Quantum logic gates for two atoms with a single resonant interaction

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A scheme is presented for realizing two-atom quantum logic gates with a single resonant interaction. The scheme does not use the cavity mode as the data bus and only requires a single resonant interaction of the atoms with a cavity mode. Thus the scheme is very simple and the interaction time is very short, which is important in view of decoherence. Quantum information can be directly transferred from one atom to another atom using this idea. The scheme can also be generalized to the ion trap system.

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I. INTRODUCTION

Recently, much attention has been paid to the quantum computers, which are based on the fundamental quantum-mechanical principle. The new type of machines can solve some problems exponentially faster than the classical computers [1]. Shor has shown that the problem of factorizing a large integer, the basis of the security of many cryptographic systems, can be solved in polynomial time using a quantum computer, while it takes exponentially increasing time to solve on a classical computer [2]. Grover has shown that a search with a quantum computer for an item from a disordered system needs much shorter time than that with a classical computer, and the advantage becomes more and more obvious with the increase of the number of the items in the system [3]. It has been shown that the building blocks of quantum computers are two-quantum-bit (qubit) logic gates [4].

The two-qubit quantum gates have been demonstrated in the cavity QED system [5]. In this experiment, the cavity acts as memories, which stores the information of an electronic system and then transfer back to this electronic system after the conditional dynamics. Thus three resonant atom-cavity interactions are required in order to achieve a quantum phase gate between two atoms. The main obstacle for the implementation of quantum information is the decoherence. We have proposed a scheme for the realizing two-qubit phase gates within a nonresonant cavity [6]. The advantage of the scheme is that the cavity decay is suppressed since the cavity field is only virtually excited during the procedure. It has been shown that the decoherence effect can also be effectively suppressed by employing adiabatic passages and decoherence-free subspace [7–10].

In this paper we present an alternative scheme for the realization of quantum phase gates between two atoms in cavity QED. The scheme does not use the cavity mode as the data bus and only requires a single resonant interaction of the atoms with a cavity mode. Thus the scheme is much simpler than the scheme of Ref. [5] and the required interaction time is shortened. The simplification of the procedure and decrease of operation time are important for suppressing decoherence. The idea can also be used to directly transfer quantum information from one atom to another atom without using the cavity mode as the memory, which is required in the previous experiment [11]. The scheme is applicable to the ion trap system.

The paper is organized as follows. In Sec. II, we propose a method for implementing quantum phase gates between two atoms with a single resonant atom-cavity interaction. In Sec. III, we discuss the phase gate with the decay being considered. In Sec. IV, we show how we can directly transfer quantum information between two atoms. In Sec. V, we generalize the idea to the ion trap system. A summary appears in Sec. VI.

II. TWO-ATOM PHASE GATE

We first consider the interaction of two two-level atoms resonantly interacting with a single-mode cavity. We here assume that the coupling strength of the first atom with the cavity is g_1 and that of the other atom with the cavity is g_2. The Hamiltonian is (assuming $\hbar=1$)

$$H_i = g_1(a^\dagger S_1^- + aS_1^+) + g_2(a^\dagger S_2^- + aS_2^+), \tag{1}$$

where $S_j^+=|e_j\rangle\langle g_j|$ and $S_j^-=|g_j\rangle\langle e_j|$ are the flipping operators, with $|e_j\rangle$ and $|g_j\rangle$ being the excited and ground states of the jth atom, $a^\dagger$ and a are the creation and annihilation operators for the cavity mode.

