# Localization of the relative position of two atoms induced by spontaneous emission

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We reexamine the back-action of emitted photons on the wave packet evolution about the relative position of two cold atoms. We show that photon recoil resulting from the spontaneous emission can induce the localization of the relative position of the two atoms through the entanglement between the spatial motion of individual atoms and their emitted photons. The obtained result provides a more realistic model for the analysis of the environment-induced localization of a macroscopic object.

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## I. INTRODUCTION

To understand the transition from the quantum world to the classical world, one of the central issues is to consider how the macroscopic object is localized in a certain spatial domain [1,2]. Superposition and its many-particle versionentanglement-are the essential features of quantum physics that permit macroscopic objects to spread across the whole space, while classical physics is based on a local realism and thus a macroscopic object in the classical world has a welldefined position. The theory of quantum decoherence is a successful description about the quantum-classical transition and it is explained that the loss of the quantum coherence of macroscopic objects is due to their coupling with the environment. Therefore perfect knowledge of the mechanism of decoherence is crucial for understanding the quantumclassical transition since delocalization usually results from quantum coherence. On the other hand, in the science of quantum information, information is mainly processed by using quatnum coherence. Knowing how decoherence destroys the wave nature of matter in the wave-particle duality makes it possible to find decoherence-free states such as the computation space [3].

Theoretical studies in this context have concerned a variety of modeled systems [4-9]. Corresponding experiments have also been done in the last years [10-12] to demonstrate the dynamic process of decoherence and the collapse and revival of the quantum coherence. We studied the phenomenon of the quantum decoherence of a macroscopic object by introducing a novel concept, adiabatic quantum entanglement between collective states [such as that of the center-of-mass (c.m)] and inner states [9,13]. In the adiabatic separation of slow and fast variables of a macroscopic object, its wave function can be written as an entangled state with correlation between adiabatic inner states and quasiclassical motional configuration of the c.m. Since the adiabatic inner states are factorized with respect to the composing parts of the macroscopic object [14], this adiabatic separation can

induce quantum decoherence of the collective motion. This observation thus provides us with a possible solution to the Schrödinger cat paradox at least at the model level. In this sense, the quantum-classical transition is just characterized by localization of the macroscopic object. When this idea was generalized to a triple system (formed by the measured system, the "pointer" of Schrödinger-cat-like matter and its inner variables) similar to that by Zurek [5,6], a consistent approach for quantum measurement was presented by Zhang, Liu, and Sun [8].

The next step is naturally to investigate actual physical systems (such as atoms interacting with the vacuum) demonstrating the essence of such environment-induced decoherence. In this context to focus on the essence of the problem we need not consider a realistic macroscopic object consisting of too many particles. In principle the localization phenomenon of the relative coordinate of two atoms induced by the environment is sufficient to account for the fundamental conception behind such a quantum decoherence problem.

Actually, physicists have studied a more realistic model involving a sequence of external scattering interactions with a system of two particles considering neither the interparticle interaction [15] nor the inner structure of the particle. It shows that the scattering interactions progressively entangle two particles and decohere their relative phase, naturally leading to localization of the particles in relative-position space. A more profound result for the measurement-induced localization has been discovered and the phenomenon of phase entanglement was first defined in Ref. [16]. It is found that there is phase entanglement only in coordinate space, which can interpret the spatial localization phenomenon of atom. They also make a Schmidt-mode analysis of the entanglement between the emitting atom and the emitted photon generated in the process of spontaneous emission and show that the localization of the phonon can be controlled by measuring the atom state [17]. Newly a model of two entangled atoms located inside two spatially separated cavities has also been investigated [18]. It is found that the local decoherence takes an infinite time while the disentanglement due to spontaneous emission may take a finite time. For applications in quantum computing, You has investigated decoherence effects due to motional degrees of freedom of trapped electronically coded atomic or ionic qubits [19].

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In this article, we continue the studies of the environmentinduced decoherence for the motion of the c.m. of a pair of atoms induced by the back-action of light emitted from the atomic inner states. Our investigation will emphasize the reality of the physical model examining the localization of the relative position due to such spontaneous emission. Under the second-order approximation we study the time evolution of the c.m. relevant state in detail by considering the realistic environment formed by the photons in spontaneous emission, which causes the atomic recoils. The corresponding localization phenomenon is characterized by the spatial reduced density matrix in real space as the vanishing of the off-diagonal elements.

This paper is organized as follows. In Sec. II, by neglecting multiphoton processes such as the higher-order approximation we present a simplified model to study the time evolution of the spatial states of the two-atom system under the back-action of emitted photons. In Sec. III the spatial decoherence induced by atomic spontaneous emission is studied by caculation of the reduced density matrix. Section IV demonstrates the localization of a macroscopic object resulting from the spatial decoherence by two simple examples, and finally conclusions are given in Sec. V.

