which resembles the Kondo resonance in the continuum, appears at the Fermi level. The characteristic temperature is always of the order of J [see inset of Fig. 4(b)], reaching the limiting value $T \sim 2J_{eff}^{K}=2J$ for $J \ll \Delta$, as predicted above.

We discuss now the experimental realization of the regimes we have studied above. First of all we need $U_f \gg U$. Since $U_f/U \sim V_0'/V_0$ the potential depth of supersites must be very large. As a consequence the energy offset $\Delta\epsilon \sim V_0'-V_0$ will be also large whereas the coupling V will be very small. For a typical value of $V_0'/V_0 \sim 10$ we obtain $J < 10^{-4}t$, which yields extremely small characteristic temperatures, completely unrealistic with current technology. In order to overcome this problem we propose the following scheme. Let us

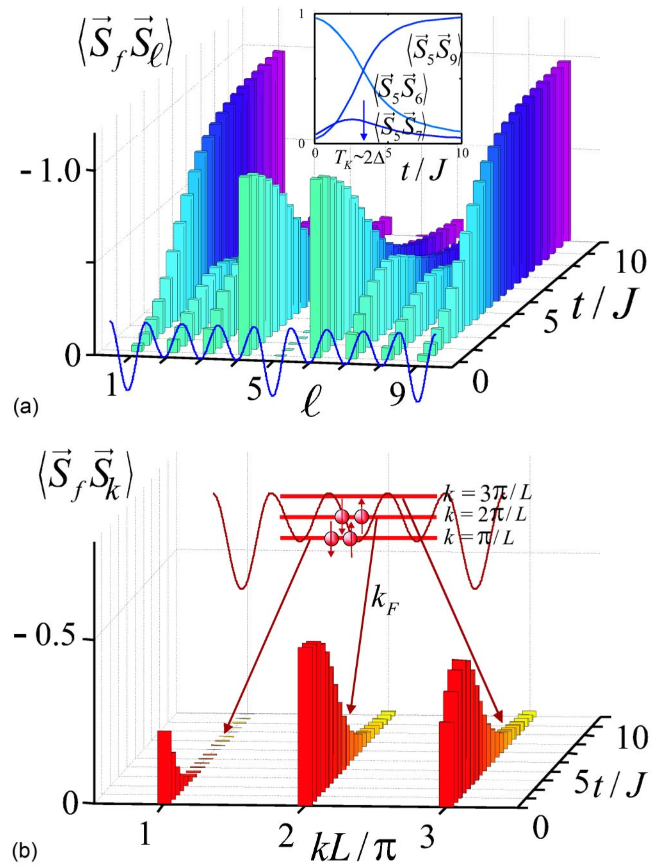

FIG. 3. (Color online) Spin-spin correlation functions of a fixed f-site ($s=5$) with the rest of sites (a) and the k-momentum states of a neighboring island (b), as a function of t/J. Parameters: $M=9$, $N=11(N_c=4)$, $U=0$, $U_f=10\Delta\epsilon$, and $\Delta\epsilon=10J$. In the inset of (a) correlations of supersite 5 with sites 6, 7, and supersite 9 are plotted.

consider an ensemble of ^{6}Li atoms, which have six hyperfine states $|F,M_F\rangle$, with a total spin $F=1/2$, or $3/2$ [18]. We consider a situation in which four of these internal states, $a\uparrow, a\downarrow, b\uparrow, b\downarrow$ are trapped (as shown in Fig. 5). We will also assume that interactions are engineered in such a way that interactions of type a-a and a-b are negligible, whereas b-b are positive and large. The goal is to use a-atoms (noninteracting) as "conducting" c-atoms, and b-atoms (strongly interacting) as localized f-atoms. One possibility is the following. Let us assume we load ^{6}Li atoms in an optical lattice, with $N_\uparrow=N_\downarrow$, $\nu_b=1$, and $\nu_a<1$ (variable). Atoms of type a will delocalize along the lattice forming a Fermi sea, whereas atoms of type b will be localized forming a Mott phase. If the superlattice is now adiabatically turned on supersites will be filled up, each supersite containing two a-atoms with opposite spins and only one b-atom. The idea is to couple a-atoms in normal sites with b-atoms in supersites by using an off-resonant laser. By tuning the intensity Ω and frequency D of the laser, the parameters $\Delta\epsilon$ and V can be tuned to the desired values. The only restriction here is that the laser must not excite other processes, as for example, transitions between a-atoms and b-atoms in normal sites. This sets the condition $(\Omega/\delta)^2 \ll 1$ (see Fig. 5). Taking this into account it is possible

