Two-color ultraslow optical solitons via four-wave mixing in cold-atom media

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We analyze a lifetime-broadened four-state, ladder-type four-wave mixing (FWM) scheme in the context of optical soliton formation. We show that a pulsed probe field and a pulsed FWM field of considerably different frequency can evolve into a pair of matched solitons with the same temporal shape and ultraslow group velocity ($V_g/c\sim 10^{-3}$), i.e., two-color ultraslow optical solitons. In addition, we show regimes where two-color superluminal ($V_g/c<0$) optical soliton propagation may occur.

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There has been significant research activity on wave propagation in highly resonant cold media during recent decades [1–4]. One of the striking features of wave propagation in such a highly resonant medium is the significant reduction of the propagation velocity [5,6] of the optical field involved. Such an ultraslow propagation of an optical field has been shown [7–17] to lead to several propagation effects in the field of fundamental physics and one could envision potential technological impact of the technique in modern optical and telecommunication engineering. In particular, motivated by recent reports using a conventional electromagnetically induced transparency [1] technique to enhance Kerr nonlinearities and to achieve efficient multiwave mixing by channel opening [7–12], we have shown that there exists a class of optical solitons, ultraslow optical solitons, in highly resonant three-state [14] and four-state [17] media. Such well-characterized and distortion-free ultraslow optical wave packets may lead to important applications such as high-fidelity optical buffers, phase shifters [7,8], transmission lines [18], switches [19], routers, wavelength converters [20], and optical gates [21].

In this paper, we describe the interesting phenomenon of *two-color ultraslow optical solitons*, i.e., two ultraslow optical solitons of considerably different carrier frequencies or colors with matched [15,22] temporal shapes, generated via four-wave mixing (FWM) in a highly resonant nonlinear optical medium composed of lifetime-broadened four-state atoms. In this regard, we note that bichromatic and trichromatic solitons [23] as well as multichromatic solitons [24] of matched shape *without* the concept of ultraslow and superluminal propagation have been predicted in stimulated Raman scattering.

We consider the FWM scheme as shown in Fig. 1 in which a pulsed FWM field can be efficiently generated by two strong continuous wave (cw) laser pump fields (B and C) and a weak pulsed probe field (p) illuminating on lifetime-broadened four-state atoms. An experimental candidate for the proposed system is ^{85}Rb atoms (for instance $|0\rangle=|5S_{1/2}\rangle$, $|1\rangle=|5P_{1/2}\rangle$, $|2\rangle=|5D_{3/2}\rangle$, and $|3\rangle=|nP_{3/2}\rangle$ with $n>10$). The respective transitions are $|0\rangle\to|1\rangle$ at 795 nm ($\gamma_1\simeq 5.9$ MHz), $|1\rangle\to|2\rangle$ at 762 nm ($\gamma_2\simeq 0.8$ MHz), and $|2\rangle\to|3\rangle$ at 1.3–1.5 μm ($\gamma_3\simeq 0.09$ MHz), all accessible with diode lasers.

The atomic equations of motion and equations for the probe and four-wave-mixing fields are

$$\frac{\partial A_1}{\partial t}=i(\Delta\omega_1+i\gamma_1)A_1+i\Omega_pA_0+i\Omega_C^*A_2, \tag{1a}$$

$$\frac{\partial A_2}{\partial t}=i(\Delta\omega_2+i\gamma_2)A_2+i\Omega_B^*A_3+i\Omega_CA_1, \tag{1b}$$

$$\frac{\partial A_3}{\partial t}=i(\Delta\omega_3+i\gamma_3)A_3+i\Omega_BA_2+i\Omega_mA_0, \tag{1c}$$

$$\frac{\partial \Omega_p}{\partial z}+\frac{1}{c}\frac{\partial \Omega_p}{\partial t}=i\kappa_{01}A_1A_0^*, \tag{1d}$$

$$\frac{\partial \Omega_m}{\partial z}+\frac{1}{c}\frac{\partial \Omega_m}{\partial t}=i\kappa_{03}A_3A_0^*, \tag{1e}$$

where A_j and γ_j are the probability amplitude and the decay rate of the state $|j\rangle$ ($j=1,2,3$), respectively, the ground state probability $|A_0|^2=1-\Sigma_{j=1}^3|A_j|^2$, $2\Omega_j$ and ω_j ($j=B,C,p,m$) are