## II. MOTION OF THE c.m. OF TWO ATOMS INFLUENCED BY A VACUUM ELECTROMAGNETIC FIELD

Our system consists of a pair of noninteracting two-level atoms of the same mass *m* and same transition frequency  $\omega_0$ placed in a vacuum electromagnetic field. Here and further on we use blackbody text to denote vector quantities for convenience. The atoms are spatially separated in the positions  $\mathbf{r}_A$  and  $\mathbf{r}_B$ , respectively, and the corresponding momenta are  $\mathbf{p}_A$  and  $\mathbf{p}_B$  (as illustrated in Fig. 1). We denote the c.m. and relative momenta, respectively, by  $\mathbf{P}=\mathbf{p}_A+\mathbf{p}_B$  and  $\mathbf{p}$  $=(\mathbf{p}_A-\mathbf{p}_B)/2$ , and similarly the c.m. and relative positions can be denoted respectively by  $\mathbf{X}$  and  $\mathbf{r}$ .

Under the rotating-wave approximation, the Hamiltonian of our system reads

$$H = \frac{\mathbf{P}^2}{2M} + \frac{\mathbf{p}^2}{2\mu} + \frac{1}{2}\hbar\omega_0 \left(\sigma_z^{(1)} + \sigma_z^{(2)}\right) + \sum_{\mathbf{k}}\hbar\omega_k a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}} + \hbar\sum_{\mathbf{k}}g(\mathbf{k})$$
$$\times \left[\left(\sigma_+^{(1)}e^{i\mathbf{k}\cdot(\mathbf{X}+\mathbf{r}/2)} + \sigma_+^{(2)}e^{i\mathbf{k}\cdot(\mathbf{X}-\mathbf{r}/2)}\right)a_{\mathbf{k}} + \text{H.c.}\right], \tag{1}$$

where M=2m and  $\mu=m/2$ . The atomic transition operators are denoted by  $\sigma_{+}^{(i)}=|e_i\rangle\langle g_i|$  and  $\sigma_{-}^{(i)}=|g_i\rangle\langle e_i|$  (*i*=1,2) with respect to the excited states  $|e\rangle$  and the ground states  $|g\rangle$  of each atom.  $a_{\mathbf{k}}^{\dagger}$  and  $a_{\mathbf{k}}$  are the annihilation and creation operators of the vacuum electromagnetic field mode  $\mathbf{k}$  with frequency  $\omega_k = ck$  ( $k = |\mathbf{k}|$ ). The coupling constant

$$\hbar g(\mathbf{k}) = \sqrt{\frac{\hbar \omega_k}{2\varepsilon_0 V}} \boldsymbol{\varepsilon}_{\mathbf{k}} \cdot \mathbf{d}$$
(2)

depends on the effective mode volume V, the polarization vector of the field vector  $\boldsymbol{\varepsilon}_{\mathbf{k}}$ , and the transition dipole moment of the atom **d**.

For simplicity, we consider the problem in twodimensional space. We assume the atoms move only along



FIG. 1. (a) Schematic illustration of two atoms *A* and *B* in the positions  $\mathbf{r}_A$  and  $\mathbf{r}_B$  interacting with a vacuum electromagnetic field coherently. (b) The vacuum light scattering in two-dimensional space: the atoms move only along the *x* axis while the photon can be emitted along the angle  $\varphi$  between the wave vector  $\mathbf{k}$  of the emitted photon and the *y* axis. The momentum kick given by the emitted photon to the atom is  $p_{\varphi} = \hbar k \sin \varphi$ .

the *x* axis while photons can be emitted along any direction into the field. We also suppose  $\varphi$  is the angle between the wave vector **k** of the emitted photon and the *y* axis. The momentum kick given by the emitted photon to the atom is  $p_{\varphi}=\hbar k \sin \varphi$ . At *t*=0 the relative momentum of the atoms is undefined and its state is a linear combination (superposition) of the eigenstates  $|p\rangle$  of the relative momentum operator **p**. The c.m. momentum state is  $|P\rangle$  which is the eigenstate of c.m. momentum operator **P** corresponding to the eigenvalue *P* and the two atoms are both in excited states initially. The initial wave function of the system can be written as a product state,

$$|\Psi(0)\rangle = |P\rangle \otimes \int d^3 p C_p |p\rangle \otimes |e_1\rangle \otimes |e_2\rangle \otimes |0\rangle, \quad (3)$$

where  $C_p$  is the distribution function corresponding to the relative momentum eigenstate  $|p\rangle$  and satisfies the normalization condition  $\int_{-\infty}^{\infty} |C_p|^2 dp = 1$ , and  $|e_1\rangle$  and  $|e_2\rangle$  represent the excited states of atoms A and B, respectively.  $|0\rangle$  means the vacuum state of the electromagnetic field. The time evolution of the state  $|\Psi(t)\rangle$  is described by the Schrödinger equation.

