Dependence of Berry's phase for atom on a sign of the *g* factor in the rotating magnetic field

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Berry's phase of the atom in the state with a positive or negative *g* factor for partial cycles of a rotating magnetic field was determined free from the dynamical phase shift using a time-domain atom interferometer. The experimental phase shift is in good agreement with the prediction of Berry's phase for partial cycles. It was found that the sense of Berry's phase depends on the sign of the magnetic quantum number, the sense of the rotating magnetic field, and the sign of the *g* factor of the state.

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Since Berry predicted that the wave function of an adiabatically changing quantum-mechanical system acquires topological (geometrical) phases for cyclic evolution of the Hamiltonian in 1984 [1], numerous experiments on Berry's phase have been carried out using polarized lights [2], neutron interferometers [3], nuclear magnetic resonances [4], atom interferometers [5], and so on [6]. Presently, Berry's phase has become well known in many physical systems, not only Berry's proposal for a particle with a spin component of m along the magnetic field whose amplitude is kept constant but whose direction is varied slowly around a circuit. Recently, the topological phase has been imprinted in the atoms of the Bose-Einstein condensate by adiabatically inverting a magnetic bias field and has succeeded in generating vortices in a Bose-Einstein condensate [7].

When a particle with a spin component of m is in a slowly rotating magnetic field around a circuit C, it acquires a geometrical phase factor $\exp\{i\gamma(C)\}$ in addition to the familiar dynamical phase factor. Berry's phase for a particle with a complete turn of the magnetic field around a cone of semiangle θ is given by

$$\gamma(C) = -2\pi m(1-\cos\theta). \tag{1}$$

In a special case in which the magnetic field rotates in the equator plane of $\theta=\pi/2$, the solid angle is reduced to geodesic arcs connecting two points. Then Berry's phase is given by [8,9]

$$\gamma = -m\varphi, \tag{2}$$

where φ is the angle between A and B. Therefore, Berry's phase becomes $-2m\pi$ for a complete turn and $-m\varphi$ for a partial cycle of the angle φ. Consequently, for a partial cycle, it is expected that Berry's phase gains or loses also according to the direction of the rotation of the magnetic field and the sign of the magnetic quantum number. On the other hand, it is well known that Berry's phase does not depend on the magnitude of the *g* factor. However, Tycko demonstrated that it also depends on the sign of the *g* factor using a single crystal in a magnetic field [10]. In atomic systems, there are states with positive *g* factors and negative *g* factors. Therefore, we will observe the sense of Berry's phase reversed depending on the *g* factor of the state.

However, most of the previous measurements were carried out under a complete turn and therefore the sign of Berry's phase has not been taken into consideration. Only the dependences of Berry's phase on the rotation angle for partial cycles have been found by Weinfurter and Badurek using the neutron spin, but the direction of the rotation was not discussed [11]. These dependences could be verified by measuring Berry's phase of atoms which have both states of positive and negative *g* factors, for partial cycles.

In order to verify the dependence of Berry's phase on the direction of the rotation, we attempted to use an atom interferometer. Atom interferometers using the interaction of atoms with light as a beam splitter, which were realized in 1991 [12,13], have become powerful tools for measuring the quantum phases, because they can determine the phase difference between two states with different quantum numbers such as an optical polarization interferometer. One of the authors (A.M.) developed a Mach-Zender-type atom interferometer using copropagating laser beams [14] and thus far we have measured the Aharonov-Casher phase using the space-domain atom interferometer with states of $m=1$ and $m=-1$ [15,16] and the scalar Aharonov-Bohm effect using the time-domain sodium atom interferometer with states of $m=2$ and $m=1$ [17]. We confirmed that the phase shift was in proportion to the variation of the strength of the resultant magnetic field during two light pulses. In that experiment, the phase change was found to be due only to the variation in magnitude of the resultant magnetic field, under the magnetic field which rotated by a certain angle in one direction and then returned to the original position. Therefore, we are very interested in determining how the phase of the state with a magnetic quantum number m changes when the magnetic field with constant amplitude rotates in one direction, whose situation is exactly equivalent to that of Berry's phase. Furthermore, since the two ground hyperfine states of a sodium atom have both a positive *g* factor and a negative *g* factor, it is appropriate to examine the dependence of Berry's phase on the direction of the rotation of the magnetic field.

In the present Brief Report, we demonstrate the measurement of Berry's phase for partial cycles using a time-domain atom interferometer and confirm that the sense of the phase shift for magnetic sublevels of $m=1$ is opposite to that for $m=-1$ for the same *g* factor. Finally, we confirm that Berry's phase depends on the direction of the rotation of the magnetic field and the sign of the *g* factor of the state in the atomic system.