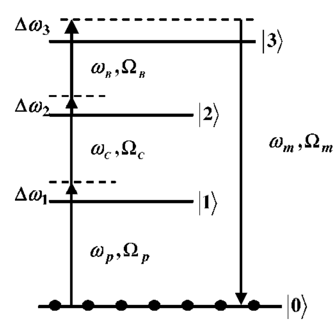

FIG. 1. A lifetime-broadened four-level atomic system that interacts with two continuous wave (cw) laser fields (frequencies ω_B and ω_C, and Rabi frequencies $2\Omega_B$ and $2\Omega_C$) and a weaker pulsed probe field (frequency ω_p and Rabi frequency $2\Omega_p$) to generate a four-wave-mixing field (frequency ω_m and Rabi frequency $2\Omega_m$).

Rabi and optical frequencies of the relevant optical fields, $\Delta\omega_1=\omega_p-\epsilon_1/\hbar$, $\Delta\omega_2=\omega_p+\omega_C-\epsilon_2/\hbar$, $\Delta\omega_3=\omega_p+\omega_C+\omega_B-\epsilon_3/\hbar$ with ϵ_j being the energy of state $|j\rangle$ ($\epsilon_0=0$), and $\kappa_{01(03)}=2N\omega_{p(m)}|D_{01(03)}|^2/(c\hbar)$ with N and D_{0j} being the concentration and dipole moment between states $|0\rangle$ and $|j\rangle$, respectively.

We assume that the two cw pump fields $\Omega_{B,C}$ are strong compared with the other two pulsed fields $\Omega_{p,m}$ ($\Omega_B\sim\Omega_C$, $\epsilon\sim\Omega_{p,m}/\Omega_C\sim\Omega_C^{-1}\partial/\partial t$, and ϵ is a small parameter) and that all the atoms are in their ground states before the probe pulse enters the medium at $t=0$ so that $A_k(t=0)=\delta_{k0}$, $k=0,1,2,3$. Substituting $A_j=\Sigma_k A_j^{(k)}$ with $A_j^{(k)}\sim O(\epsilon^k)$ into Eqs. (1a)–(1e) and noting $A_k^{(0)}=\delta_{k0}$, we then, after some tedious but straightforward manipulation, obtain the following equations for the pulsed fields:

$$\left(i\frac{\partial}{\partial z}+K_+(\hat\omega)\right)\left(i\frac{\partial}{\partial z}+K_-(\hat\omega)\right)\Omega_p =-i\,n^{(2)}\left(2i\frac{\partial}{\partial z}+K_+(0)+K_-(0)\right)\frac{\partial\Omega_p}{\partial z}+O(\epsilon^4), \tag{2a}$$

$$\Omega_m=\left(-i\,D(\hat\omega)\frac{\partial}{\partial z}+\kappa_{01}D_1(\hat\omega)\right)\frac{\Omega_p}{\kappa_{01}\Omega_B^*\Omega_C^*}+O(\epsilon^3), \tag{2b}$$

where $\hat\omega=i\partial/\partial t$ is a differential operator, and

$$K_\pm(\omega)=\frac{\omega}{c}+\frac{-[\kappa_{03}D_3(\omega)+\kappa_{01}D_1(\omega)]\pm\sqrt{G(\omega)}}{2D(\omega)}, \tag{3}$$