Under the second-order approximation about the weak coupling characterized by  $g(\mathbf{k})$ , the state vector  $|\Psi(t)\rangle$  can be calculated as

$$\begin{split} |\Psi(t)\rangle &= \exp(-i\omega_{0}t) \Bigg[ \int dp A(p,t) |P,p,e_{1},e_{2},0\rangle \\ &+ \sum_{\mathbf{k}} \int dp B_{\mathbf{k},p}(t) \Bigg[ \left| P - p_{\varphi},p - \frac{1}{2}p_{\varphi},g_{1},e_{2},1_{\mathbf{k}} \right\rangle \\ &+ \left| P - p_{\varphi},p + \frac{1}{2}p_{\varphi},e_{1},g_{2},1_{\mathbf{k}} \right\rangle \Bigg] \\ &+ \sum_{\mathbf{k},\mathbf{k}'} \int dp D_{\mathbf{k},\mathbf{k}',p}(t) |P - p_{\varphi} - p_{\varphi}'\rangle \\ &\otimes \left| p - \frac{1}{2}(p_{\varphi} - p_{\varphi}') \right\rangle \otimes |g_{1},g_{2}\rangle \otimes |1_{\mathbf{k}}1_{\mathbf{k}'}\rangle \Bigg], \qquad (4) \end{split}$$

with  $p'_{\varphi} = \hbar k' \sin \varphi'$  and  $\int dp$  roughly denotes the definite integral of  $\int_{-\infty}^{\infty} dp$ . We notice that  $\exp(-i\omega_0 t)$  is a common phase factor, and the first term in Eq. (4) means that both atoms are in the excited states while the field is in the state of vacuum. The second term denotes one of the atoms decaying to the ground state  $|g_i\rangle$  (*i*=1,2) from the excited states  $|e_i\rangle$  with a photon of momentum  $\hbar \mathbf{k}$  emitted simultaneously. The last term describes the situation when both atoms jump down to the ground states emitting two photons with momenta  $\hbar \mathbf{k}$  and  $\hbar \mathbf{k}'$ , respectively.

The time-dependent coefficients A(p,t),  $B_{\mathbf{k},p}(t)$ , and  $D_{\mathbf{k},\mathbf{k}',p}(t)$  can be calculated by directly solving the Schrödinger equation. The obtained system of equations is

$$\dot{A}_p + i(\omega_A - \omega_0)A_p = -2i\sum_{\mathbf{k}} B_{\mathbf{k},p}(t)g(\mathbf{k}), \qquad (5)$$

$$\dot{B}_{\mathbf{k},p} + [i\omega_B(\mathbf{k}) - \omega_0]B_{\mathbf{k},p} = -iA_pg(\mathbf{k}) - i\sum_{\mathbf{k}'} D_{\mathbf{k},\mathbf{k}',p}g(\mathbf{k}'),$$
(6)

and

$$\dot{D}_{\mathbf{k},\mathbf{k}',p} + [i\omega_D(\mathbf{k},\mathbf{k}') - \omega_0]D_{\mathbf{k},\mathbf{k}',p} = -iB_{\mathbf{k},p}g(\mathbf{k}') - iB_{\mathbf{k}',p}g(\mathbf{k}),$$
(7)

where the coefficients

$$\hbar \omega_A = P^2 / 2M + p^2 / 2\mu + \hbar \omega_0, \qquad (8)$$

$$\hbar\omega_B(\mathbf{k}) = \frac{(P - p_{\varphi})^2}{2M} + \frac{\left(p - \frac{1}{2}p_{\varphi}\right)^2}{2\mu} + \hbar\omega_k \tag{9}$$

describe the scattering processes with photon recoil while

$$\hbar \omega_{D}(\mathbf{k}, \mathbf{k}') = \frac{(P - p_{\varphi} - p_{\varphi}')^{2}}{2M} + \frac{\left(p - \frac{1}{2}p_{\varphi} + \frac{1}{2}p_{\varphi}'\right)^{2}}{2\mu} + \hbar \omega_{k} + \hbar \omega_{k'} - \hbar \omega_{0}$$
(10)

means that two-photon scattering will induce the momentum transfer. Notice that in the above calculation we have ignored the higher-order multiphoton processes.