Our strategy for examining the phase shift between states

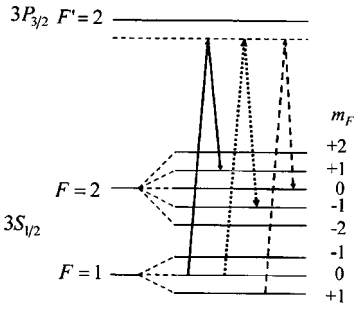

FIG. 1. Zeeman energy levels of the sodium ground hyperfine states and some two-photon Raman transitions connecting them.

with different magnetic quantum numbers under the rotation of the magnetic field was as follows. The time-domain sodium atom interferometer with two copropagating stimulated Raman pulses was used to measure the phase difference between two ground hyperfine states under the rotation of the magnetic field. The relative direction of the phase shift can be observed from the phase of the interference fringes, with some offset phase without any rotation field. Figure 1 shows the energy levels of the two ground hyperfine states of sodium which were used as arms of the interferometer and Raman transitions which connected both states, under the influence of the external magnetic field. The atom in the $S_{1/2}$ and $F=1$ state has a negative g factor of $-1/2$ and the energies of the magnetic substates are in the sequence of -1, 0, 1 at an equal interval from the highest energy. On the other hand, the atom in the $S_{1/2}$ and $F=2$ state has a positive g factor of $1/2$ and the magnetic states are in the reverse sequence, but with the same interval as that of $F=1$. Therefore, for the same direction of the rotation of the magnetic field, the phase differences due to Eq. (2) between the upper state of $|S_{1/2},F=2,m_F=1\rangle$ and the lower state of $|S_{1/2},1,0\rangle$—that is, $\Delta m=1$—are opposite to that between the $|S_{1/2},2,-1\rangle$ and $|S_{1/2},1,0\rangle$—that is, $\Delta m=-1$. However, the former phase difference will be in the same direction as that between $|S_{1/2},2,0\rangle$ and $|S_{1/2},1,1\rangle$—that is, $\Delta m=-1$—for the same direction of rotation, since the g factor has an opposite sign.

The measurements of Berry's phase for the above states were carried out as shown schematically in Fig. 2. The atom interferometer used was almost the same as that used for the

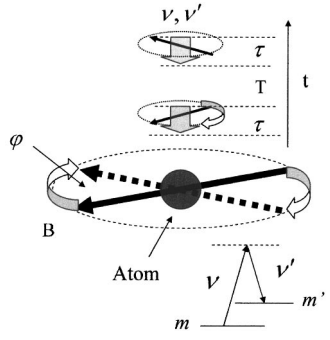

FIG. 2. Schematic diagram of Berry's phase for atoms in the rotating magnetic field B. A time-domain atom interferometer is composed of a couple of circularly polarized two-photon Raman pulses separated by T.

study of the scalar Aharonov-Bohm effect using cold sodium atoms [17]. In this Brief Report, we describe briefly the essential steps in the measurement of Berry's phase. The experimental details will be described in another paper [18].

Sodium atoms of the $F=2$ state were trapped in a magneto-optical trap with a temperature of less than 1 mK. At 3 ms after free expansion of trapped atoms, the sodium atoms were initialized perfectly by optical pumping to the $F=1$ state. Then a quantization magnetic field was applied to sodium atoms, and two-photon Raman laser pulses with a pulse width τ of 20 μs were applied to them with a pulse separation of 80 μs to compose the atom interferometer. The two Raman pulse beams have right-handed circular polarization. The laser frequency was detuned to approximately 500 MHz below the resonance frequency of the state $|S_{1/2},1,0\rangle$ to $|P_{3/2},2,m'\rangle$ and was used as one of the Raman beams. The other frequency of the Raman transition was produced through an electro-optical modulator driven at around 1.77 GHz using a synthesizer, which was tuned to the frequency of the states of $|S_{1/2},1,0\rangle$ to $|S_{1/2},2,1\rangle$ and is equal to the frequency of $|S_{1/2},1,1\rangle$ to $|S_{1/2},2,0\rangle$. The laser beam whose propagation axis is orthogonal to the rotation plane of the magnetic field was used to irradiate the cold sodium atoms. The quantization axis lies on the rotation plane at an arbitrary time and the circular polarization decomposes into two linear π and σ polarizations with the same amplitudes, which are parallel and orthogonal to the quantization axis, respectively. With these two photons, the sodium atom in the $|S_{1/2},1,0\rangle$ is excited to the $|S_{1/2},2,1\rangle$ via $|P_{3/2},2,m'\rangle$ by σ and π polarizations or π and σ polarizations. Then the atom interferometers were composed of the upper state of $|S_{1/2},2,1\rangle$ and the lower state of $|S_{1/2},1,0\rangle$ as two arms of the interferometer.