$$\begin{aligned}D(\omega)&=|\Omega_C|^2(\omega+\Delta_3)+|\Omega_B|^2(\omega+\Delta_1)\\&\quad-(\omega+\Delta_1)(\omega+\Delta_2)(\omega+\Delta_3),\\ D_1(\omega)&=|\Omega_B|^2-(\omega+\Delta_2)(\omega+\Delta_3),\\ D_3(\omega)&=|\Omega_C|^2-(\omega+\Delta_1)(\omega+\Delta_2),\\ G(\omega)&=[\kappa_{03}D_3(\omega)-\kappa_{01}D_1(\omega)]^2+4\kappa_{03}\kappa_{01}|\Omega_B|^2|\Omega_C|^2,\end{aligned}$$

$\Delta_j=\Delta\omega_j+i\gamma_j$, and $n^{(2)}=\Sigma_{j=1}^3|A_j^{(1)}|^2$ is an $O(\epsilon^2)$ term with

$$A_j^{(1)}=\frac{[\Omega_C(\Omega_B\delta_{j3}-\Delta_3\delta_{j2})-D_j(0)\delta_{j1}]}{D(0)}\Omega_p+\frac{[\Omega_B^*(\Omega_C^*\delta_{j1}-\Delta_1\delta_{j2})-D_j(0)\delta_{j3}]}{D(0)}\Omega_m. \tag{4}$$

Before considering the nonlinear evolution of the pulsed probe and FWM fields according to Eqs. (2a) and (2b), let us first discuss the corresponding linearized results obtained by neglecting its nonlinear term(s) in the right-hand side of Eq. (2a). Obviously there exist two modes ($K_\pm$ modes) described by the linearized dispersion relations $K=K_+(\omega)$ and $K=K_-(\omega)$, respectively [13]. Using $K_\pm(\hat\omega)=K_\pm(0)+\hat\omega/V_{g\pm}+O(\hat\omega^2)$ (the group velocity dispersion term will be taken into consideration later), we readily obtain from Eqs. (2a) and (2b) the pulsed probe and FWM fields,

$$\Omega_k(z,t)=\Omega_k^{(+)}(z,t)+\Omega_k^{(-)}(z,t),\quad k=p,m, \tag{5a}$$

$$\Omega_p^{(\pm)}(z,t)=F_\pm\left(t-\frac{z}{V_{g\pm}}\right)\exp[iK_\pm(0)z], \tag{5b}$$

$$\Omega_m^{(\pm)}(z,t)=W_\pm\Omega_p^{(\pm)}(z,t), \tag{5c}$$

where $V_{g\pm}$ are group velocities determined by $1/V_{g\pm}=[\partial K_\pm(\omega)/\partial\omega]_{\omega=0}$, and

$$W_\pm=\frac{\kappa_{01}D_1(0)-\kappa_{03}D_3(0)\pm\sqrt{G(0)}}{2\kappa_{01}\Omega_B^*\Omega_C^*}, \tag{6a}$$

$$F_\pm(t)=\frac{\pm\Omega_m(0,t)\mp W_\mp\Omega_p(0,t)}{W_+-W_-}. \tag{6b}$$

Equations (5a)–(5c), (6a), and (6b) demonstrate that when the given input fields $\Omega_{p,m}(0,t)$ satisfy the condition $\Omega_m(0,t)/\Omega_p(0,t)=W_s$, leading to $F_{\bar s}(t)\equiv 0$ ($\bar s=-s$ for $s=\pm$, i.e., $\bar s=+$ for $s=-$, and $\bar s=-$ for $s=+$), there exists no excitation for the $K_{\bar s}$ mode and the probe and FWM fields in the atomic medium are $\Omega_p(z,t)=\Omega_p(0,t-z/V_{gs})\exp[i\,K_s(0)z]$ and $\Omega_m(z,t)=W_s\Omega_p(z,t)$. In this paper, we are interested in the situation where the FWM field is generated with no input FWM field, i.e., $\Omega_m(0,t)=0\Rightarrow\Omega_m(0,t)/\Omega_p(0,t)\neq W_\pm$, and hence both K_- and K_+ modes will simultaneously be excited in the atomic medium. However, even in this situation, there exist parameter regimes in which the absorption coefficients $\alpha_\pm=2\,\mathrm{Im}[K_\pm(0)]$ differ significantly from each other, and one of the modes can thus be neglected after a short propagation distance. For instance, in the parameter regime as shown in Fig. 2, the K_- mode decays very quickly and can well be neglected within a millimeter, a very short propagation distance indeed.