We perform the Laplace transformation on Eqs. (5)–(7) with the initial conditions  $A(p,0)=C_p$ ,  $B_{\mathbf{k},p}(0)=0$ , and

 $D_{\mathbf{k},\mathbf{k}',p}(0)=0$ . The explicit solutions to  $A_p(p,t)$ ,  $B_{\mathbf{k},p}(t)$ , and  $D_{\mathbf{k},\mathbf{k}',p}(t)$  can be obtained in the Weisskopf-Wigner approximation [20] (see the Appendix for detailed calculations). For the purpose of this paper we need not write them down here for arbitrary time *t*. In the limit  $t \to \infty$ , we have  $A_p(\infty) \to 0$ ,  $B_{\mathbf{k},p}(\infty) \to 0$ , and

$$D_{\mathbf{k},\mathbf{k}',p}(\infty) \to \frac{g(\mathbf{k}')g(\mathbf{k})C_p}{i[\omega_B(\mathbf{k}) - \omega_D(\mathbf{k},\mathbf{k}')] + \frac{\Gamma}{2}} \times \frac{e^{-i[\omega_D(\mathbf{k},\mathbf{k}') - \omega_0]t}}{i[\omega_B(\mathbf{k}') - \omega_D(\mathbf{k},\mathbf{k}')] + \frac{\Gamma}{2}}, \qquad (11)$$

where  $\Gamma$  is the decay rate of an atom from state  $|e\rangle$  to state  $|g\rangle$ . Neglecting the small recoil energies, we have

$$D_{\mathbf{k},\mathbf{k}',p}(\infty) \to \frac{g(\mathbf{k}')g(\mathbf{k})C_p}{i\left[\omega_0 - \omega_{k'} - \left(\frac{p}{2\mu} - \frac{P}{M}\right)\frac{p'_{\varphi}}{\hbar}\right] + \frac{\Gamma}{2}} \times \frac{e^{-i[\omega_D(\mathbf{k},\mathbf{k}') - \omega_0]t}}{i\left[\omega_0 - \omega_k - \left(\frac{p}{2\mu} - \frac{P}{M}\right)\frac{p_{\varphi}}{\hbar}\right] + \frac{\Gamma}{2}}.$$
 (12)

In general the c.m. momentum of a hot atom is very large and so its momentum exchange with the electromagnetic field can be neglected. In this sense the electromagnetic field does not influence its c.m. state nearly. However, it is not the case for the ultracold atoms because their c.m. momenta are very small. The influence of the interaction between the atoms and electromagnetic field on the spatial motion of the atoms becomes very important. Therefore how the spatial states of the ultracold atoms are affected by the electromagnetic field is a crucial issue. In the following section, we will go on to study how spontaneous emission affects the distribution of the atomic relative position.

#### III. SPATIAL DECOHERENCE INDUCED BY INCOHERENT SPONTANEOUS EMISSION

According to the above analysis, after a sufficiently long time  $t \ge 1/\Gamma$ , the state of the system becomes

$$\begin{split} |\Psi\rangle &\to \sum_{\mathbf{k},\mathbf{k}'} \int dp D_{\mathbf{k},\mathbf{k}',p}(\infty) |P - p_{\varphi} - p_{\varphi}'\rangle \otimes \left| p - \frac{1}{2} (p_{\varphi} - p_{\varphi}') \right\rangle \\ &\otimes |g_{1},g_{2}\rangle \otimes |1_{\mathbf{k}} 1_{\mathbf{k}'}\rangle. \end{split}$$
(13)

Supposing that the modes of the field are closely spaced in the frequency domain, we can replace  $\Sigma_{\mathbf{k},\mathbf{k}'}$  by the integral of

$$\frac{V^2}{(2\pi)^4} \int_0^\infty k dk \int_0^{2\pi} d\varphi \int_0^\infty k' dk' \int_0^{2\pi} d\varphi' \,.$$

Considering that the velocity of a realistic atom is far smaller than the light velocity in vacuum, we can further simplify Eq. (13) in the representation of the c.m. relative coordinates (*X* and *x*):

$$\begin{split} |\Psi\rangle &= N \int d\xi e^{(i/\hbar)px} e^{-(i/2\hbar)x(p_{\varphi}-p_{\varphi}')} |X,x,g_1,g_2,1_{k,\varphi}1_{k',\varphi'}\rangle \\ &\times \frac{C_p e^{(i/\hbar)PX} e^{-(i/\hbar)X(p_{\varphi}'+p_{\varphi})} e^{-i[\omega_D(\mathbf{k},\mathbf{k}')-\omega_0]t}}{\left[i(\omega_0-\omega_{k'})+\frac{\Gamma}{2}\right] \left[i(\omega_0-\omega_{k})+\frac{\Gamma}{2}\right]}, \end{split}$$
(14)

where  $\int d\xi$  roughly denotes the definite multi-integral of

$$\int_0^\infty dk \int_0^{2\pi} d\varphi \int_0^\infty dk' \int_0^{2\pi} d\varphi' \int_{-\infty}^\infty dx \int_{-\infty}^\infty dX \int_{-\infty}^\infty dp$$

and N is a normalization factor including the slowly varying  $g(\mathbf{k}')$  and  $g(\mathbf{k})$ .