During two Raman pulses, the quantization magnetic field was rotated while keeping the amplitude of the magnetic field constant with a frequency of f. The rotating magnetic field was produced by two mutually orthogonal pairs of Helmholtz coils which were driven by alternating currents with a relative phase shift of 90° and with the same field strength. In order to maintain a constant magnetic field strength during rotation, each amplitude of the alternating current was determined so as to generate the same Zeeman frequency shift of the resonance, within 1 kHz, for both axes of the Helmholtz coil. The constant magnetic field was typically 0.20 G, which corresponds to a Larmor frequency of $\nu_R=140$ kHz. Therefore, the rotation frequency of the magnetic field was kept below 10 kHz, to fulfill the adiabatic condition. During the pulse separation of T, the resultant magnetic field was rotated with a constant frequency of f (dc$\sim$10 kHz), and then it was rotated by an angle of $2\pi/\{f(T+\tau)\}$.

At the second pulse after T, the sodium atom interacts with the same polarized lights as the first pulse, but they are the same delay in the optical phase from the first ones. However, the optical phases have canceled out by simultaneous stimulated absorption and emission. Therefore, if we measure the population of $|S_{1/2},2\rangle$ after the second pulses, we will see the differences between the phase of $|S_{1/2},2,1\rangle$ including Berry's phase and that of $|S_{1/2},1,0\rangle$, which corresponds to the reference. However, with the same frequencies

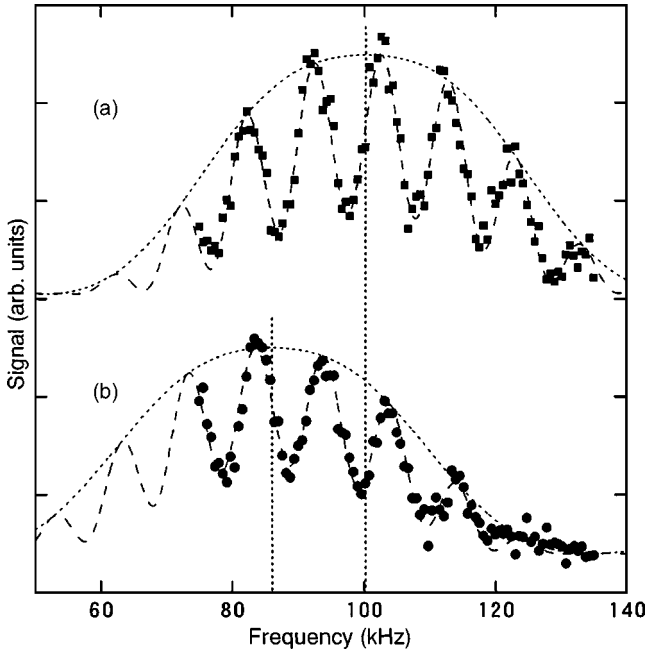

FIG. 3. Ramsey fringes obtained at a pulse width of 20 μs and pulse separation of 80 μs under a static magnetic field (a) and a rotating magnetic field of 5 kHz (b). Dashed lines are fitted curves.

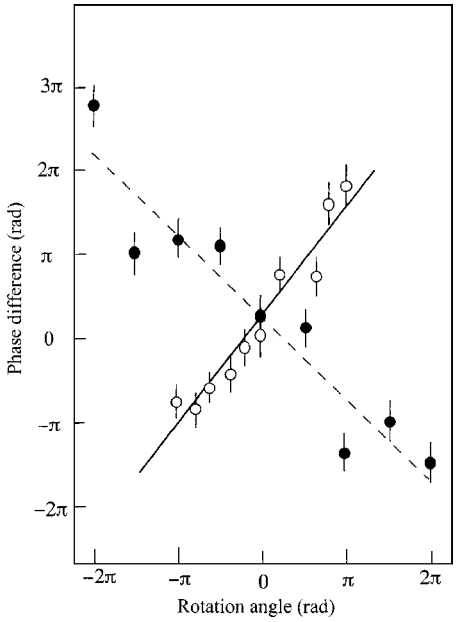

FIG. 4. Observed phase differences as a function of the rotation angle. Solid circle: the phase difference between the states of $F=2$, $m_F=1$ and $F=1$, $m_F=0$, together with that between the states of $F=2$, $m_F=0$ and $F=1$, $m_F=1$. The dashed line is a fitted line with a slope of -0.97 ± 0.13. Open circle: the phase difference between the states of $F=2$, $m_F=-1$ and $F=1$, $m_F=0$ together with that between the states of $F=2$, $m_F=0$ and $F=1$, $m_F=-1$. The solid line is a fitted line with a slope of 1.2 ± 0.2.