Now we are ready to study the nonlinear evolution of the pulsed probe and FWM fields in the situation of no input FWM field, i.e., $\Omega_m(0,t)=0$, but in the parameter regimes in which the K_- mode decays very quickly and can well be neglected after a very short propagation distance as shown in Fig. 2. The pulsed probe field under these conditions has the form $\Omega_p(z,t)=\Omega_p^{(+)}(z,t)+\Omega_p^{(-)}(z,t)\approx\Omega_p^{(+)}(z,t)=F\exp[i\,K_+(0)z]$. Here F is a slowly varying function so that $[i\partial/\partial z+K_+(\hat\omega)][i\partial/\partial z+K_-(\hat\omega)]\Omega_p\simeq\exp[i\,K_+(0)z]\times[K_-(0)-K_+(0)][i\partial/\partial z+K_+(\hat\omega)-K_-(0)]F$. Substituting this form of the probe field into Eqs. (2a) and (2b) and noting that $K_+(\hat\omega)=K_+(0)+\hat\omega/V_g+K_2\hat\omega^2+O(\hat\omega^3)$ and $\hat\omega=i\partial/\partial t$, we readily obtain

$$i\frac{\partial}{\partial\xi}F-K_2\frac{\partial^2}{\partial\eta^2}F=U\,e^{-\alpha\,\xi}|F|^2F, \tag{7a}$$

$$\Omega_m(z,t)/W_+=\Omega_p(z,t)=F\exp[i\,K_+(0)z], \tag{7b}$$

where $\xi=z$, $\eta=t-z/V_g$, $\alpha=2\mathrm{Im}[K_+(0)]$ is the absorption coefficient, $V_g=1/K_1$ and K_2 denote the group velocity and group velocity dispersion, respectively, $K_j=j[\partial^jK_+(\omega)/\partial\omega^j]_{\omega=0}$ ($j=1,2$), $K_+(\omega)$ and W_+ are given by Eqs. (3) and (6a), respectively, and