In order to realize the two-qubit quantum gate we use a third atomic state $|i\rangle$, which is not affected during the atom-cavity interaction. The level configuration of the atoms is shown in Fig. 1. The quantum information of the control qubit is encoded on the states $|e_1\rangle$ and $|g_1\rangle$, while the quantum information of the controlled qubit is encoded on the states $|g_2\rangle$ and $|i_2\rangle$. Assume that the cavity mode is initially in the vacuum state $|0\rangle$. If the two atoms are initially in the state $|e_1\rangle|g_2\rangle$, the evolution of the system is

$$\begin{aligned}|\psi(t)\rangle = N\Bigg[&\frac{1}{E_2}\left(g_1\cos(E_2 t) + \frac{g_2^2}{g_1}\right)|e_1\rangle|g_2\rangle|0\rangle \\ &+ \frac{1}{E_2}g_2[\cos(E_2 t) - 1]|g_1\rangle|e_2\rangle|0\rangle \\ &- i\sin E_2 t|g_1\rangle|g_2\rangle|1\rangle\Bigg],\end{aligned} \tag{2}$$

where

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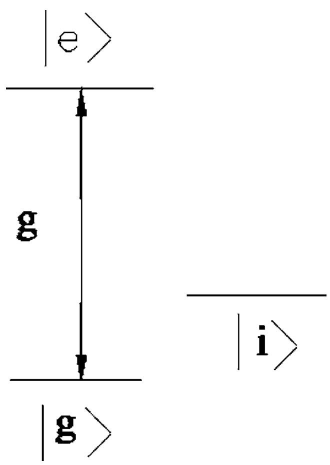

FIG. 1. The level configuration of the atoms. The cavity mode resonantly couples the state $|g\rangle$ to $|e\rangle$. The state $|i\rangle$ is not affected by the cavity mode.

$$E_2=\sqrt{g_1^2+g_2^2}, \tag{3}$$

and

$$N=\frac{g_1}{\sqrt{g_1^2+g_2^2}}. \tag{4}$$

With the choice

$$E_2 t=2\pi, \tag{5}$$

we obtain

$$|\psi(t)\rangle=|e_1\rangle|g_2\rangle|0\rangle. \tag{6}$$

In this case the system returns to the initial state.

If the two atoms are initially in the state $|e_1\rangle|i_2\rangle$, the second atom does not couple with the cavity mode. In this case the evolution of the system is

$$|\psi'(t)\rangle=[\cos(g_1t)|e_1\rangle|0\rangle-i\sin(g_1t)|g_1\rangle|1\rangle]|i_2\rangle. \tag{7}$$

Choosing

$$g_1t=\pi, \tag{8}$$

we have

$$|\psi'(t)\rangle=-|e_1\rangle|i_2\rangle|0\rangle. \tag{9}$$

Thus the system returns to the initial state with an additional phase shift π. We can satisfy Eqs. (5) and (8) by choosing the ratio between the two coupling strengths and interaction time appropriately so that $g_2=\sqrt{3}g_1$ and $t=\pi/g_1$.

On the other hand, the states $|g_1\rangle|g_2\rangle|0\rangle$ and $|g_1\rangle|i_2\rangle|0\rangle$ do not undergo any transition since $H_i|g_1\rangle|g_2\rangle|0\rangle=H_i|g_1\rangle|i_2\rangle|0\rangle=0$. Therefore we have

$$|g_1\rangle|g_2\rangle|0\rangle\to|g_1\rangle|g_2\rangle|0\rangle,$$

$$|g_1\rangle|i_2\rangle|0\rangle\to|g_1\rangle|i_2\rangle|0\rangle,$$

$$|e_1\rangle|g_2\rangle|0\rangle\to|e_1\rangle|g_2\rangle|0\rangle,$$

$$|e_1\rangle|i_2\rangle|0\rangle\to-|e_1\rangle|i_2\rangle|0\rangle. \tag{10}$$

In this way we obtain a quantum phase gate for the two atoms with the cavity mode left in the vacuum state.