and polarization of the laser, the sodium atom in the state $|S_{1/2},1,1\rangle$ is also excited to the $|S_{1/2},2,0\rangle$ via $|P_{3/2},2\rangle$ by π and σ polarizations or σ and π polarizations. Therefore, the atom interferometers with the $|S_{1/2},1,1\rangle$ and $|S_{1/2},2,0\rangle$ are also formed simultaneously. Thus we observe the overlapped interference fringes of two atom interferometers which are the combination of $|S_{1/2},2,1\rangle$ and $|S_{1/2},1,0\rangle$ and the combination of $|S_{1/2},2,0\rangle$ and $|S_{1/2},1,1\rangle$. The magnitudes of Berry's phases for the two interferometers are the same since $|\Delta m|=1$. Therefore, if the Berry's phases of $|S_{1/2},1,1\rangle$ have the same sign as those of $|S_{1/2},2,1\rangle$, the interference fringes must be decreased without a phase shift. Otherwise, the interference fringes will be overlapped cooperatively.

The frequency was swept by a frequency synthesizer in order to observe the Ramsey fringes. The probability of atoms excited to the state of $F=2$ after two pulses was proved by the absorption coefficients of the laser beams which is resonant from $F=2$ in the state $S_{1/2}$ to an excited state $P_{3/2}$. The time sequence was repeated every 10 ms and the excitation probabilities for each run were accumulated and averaged in a computer. In order to measure the phase shift, the population probability of the excited state was monitored as a function of detuning frequency.

Figure 3(a) shows typical Ramsey fringes obtained at a pulse width of 20 μs and pulse separation of 80 μs. Within a spectrum width of 40 kHz, Ramsey fringes with a cycle of 10 kHz are clearly seen with a visibility of 0.42. Figure 3(b) shows the Ramsey resonance under a rotating magnetic field with a frequency of 5 kHz under the same excitation pulses. The visibility is almost the same as that shown in Fig. 3(a) with the same fringe cycles. However, we can find the shift of the center frequency of the envelope and the shift between the two phases at the center frequency. This frequency shift will occur due to the inevitable variation of the magnetic field at the irradiation times of the two pulses. The phase shift observed at the center is Berry's phase. The fact that the visibility under the rotation is the same degree of that without rotation of the magnetic field shows that Berry's phase for $|S_{1/2},1,1\rangle$ with a negative g factor has a sign opposite to that for $|S_{1/2},2,1\rangle$ with a positive g factor for the same rotation. Thus, it is verified that the sign of the Berry's phase depends on the sign of the g factor—that is, the sense of a precession motion like a right-handed or left-handed circular polarization in the optical fiber [2].

The phase shift was measured for several rotation frequencies up to 10 kHz for clockwise rotation and counterclockwise rotation of the magnetic field. The angle of a counterclockwise rotation was assumed to be a negative sign for convenience. At 10 kHz, the magnetic field turns a complete circle of 2π for $T=100$ μs. Figure 4 shows the phase shift as a function of rotation angle for the resonance between the states of $F=1$, $m=0$ and $F=2$, $m=1$. The phase changed along a straight line from a positive phase to a negative phase as the rotation angle varies from negative to positive. It shows that Berry's phase is reversed by reversing the direction of rotation. The slope of the phase shift is -0.97 ± 0.13, which is in agreement with the prediction of Berry's phase for partial cycles given in Eq. (2). Furthermore, the same slope for both rotation directions will confirm that the dynamical phase shift is not included in the phase. In order to test the dependence of the magnetic quantum number, the phase shift for $\Delta m=-1$ between the states of $F=1$, $m=0$ and $F=2$, $m=-1$ was also examined. The result is also shown in Fig. 4. As we expect according to the

theory, the inclination of the phase shift was opposite to that of $\Delta m=1$ and the slope is 1.2 ± 0.2. This result verified clearly the dependence of Berry's phase on the magnetic quantum number.

In conclusion, we demonstrated Berry's phase for magnetic field rotation using a time-domain atom interferometer free from a dynamical phase shift. The results show that the phase is dependent on the magnetic quantum number multiplied by the rotation angle for partial cycles and that the sense of the phase shift depends on the direction of rotation, the sign of the magnetic quantum number, and the sign of the g factor. In order to confirm the dependence of the Berry's phase on the sign of the g factor more clearly, a selection of the magnetic substate is expected. Especially, measurement of the phase difference between the states of $|S_{1/2},2,2\rangle$ and $|S_{1/2},1,1\rangle$ will be of interest, because Berry's phase of the former state will shift to a direction opposite to that of the latter one.

Note added in proof. Recently we succeeded in measuring the phase difference between the states of $|S_{1/2},2,2\rangle$ and $|S_{1/2},1,1\rangle$ for rotation of the magnetic field and found that the slope of the phase shift is -3.0 ± 0.3. This result strongly confirms the dependence of Berry's phase on the sign of the g factor.

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