Tracing over the electromagnetic field, the inner states of the atoms, and the c.m. motion, one can obtain the reduced density matrix

$$\rho(x,x',t) = N' \int d\varphi \, d\varphi' \, e^{-i(\omega_0/2c)(x-x')\sin\varphi} e^{i(\omega_0/2c)(x-x')\sin\varphi'} \\ \times \psi \left( x + \frac{\hbar \omega_0 t(\sin\varphi + \sin\varphi')}{2\mu c}, t \right) \\ \times \psi^* \left( x' + \frac{\hbar \omega_0 t(\sin\varphi + \sin\varphi')}{2\mu c}, t \right) \\ \approx N' \psi(x,t) \psi^*(x',t) J_0 \left[ \frac{\omega_0}{2c}(x-x') \right]^2, \quad (15)$$

where N' is the normalization factor and

$$\psi(x,t) = \int_{-\infty}^{\infty} C_p \exp\left[\frac{i}{\hbar}\left(px - \frac{p^2}{2\mu}t\right)\right] dp.$$
(16)

Considering that the term of  $\hbar \omega_0 t(\sin \varphi + \sin \varphi')/(2\mu c)$  in Eq. (15) means the small offset induced by atomic spontaneous emission of the relative position between the two atoms, we have expanded  $\psi(x+\hbar\omega_0 t(\sin \varphi + \sin \varphi')/(2\mu c), t)$  around x to the first-order approximation and also supposed  $\omega_0 \ge \Gamma$ .  $J_0(z)$  is a Bessel function of the first kind. Here we have used the following integral formula (*a* is real number):

$$\int_{0}^{2\pi} \cos[a \sin(z)] dz = 2\pi J_0(a).$$

Now we can define the decoherence factor F(x, x') as

$$F(x,x') = J_0^2 \left(\frac{\pi}{\lambda}(x-x')\right).$$
(17)

Here  $\lambda = 2\pi c / \omega_0$  is the wavelength of atomic radiation. The elements of the reduced density matrix can be rewritten as

$$\rho(x, x', t) = N' \psi(x, t) \psi^*(x', t) F(x, x').$$
(18)

In Fig. 2, we draw the schematic curve of F(x,x'). It is illustrated that the off-diagonal elements of the reduced density matrix vanish when  $t \ge 1/\Gamma$  and thus the quantum coherence of the system is lost. And the diagonal elements of the reduced density matrix are suppressed with the ultimate breadth  $\lambda/(\pi e)$ . The result also shows that the quantum interference becomes more clear as the wavelength of the pho-



FIG. 2. Schematics of the decoherence factor F(x,x') (the unit of x and x' is  $\lambda/\pi$ ). When  $(\pi/\lambda)(x-x') < e^{-1}$ , there exists a perfect quantum interference. When  $F(x,x') > e^{-3}$ , the quantum coherence disappears.

ton emitted becomes larger or the the distance of the atoms becomes smaller.

## IV. FROM DECOHERENCE TO LOCALIZATION OF A MACROSCOPIC OBJECT

Now we take a simple example to illustrate how the above-discussed decoherence can result in the localization of a macroscopic object. We take the initial state as a superposition,

$$\Psi(0) = \psi(x) = \frac{1}{\sqrt{2}} [G_{-}(x) + G_{+}(x)], \qquad (19)$$

of two Gaussian wave packets

$$G_{\pm}(x) = \frac{1}{\sqrt[4]{2\pi d^2}} \exp\left[-\frac{(x\pm a)^2}{4d^2}\right].$$
 (20)

As pointed out in Ref. [1], models with this initial state may arise in the double-slit experiment. Now we study the dynamical evolution of the wave packets when  $t \ge 1/\Gamma$ . In the coordinate picture the elements of the corresponding reduced density matrix can be expressed as

$$\rho(x,x',t) = N' \psi(x,t) \psi^*(x',t) F(x,x')$$
  
=  $N' J_0 \left(\frac{\pi}{\lambda} (x-x')\right)^2 \{G_-(x,t) G_-(x',t)$   
+  $G_-(x,t) G_+(x',t) + G_+(x,t) G_-(x',t)$   
+  $G_+(x,t) G_+(x',t)\},$  (21)

where

$$G_{\pm}(x,t) = \frac{1}{(2\pi)^{1/4} \sqrt{d + it\hbar/(2\mu d)}} \\ \times \exp\left(-\frac{\left(1 - \frac{i\hbar}{2\mu d^2}t\right)(x \pm a)^2}{4d^2 + \frac{t^2\hbar^2}{\mu^2 d^2}}\right).$$
(22)