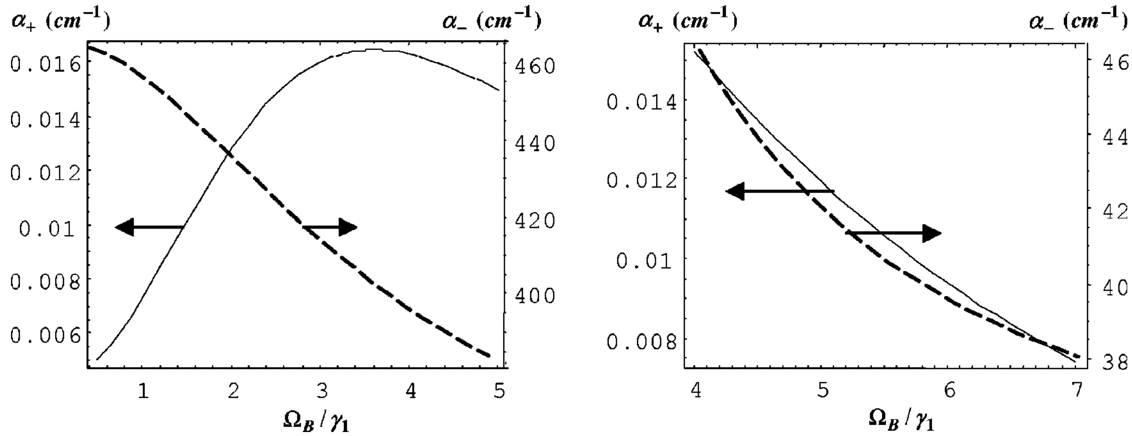

FIG. 2. Absorption coefficients $\alpha_\pm$ versus dimensionless Rabi frequency Ω_B/γ_1 for $\kappa_{01}=100\kappa_{03}=10^9$ cm^{-1} s^{-1}, $\Omega_C=\gamma_1=5.9$ MHz, $\gamma_2/\gamma_1=0.8/5.9$, and $\gamma_3/\gamma_1=0.09/5.9$. The left panel has detunings $\Delta\omega_1=-\gamma_1$, $\Delta\omega_2=-3\gamma_1$, and $\Delta\omega_3=5\gamma_1$, while the right one corresponds to $\Delta\omega_1=-3\gamma_1$, $\Delta\omega_2=-0.6\gamma_1$, and $\Delta\omega_3=10\gamma_1$.

$$U=\left[|A|^2+\frac{|A\Delta_1+1|^2}{|\Omega_C|^2}+\frac{|\Delta_2-D_{30}A|^2}{|\Omega_B|^2|\Omega_C|^2}\right]K_+(0), \qquad (8)$$

with $A=K_+(0)/\kappa_{01}$, $D_{30}=|\Omega_C|^2-\Delta_1\Delta_2$, and $\Delta_j=\Delta\omega_j+i\gamma_j$.

Inspection of Eq. (7a) shows that if a reasonable and realistic set of parameters can be found so that $\exp(-\alpha L)\simeq 1$ (L is the length of the atomic system), $K_2=K_{2r}+iK_{2i}\simeq K_{2r}$, and $U=U_r+iU_i\simeq U_r$, then Eq. (7a) can be reduced to the standard nonlinear Schrödinger equation,

$$i\frac{\partial}{\partial\xi}F-K_{2r}\frac{\partial^2}{\partial\eta^2}F=U_r|F|^2F, \qquad (9)$$

which admits [25–28] solutions describing dark ($K_{2r}U_r<0$) and bright ($K_{2r}U_r>0$) solitons including n-solitons ($n=1,2,3,\ldots$) for dark [26] and bright [27] solitons. The one-soliton and n-soliton ($n\geqslant 2$) are also called the fundamental soliton and the soliton of nth order, respectively.

The fundamental dark soliton [26], fundamental bright soliton, and bright two-soliton (bright soliton of second order) [27,28] of Eq. (9) are given, respectively, by

$$F(\xi,\eta)=F_0\tanh(\eta/\tau)\exp(-i|F_0|^2U_r\xi), \qquad (10)$$

$$F(\xi,\eta)=F_0\mathrm{sech}(\eta/\tau)\exp(-i\xi U_r|F_0|^2/2), \qquad (11)$$

$$F(\xi,\eta)=F_0\frac{4[\cosh(3\eta/\tau)+3\exp(-8\,i\,K_{2r}\xi/\tau^2)\cosh(\eta/\tau)]\exp(-i\,K_{2r}\xi/\tau^2)}{\cosh(4\eta/\tau)+4\cosh(2\eta/\tau)+3\cos(8K_{2r}\xi/\tau^2)}. \qquad (12)$$

The probe and FWM fields relate to these solitons by Eq. (7b), i.e., $\Omega_m(z,t)/W_+=\Omega_p(z,t)=F(\xi,\eta)$. Here the (complex) amplitude F_0 and (real and positive) width τ are arbitrary constants, subject only to the constraint $|\tau F_0|^2=-2K_{2r}/U_r>0$ for Eq. (10), and $|\tau F_0|^2=2K_{2r}/U_r>0$ for Eqs. (11) and (12).