III. PHASE GATE INCLUDING DECAY

We now consider the atomic spontaneous emission and cavity decay. Under the condition that no photon is detected either by the spontaneous emission or by the leakage of a photon by the cavity mirrors, the evolution of the system is governed by the conditional Hamiltonian

$$H_{\mathrm{con}}=g_1(a^\dagger S_1^-+aS_1^+)+g_2(a^\dagger S_2^-+aS_2^+)-i\frac{\kappa}{2}a^\dagger a-i\frac{\Gamma}{2}\sum_{j=1}^{2}|e_j\rangle \times\langle e_j|, \tag{11}$$

where κ is the cavity decay rate and Γ is the atomic spontaneous emission rate. If the two atoms are initially in the state $|e_1\rangle|g_2\rangle$, the evolution of the system is

$$\begin{aligned}|\psi_{\mathrm{con}}(t)\rangle=&\frac{1}{g_1^2+g_2^2}\left[g_1^2e^{-(\kappa+\Gamma)t/4}\left(\cos(\alpha t)+\frac{(\kappa-\Gamma)}{4\alpha}\sin(\alpha t)\right)\right.\\&\left.+g_2^2e^{-\Gamma t/2}\right]|e_1\rangle|g_2\rangle|0\rangle+\frac{g_1g_2}{g_1^2+g_2^2}\left[e^{-(\kappa+\Gamma)t/4}\left(\cos(\alpha t)\right.\right.\\&\left.\left.+\frac{(\kappa-\Gamma)}{4\alpha}\sin(\alpha t)\right)-e^{-\Gamma t/2}\right]|g_1\rangle|e_2\rangle|0\rangle\\&-i\frac{g_1}{\alpha}e^{-(\kappa+\Gamma)t/4}\sin(\alpha t)|g_1\rangle|g_2\rangle|1\rangle,\end{aligned} \tag{12}$$

where

$$\alpha=\sqrt{g_1^2+g_2^2-(\kappa-\Gamma)^2/16}. \tag{13}$$

On the other hand, if the two atoms are initially in the state $|e_1\rangle|i_2\rangle$ the evolution of the system is

$$\begin{aligned}|\psi'_{\mathrm{con}}(t)\rangle=&e^{-(\kappa+\Gamma)t/4}\left[\left(\cos(\beta t)+\frac{(\kappa-\Gamma)}{4\beta}\sin(\beta t)\right)|e_1\rangle|g_2\rangle|0\rangle\right.\\&\left.-i\frac{g_1}{\beta}\sin(\beta t)|g_1\rangle|g_2\rangle|1\rangle\right],\end{aligned} \tag{14}$$

where

$$\beta=\sqrt{g_1^2-(\kappa-\Gamma)^2/16}. \tag{15}$$

Set $\kappa=\Gamma=0.1g_1$, $g_2=\sqrt{3}g_1$, and $t=\pi/g_1$. Then we have

$$|g_1\rangle|g_2\rangle|0\rangle\to|g_1\rangle|g_2\rangle|0\rangle,$$

$$|g_1\rangle|i_2\rangle|0\rangle\to|g_1\rangle|i_2\rangle|0\rangle,$$

$$|e_1\rangle|g_2\rangle|0\rangle\to e^{-\pi/20}|e_1\rangle|g_2\rangle|0\rangle,$$

$$|e_1\rangle|i_2\rangle|0\rangle\to-e^{-\pi/20}|e_1\rangle|i_2\rangle|0\rangle. \tag{16}$$

Suppose that the two atoms are initially in the state

$$|\phi_a\rangle = \frac{1}{2}(|g_1\rangle + |e_1\rangle)(|g_2\rangle + |i_2\rangle). \tag{17}$$

The ideal phase gate produces the maximally entangled state

$$|\phi_i\rangle = \frac{1}{2}[|g_1\rangle(|g_2\rangle + |i_2\rangle) + |e_1\rangle(|g_2\rangle - |i_2\rangle)], \tag{18}$$

while the conditional Hamiltonian results in

$$|\phi_{\rm con}\rangle = \sqrt{\frac{1}{2(1+e^{-\pi/10})}}[|g_1\rangle(|g_2\rangle + |i_2\rangle) + e^{-\pi/20}|e_1\rangle(|g_2\rangle - |i_2\rangle)]. \tag{19}$$