In fact, the spreading speed of the wave packets may be faster than the speed of the decay of the atoms when the



FIG. 3. Schematics of the evolution of the system state at three different times  $t_1=100/\Gamma$ ,  $t_2=200/\Gamma$ , and  $t_3=1000/\Gamma$  (the unit of *x* and *x'* is  $\lambda/\pi$ ). (a1), (b1), and (c1) demonstrate the state evolution when there is no spontaneous emission while (a2), (b2), and (c2) represent the case when spontaneous emission exists. (c1) and (c2) correspond to the case when the two packets have overlapped. These schematics illustrate that the quantum coherence of the system is lost in the presence of spontaneous emission and thus the spreading wave packets are suppressed.

distance of the two atoms is far smaller because the spreading speed is fast in this case. The wave packets have overlapped before the atoms decay. We will discuss this case in the following and here we suppose the distance of the atoms is large enough so that we can say the two packets have not overlapped when we study it at some time  $t \ge 1/\Gamma$ . We give the schematics of the evolution of the reduced density matrix at three different times and in two cases in Fig. 3: one is when there is no spontaneous emission and the other is when there exists spontaneous emission. It demonstrates the evolution of the localization of the two initial Gaussian wave packets.

Actually, about ten years ago [21], we studied generally a quite simple dynamical problem: the motion of the wave packet for a "free" particle of one dimension in present in a dissipative environment. An interesting result is that the dissipation suppresses the spreading of the wave packet if the breadth of the initial wave packet is so wide that the effect of Brownian motion can be ignored. However, for the case with dissipation, there appears to be a significant difference about the wave packet spreading. This suppression of the wave packet spreading by dissipation possibly provides a mechanism to localize the macroscopic object. It might be of interest to note that the finite value of the width of the damped particle wave packet for  $t \rightarrow \infty$  leads to exactly the same final value for the uncertainty product of the damped free particle,

also found by Schuch using a nonlinear Schrödinger equation [22]. In the following, we will demonstrate such a localization in our present realistic model.

We take the initial state as a narrow Gaussian wave packet  $G_+(x-a)$  for the relative position representation of the twoatom system. Obviously the narrowness of the wave packet implies that the two-atom system is initially in localization. If spontaneous emission did not exist, the Gaussian wave packet would spread into the full space infinitely and the localization of the wave packet is lost during the evolution of the system. Its breadth increases to infinity while its height decreases from its initial value to zero. In presence of the spontaneous emission, we calculate the time evolution of

$$G_{+}(x-a,t=0) = \int_{-\infty}^{\infty} C_{p} e^{i(p/\hbar)x} \, dp, \qquad (23)$$

$$C_{p} = \frac{2d^{2}}{\pi\sqrt{2\pi}}e^{-(d^{2}/\hbar)p^{2}}.$$
 (24)

According to Eqs. (16) and (18), the reduced density matrix at time  $t \ge 1/\Gamma$  is

$$\rho(x,x',t) = N'G_{+}(x-a,t)G_{+}^{*}(x'-a,t)F(x,x'), \quad (25)$$

where

$$G_{+}(x-a,t) = \frac{1}{(2\pi)^{1/4}\sqrt{d+it\hbar/(2\mu d)}} \\ \times \exp\left(-\frac{\left(1-\frac{i\hbar}{2\mu d^{2}}t\right)x^{2}}{4d^{2}+\frac{t^{2}\hbar^{2}}{\mu^{2}d^{2}}}\right).$$
(26)

According to Eq. (25), we can conclude that the breadth of the spreading wave packet is suppressed as can also be seen in Fig. 4. We give the schematics of the evolution of the wave packet at three different times and in two cases: one is when there is no spontaneous emission and the other is when there exists spontaneous emission. From Eq. (25) and Fig. 4, we can see that the wave packet spreading is suppressed and the ultimate breadth is related to the wavelength of the atomic radiation. The longer is the atomic radiative wave length, the wider is the breadth of the ultimate wave packet.

#### **V. CONCLUSIONS**

In summary, we have investigated the atomic spontaneous-emission-induced quantum decoherence phenomenon in association with the localization of the relative position of a two-atom system. The spontaneous emission or interaction with the vacuum electromagnetic field may be a fundamental process destroying quantum effects in macroscopic objects. By analyzing two simple examples, we demonstrate how spontaneous emission suppresses the spreading wave packet and thus localizes a macroscopic object.