Taking $\kappa_{01}=100\kappa_{03}=10^9$ cm^{-1} s^{-1}, $\Omega_C=\Omega_B=\gamma_1=5.9$ MHz, $\gamma_2/\gamma_1=0.8/5.9$, $\gamma_3/\gamma_1=0.09/5.9$, $\Delta\omega_1=-\gamma_1$, $\Delta\omega_2=-3\gamma_1$, and $\Delta\omega_3=4\gamma_1$, we have $U=U_r+iU_i=(-1.13+0.008\,i)\times 10^{-13}$ s^2/cm $\simeq U_r$, $K_2=K_{2r}+iK_{2i}=(-6.4+0.14\,i)\times 10^{-16}$ s^2/cm $\simeq K_{2r}$, $\alpha=0.0055$/cm, and $V_g/c\simeq 1.96\times 10^{-3}$. These results and Fig. 3 indicate that the standard nonlinear Schrödinger equation (9) admitting ultraslow ($V_g/c\simeq 10^{-3}$) bright solitons ($K_{2r}/U_r>0$) represents a very accurate approximation to Eq. (7a) for the typical transition parameters of ^{85}Rb atoms and appropriately chosen pump lasers. Figure 3 depicts the fundamental bright soliton (upper panels) and the bright soliton of second order (lower panels) for this set of parameters. As can be seen in Fig. 3, these parameters and results show again that the standard nonlinear Schrödinger equation (9) with $K_{2r}/U_r>0$ is well characterized and the formation of bright solitons occurs with negligible probe field attenuation at least for a propagation distance of $\xi=20$ cm for the fundamental bright soliton and the bright two-soliton. This is a remarkable propagation effect in such a highly resonant system.

With $\Omega_B=5\gamma_1$, $\Delta\omega_1=-3\gamma_1$, $\Delta\omega_2=-0.6\gamma_1$, and $\Delta\omega_3=10\gamma_1$, and keeping other parameters unchanged, we have $U=U_r+iU_i=(-9.7+1.7\,i)\times 10^{-16}$ s^2/cm $\simeq U_r$, $K_2=K_{2r}+iK_{2i}=(3.5+0.58\,i)\times 10^{-16}$ s^2/cm $\simeq K_{2r}$, $\alpha=0.012$/cm, and $V_g/c\simeq 4.4\times 10^{-3}$. This indicates that there may also exist