The fidelity is given by

$$F = |\langle\phi_{\rm con}|\phi_i\rangle|^2 = 0.994. \tag{20}$$

The probability of success is

$$P = \frac{1}{2}(1 + e^{-\pi/10}) = 0.865. \tag{21}$$

The fidelity and success probability are almost the same as those of the scheme of Ref. [9]. However, the scheme of Ref. [9] requires four laser fields, which are unnecessary in our scheme. Thus our scheme is much simpler. Furthermore, with the fluctuations of the phases and amplitudes of the laser fields being considered, the fidelity of the scheme of Ref. [9] might be lower than that of our scheme.

We briefly address the experimental feasibility of the proposed scheme. The scheme requires that the two atoms have different coupling strengths with the cavity mode. The coupling between the atoms and the cavity depends upon the atomic positions: $g=\Omega e^{-r^2/w^2}$, where Ω is the coupling strength at the cavity center, w is the waist of the cavity mode, and r is the distance between the atom and the cavity center [12]. We can satisfy the condition $g_2=\sqrt{3}g_1$ if we locate the second atom at the center of the cavity and locate the other atom at the position $r=w\ \ln^{1/2}\sqrt{3}$. In recent experiments [13,14], a single and more Cs atoms were trapped in an optical cavity, and the $6S_{1/2}$, $F=4\rightarrow 6P_{3/2}$, $F=4$ transition was coupled to the cavity mode. In this case the level $6S_{1/2}$, $F=3$ can be used for $|i\rangle$. The corresponding coupling strength is $g_2=g=2\pi\times 34$ MHz. The decay rate for the atomic excited state and the cavity mode are $2\pi\times 2.6$ MHz and $2\pi\times 4.1$ MHz, respectively. Therefore the condition $g^2/(\kappa\Gamma)\simeq 100$ can be satisfied.

It should be noted that one needs to reach the Lamb-Dicke regime in order to perform the gate successfully. For the initial state of Eq. (17), in the Lamb-Dicke regime the infidelity caused by the spatial extension of the atomic wave function is about $\Delta\simeq(ka)^2\pi$, where k is the wave vector of the cavity mode and a is the spread of the atomic wave function. Setting $\Delta=0.01$ then we have $a\simeq 0.01\lambda$, where λ is the wavelength of the cavity mode. In the experiment of Ref. [14], the wave length of the cavity field is 852.4 nm. This requires the spatial extension not to be larger than 8.5 nm. However, according to the results of Ref. [14], the spatial extension in the axial direction is 33 nm and that in the direction tranverse to the cavity axis is 3.9 μm. Therefore the atomic location, especially in the tranverse direction, is far from the required Lamb-Dicke regime. One solution for this problem is the combination of ion trapping and cavity QED. In a recent experiment [15], the quadrupole transition $S_{1/2}\rightarrow D_{5/2}$ of a single trapped Ca^+ ion is coupled to a mode of an optical cavity. The spatial precision is about 0.01λ, within the required Lamb-Dicke regime.

IV. QUANTUM INFORMATION TRANSFER

We note the idea can also be used to transfer quantum information between two atoms. Assume that the first atom is initially in the state $\alpha|g_1\rangle+\beta|e_1\rangle$, while the second atom initially in the state $|g_2\rangle$. The cavity mode is initially in the vacuum state. The evolution of the state $|e_1\rangle|g_2\rangle|0\rangle$ is given by Eq. (2). Choosing $E_2t=\pi$ and $g_1=g_2$ we obtain

$$|e_1\rangle|g_2\rangle|0\rangle \rightarrow -|g_1\rangle|e_2\rangle|0\rangle. \tag{22}$$

Performing the trivial single-qubit rotation $|e_2\rangle\rightarrow-|e_2\rangle$ we have

$$|e_1\rangle|g_2\rangle|0\rangle \rightarrow |g_1\rangle|e_2\rangle|0\rangle. \tag{23}$$