In forthcoming research along this direction we will deal with a more realistic macroscopic object consisting of interacting particles. In this practical case the interaction between



FIG. 4. Schematics of the suppression of a wave packet spreading (the unit of x and x' is  $10\lambda/\pi$ ). (a1), (b1), and (c1) correspond to the evolution of the single wave packet at three different instances when there is no spontaneous emission and (a2), (b2), and (c2) represent the cases when the spontaneous emissions exist. Here  $t_1=2/\Gamma$ ,  $t_2=3/\Gamma$ , and  $t_3=5/\Gamma$ . They demonstrate how the atomic spontaneous emission suppresses the spreading of the wave packet.

the c.m. variable and inner degrees of freedom will also serve as another source of quantum decoherence of the c.m. motion together with the environment of a vacuum electromagnetic field.

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# APPENDIX: SOLUTIONS OF THE SCHRÖDINGER EQUATION

In this appendix we give the calculations on solving the system of equations consisting of Eqs. (5)–(7). We take the Laplace transformation of Eqs. (5)–(7) and obtain

$$[L+i(\omega_A-\omega_0)]A(p,L) = -2i\sum_{\mathbf{k}} B_{\mathbf{k},p}(L)g(\mathbf{k}) + C_p,$$
(A1)

$$\{L + i[\omega_B(\mathbf{k}) - \omega_0]\}B_{\mathbf{k},p}(L)$$
  
=  $-iA(p,L)g(\mathbf{k}) - i\sum_{\mathbf{k}} D_{\mathbf{k},\mathbf{k}',p}(L)g(\mathbf{k}'),$  (A2)

$$\{L + i[\omega_D(\mathbf{k}, \mathbf{k}') - \omega_0]\}D_{\mathbf{k}, \mathbf{k}', p}(L)$$
$$= -iB_{\mathbf{k}, p}(L)g(\mathbf{k}') - iB_{\mathbf{k}, p}(L)g(\mathbf{k}).$$
(A3)

From Eq. (A3), we have

$$D_{\mathbf{k},\mathbf{k}',p}(L) = \frac{-iB_{\mathbf{k},p}(L)g(\mathbf{k}') - iB_{\mathbf{k},p}(L)g(\mathbf{k})}{L + i[\omega_D(\mathbf{k},\mathbf{k}') - \omega_0]}.$$
 (A4)

Substituting Eq. (A4) into Eq. (A2), we can rewrite Eq. (A2) as

$$L + i[\omega_{B}(\mathbf{k}) - \omega_{0}] B_{\mathbf{k},p}(L)$$

$$= -iA(p,L)g(\mathbf{k}) - B_{\mathbf{k},p}(L) \sum_{\mathbf{k}'} \frac{g^{2}(\mathbf{k}')}{L + i[\omega_{D}(\mathbf{k},\mathbf{k}') - \omega_{0}]}$$

$$- \sum_{\mathbf{k}'} \frac{B_{\mathbf{k}',p}(L)g(\mathbf{k}')g(\mathbf{k})}{L + i[\omega_{D}(\mathbf{k},\mathbf{k}') - \omega_{0}]}.$$
(A5)

Since we have ignored the higher-order multiphoton processes, we can omit the last term on the right of Eq. (A5). Then we obtain

$$B_{\mathbf{k},p}(L) = \frac{-iA(p,L)g(\mathbf{k})}{L + i(\omega_B(\mathbf{k}) - \omega_0) + \sum_{\mathbf{k}'} \frac{g^2(\mathbf{k}')}{L + i(\omega_D(\mathbf{k},\mathbf{k}') - \omega_0)}}.$$
(A6)

According to the Weisskopf-Wigner approximation [20], we can obtain

$$\frac{\Gamma}{2} + i\Delta\omega = \sum_{\mathbf{k}'} \frac{g^2(\mathbf{k}')}{L + i[\omega_D(\mathbf{k}, \mathbf{k}') - \omega_0]}$$

where  $\Gamma = \omega_0^2 |\mathbf{d}|^2 / (4\varepsilon_0 \hbar c^2)$  is the decay rate of an atom from state  $|e\rangle$  to state  $|g\rangle$  and  $\Delta\omega$  is the Lamb shift which is omitted in our following calculations since it can be merged into the transition frequency  $\omega_0$ . Equation (A6) can be simplified as

$$B_{\mathbf{k},p}(L) = \frac{-iA(p,L)g(\mathbf{k})}{L + i[\omega_B(\mathbf{k}) - \omega_0] + \frac{\Gamma}{2}}.$$
 (A7)