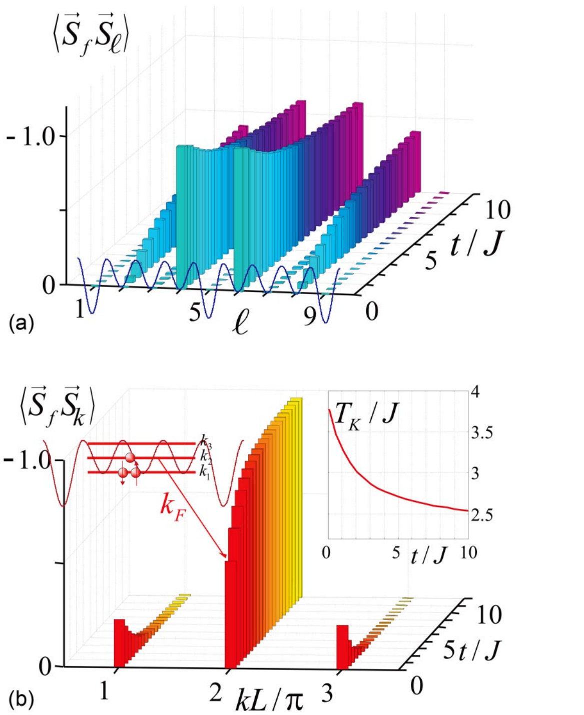

FIG. 4. (Color online) Same as in Fig. 3 with $N=9(N_c=3)$. The Kondo temperature is plotted in the inset of (b).

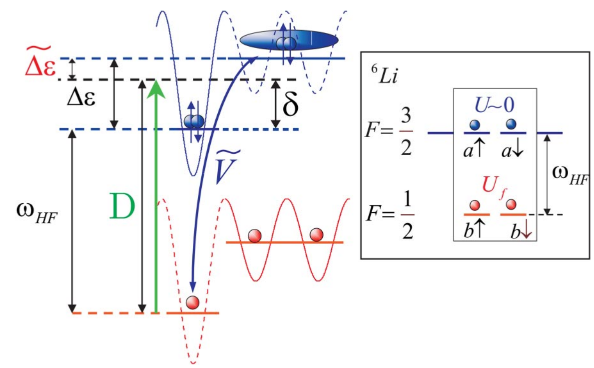

FIG. 5. (Color online) Experimental scheme (see text). The renormalized parameters are $\widetilde{\Delta\epsilon}=\omega_{\rm HF}+\Delta\epsilon-D$ (with $\omega_{\rm HF}$ the hyperfine frequency), and $\widetilde{V}=\Omega V$.

to make $0.5<t/J<20$, with values of $J\sim 0.1E_R$. This yields temperatures $T\sim\mu K$, which are of the order of the ones required to observe superfluidity for fermionic atoms in optical lattices [5]. Finally, we discuss realization of the superlattice. Each of the Fourier components of the superlattice ($e^{ik_n x}$, with $k_n=kn/L$, n integer) can be realized experimentally by two counterpropagating laser beams with wave vector k forming an appropriate angle θ_n. Therefore the number of lasers required for a good realization of the superlattice increases with L (for $V_0'/V_0\sim 4$ a set of $\sim 2L$ lasers is needed); but, as we have shown, systems with $L=3$, 4 already display the strongly correlated phenomena we have predicted.

Experimental observation. One of the most clear manifestations of the localization of the Kondo singlet at the Fermi level is the appearance of Kondo-induced resonances in several physical magnitudes, as, for instance, spin-spin correlation functions. These correlations may be, however, hard to measure in actual experiments, since they involve two particle correlations. Instead, we propose to measure the one-particle correlation function $\langle A^\dagger_{k\sigma} f_\sigma\rangle$. From Eq. (3) we get $\langle A^\dagger_{k\sigma} f_\sigma\rangle \sim \beta\beta_k$, so that the localization of the singlet in the vicinity of the Fermi level (Kondo resonance) will show up in this quantity. This correlation can be obtained in the following way. Both $\langle c^\dagger_{\sigma\ell} c_{\sigma\ell'}\rangle$ and $\langle f^\dagger_{\sigma s} f_{\sigma s'}\rangle$ can be detected in the interference pattern measured after free expansion of a- and b-atoms, respectively. As well, by applying a $\pi/4$ laser pulse between the internal states a and b right before measurement, $\langle (f^\dagger + c^\dagger)(f+c)\rangle$ can be obtained. Combining these three we obtain the desired correlation. Finally, magnetism between impurities can be detected by using spin-dependent Bragg scattering, which will show up the antiferromagnetic or ferromagnetic order of the localized spins.

In conclusion we have shown that fermionic atoms in optical superlattices exhibit strongly correlated phases, from Kondo singlet formation to magnetism of localized spins. Characteristic features of this system are the finite size of the conducting islands coupled to supersites, which strongly influence the competition between Kondo effect and magnetism. These entangled phases could be used for atomic spintronics and spin-filtering.

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