Thus we have $\alpha|g_1\rangle+\beta|e_1\rangle|g_2\rangle\rightarrow|g_1\rangle(\alpha|g_2\rangle+\beta|e_2\rangle)$. By this way the quantum information of the first atom is transferred to the second atom. In order to transfer quantum information between two atoms, the experiment reported in Ref. [11] used the cavity mode as the memory. In the experiment, the quantum information of an atom is first transferred to the cavity mode, and then to another atom. Our scheme does not use the cavity as the memory and the quantum information is directly transferred from one atom to another atom.

V. PHASE GATE WITH TRAPPED IONS

We note that the idea can also be used to trapped ion system. Assume that two ions are confined in a linear trap. We assume that the first ion is excited a laser and the other ion is excited by another laser. Both lasers are tuned to the first lower vibrational sideband. In the Lamb-Dicke limit the Hamiltonian is

$$H_i = i\eta\Omega_1(e^{i\phi_1}a^+S_1^- - e^{-i\phi_1}aS_1^+) + i\eta\Omega_2(e^{i\phi_2}a^+S_2^- - e^{-i\phi_2}aS_2^+), \tag{24}$$

where a^+ and a are the creation and annihilation operators of the collective motion of the trapped ions, and Ω_j and ϕ_j ($j=1,2$) are the Rabi frequencies and phases of the laser fields. In the case that $\phi=-\pi/2$ the Hamiltonian has the same form of Eq. (1) with $g_j=\eta\Omega_j$. We again assume that the quantum information of the control qubit is encoded on the states $|e_1\rangle$ and $|g_1\rangle$, while the quantum information of the controlled qubit is encoded on the states $|g_2\rangle$ and $|i_2\rangle$. The vibrational mode is initially cooled to the ground state $|0\rangle$. Under the condition $g_2=\sqrt{3}g_1$ and $g_1t=\pi$, the evolution of the trapped ions is has the form of Eq. (10). The Cirac-Zoller scheme [16], also based on the resonant sideband excitation, uses the vibrational mode as the data bus and requires three interactions. In comparison, the present scheme does not use the vibrational mode as the data bus and only requires one resonant interaction. According to the recent experiment of trapped $^{40}Ca^{+}$ ions [17], it is possible to reach the Lamb-Dicke regime and obtain precise different couplings for the two ions. In the experiment of Ref. [17], $S_{1/2}(m_j=-1/2)$ and $D_{5/2}(m_j=-1/2)$ of $^{40}Ca^{+}$ ions are used for the states $|g\rangle$ and $|e\rangle$, respectively. In this case $S_{1/2}(m_j=1/2)$ can be used for the state $|i\rangle$.

A variety of fast ion trap quantum computation schemes have also been proposed [18–20]. The schemes of Refs. [18,19] require a number of laser-ion interactions, while the present scheme only requires a single interaction. The scheme of Ref. [20] requires two distinct harmonic wells and uses internal-state-selective and time-dependent pushing force, which has not yet been experimentally achieved. The techniques required by the present scheme are within the scope of what can be obtained in the ion trap setup [17].

VI. SUMMARY

In conclusion, we have proposed a simple scheme for implementation of two-qubit quantum phase gates for two atoms. The scheme does not use the cavity mode as the quantum memory and only requires a single resonant interaction of the atoms with a cavity mode. Therefore the scheme is very simple and required interaction time is very short. With the atomic spontaneous emission and cavity decay being included, the gate fidelity and success rate are approximately equal to those of the scheme of Ref. [9]. However, our scheme does not include the laser fields, which are required by Ref. [9]. With the fluctuations of the phases and amplitudes of the laser fields being considered, our scheme might work better than the scheme of Ref. [9]. The scheme can be used to directly transfer quantum information between two atoms. The idea can also be applied to the ion trap system.

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