Submitting Eq. (A7) into Eq. (A1), we have

$$A(p,L) = \frac{C_p}{L + i(\omega_A - \omega_0) + 2\sum_{\mathbf{k}} \frac{g^2(\mathbf{k})}{L + i[\omega_B(\mathbf{k}) - \omega_0] + \frac{\Gamma}{2}}}.$$
(A8)

In the Weisskopf-Wigner approximation, we can also obtain

$$\frac{\Gamma}{2} + i\Delta\omega = \sum_{\mathbf{k}} \frac{g^2(\mathbf{k})}{L + i[\omega_B(\mathbf{k}) - \omega_0] + \frac{\Gamma}{2}}$$

So Eq. (A8) can be written as

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$$A(p,L) = \frac{C_p}{L + i(\omega_A - \omega_0) + \Gamma}.$$
 (A9)

Combining Eqs. (A4), (A7), and (A9), we can obtain

$$B_{\mathbf{k},p}(L) = \frac{-iC_p g(\mathbf{k})}{L + i[\omega_B(\mathbf{k}) - \omega_0] + \frac{\Gamma}{2}} \frac{1}{L + i(\omega_A - \omega_0) + \Gamma},$$

(A10)

$$D_{\mathbf{k},\mathbf{k}',p}(L) = \frac{-g(\mathbf{k}')g(\mathbf{k})C_p}{L+i[\omega_D(\mathbf{k},\mathbf{k}')-\omega_0]} \frac{1}{L+i(\omega_A-\omega_0)+\Gamma} \\ \times \left(\frac{1}{L+i[\omega_B(\mathbf{k})-\omega_0]+\frac{\Gamma}{2}} + \frac{1}{L+i[\omega_B(\mathbf{k}')-\omega_0]+\frac{\Gamma}{2}}\right).$$
(A11)

Taking the inverse Laplace transformation of the above three equations, we obtain the solutions to Eqs. (5)-(7):

$$A(p,t) = C_p e^{-\Gamma t} e^{-i(\omega_A - \omega_0)t},$$
(A12)

$$B_{\mathbf{k},p}(t) = -ig(\mathbf{k})C_p \frac{e^{i\omega_0 - (\Gamma/2)t}(e^{-i\omega_B t} - e^{-i\omega_A t})}{i[\omega_A - \omega_B(\mathbf{k})] + \frac{\Gamma}{2}},$$
(A13)

$$D_{\mathbf{k},\mathbf{k}',p}(t) = \frac{g(\mathbf{k}')g(\mathbf{k})C_{p}}{i[\omega_{A} - \omega_{D}(\mathbf{k},\mathbf{k}')] + \Gamma} \left[ \frac{e^{-i[\omega_{D}(\mathbf{k},\mathbf{k}') - \omega_{0}]t}}{i[\omega_{B}(\mathbf{k}) - \omega_{D}(\mathbf{k},\mathbf{k}')] + \frac{\Gamma}{2}} - \frac{e^{-i(\omega_{A} - \omega_{0})t - \Gamma t}}{i[\omega_{B}(\mathbf{k}) - \omega_{A}] - \frac{\Gamma}{2}} \right] + \frac{e^{-i[\omega_{B}(\mathbf{k}) - \omega_{0}]t - (\Gamma/2)t}}{\left\{ i[\omega_{D}(\mathbf{k},\mathbf{k}') - \omega_{B}(\mathbf{k})] - \frac{\Gamma}{2} \right\} \left\{ i[\omega_{A} - \omega_{B}(\mathbf{k})] + \frac{\Gamma}{2} \right\}} + \frac{e^{-i[\omega_{D}(\mathbf{k},\mathbf{k}') - \omega_{0}]t}}{i[\omega_{B}(\mathbf{k}') - \omega_{D}(\mathbf{k},\mathbf{k}')] + \frac{\Gamma}{2}} - \frac{e^{-i(\omega_{A} - \omega_{0})t - \Gamma t}}{i[\omega_{B}(\mathbf{k}') - \omega_{D}(\mathbf{k},\mathbf{k}')] + \frac{\Gamma}{2}} - \frac{e^{-i(\omega_{A} - \omega_{0})t - \Gamma t}}{i[\omega_{B}(\mathbf{k}') - \omega_{A}] - \frac{\Gamma}{2}} + \frac{e^{-i[\omega_{B}(\mathbf{k}) - \omega_{0}]t - (\Gamma/2)t}}{\left\{ i[\omega_{D}(\mathbf{k},\mathbf{k}') - \omega_{B}(\mathbf{k}')] - \frac{\Gamma}{2} \right\} \left\{ i[\omega_{A} - \omega_{B}(\mathbf{k}')] + \frac{\Gamma}{2} \right\}} \right].$$
(A14)

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