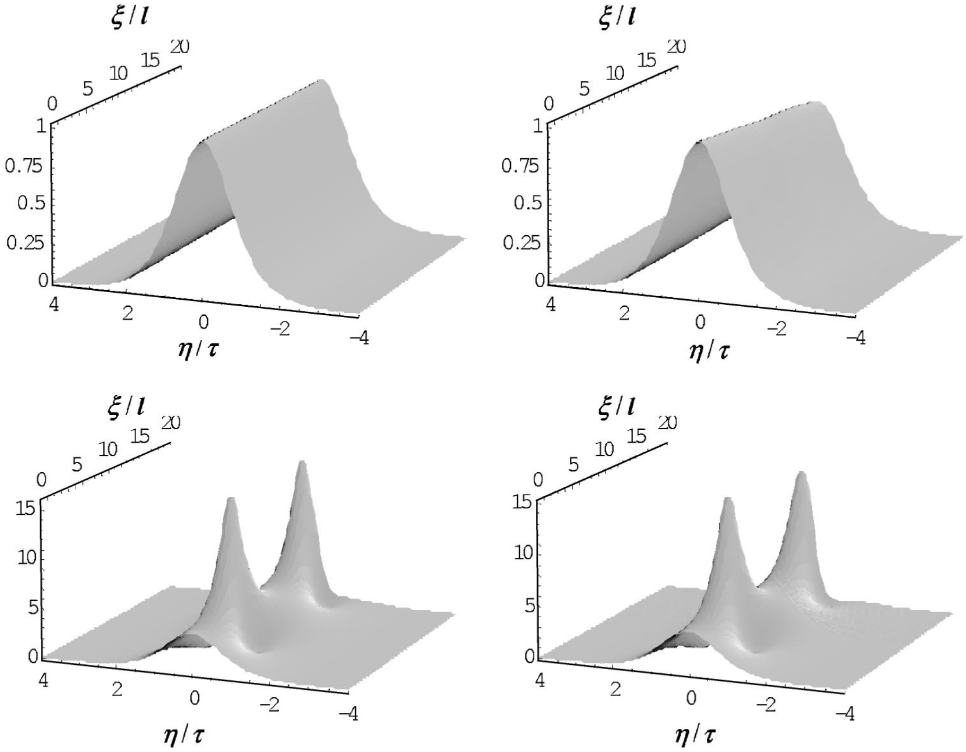

FIG. 3. Surface plots of the relative intensities of probe and FWM fields versus dimensionless time η/τ and distance ξ/l for the fundamental bright soliton (upper panels) and the bright soliton of second order (lower panels). The right panels are $|F/F_0|^2\exp(-\alpha\,\xi)$ with F being the numerical solution to Eq. (7a) while the left ones correspond to $|F/F_0|^2$ with F being the exact solutions (11) and (12), respectively, to the standard nonlinear Schrödinger equation (9). Here $l=1$ cm, $\tau=1\times10^{-7}$ s, and other parameters are given in the main text. Notice that the probe and FWM fields are related to F by Eq. (7b).

ultraslow ($V_g/c\simeq10^{-3}$) optical dark ($K_{2r}/U_r<0$) solitons for the probe and FWM fields as well.

In addition, we point out that solitons can also propagate with a superluminal group velocity ($V_g/c<0$) in certain parameter regimes. For instance, taking $\Delta\omega_1=3.8\gamma_1$, $\Delta\omega_2=0$, and $\Delta\omega_3=13\gamma_1$ and keeping other parameters unchanged, we have $U=(-2.87+1.64\,i)\times10^{-15}$ s^2/cm, $K_2=(-7.7+0.32\,i)\times10^{-14}$ s^2/cm, $\alpha=0.11$/cm, and $V_g/c\simeq-9.4\times10^{-4}$, indicating that the bright solitons ($K_{2r}/U_r>0$) propagate with a superluminal group velocity ($V_g/c\simeq-10^{-3}<0$).

To sum up, we have shown that the initially zero pulsed FWM field (corresponding to $|3\rangle\rightarrow|0\rangle$ transitions at wavelengths $\sim$300 nm) can be generated by the wave mixing of two strong cw pump lasers and a pulsed probe field (corresponding to $|1\rangle\rightarrow|0\rangle$ transitions at wavelenths $\sim$795 nm) in cold Rb atoms, and the coupling of the pulsed probe and pulsed FWM fields via two cw pumps leads to the two modes ($K_\pm$ modes) propagating with different group velocities $V_{g\pm}$ and different absorption coefficients $\alpha_\pm$. We have also shown that for a reasonable and realistic set of parameters, the K_- mode may be neglected after a very short characteristic propagation distance z_c ($z_c<1$ for the typical parameters of Figs. 2 and 3) and hence the pulsed FWM field of the carrier frequency $\sim c/(300$ nm$)$ matches the pulsed probe field of the carrier frequency $\sim c/(795$ nm$)$ in both the temporal profile and ultraslow group velocity $V_g=V_{g+}\sim10^{-3}\,c$. We have further shown that the ultraslow matched pulses of the probe and FWM fields are governed approximately by the standard nonlinear Schrödinger equation that admits both bright and dark solitons for the same realistic set of parameters. In other words, we have revealed the interesting phenomenon of two-color ultraslow optical solitons, i.e., two ultraslow optical solitons of considerably different carrier frequencies or colors with matched temporal shape, generated via the FWM in a highly resonant nonlinear optical medium composed of lifetime-broadened four-state atoms. Finally, we have pointed out the possibility that two-color bright and dark solitons may propagate with superluminal group velocities ($V_g/c<0$) in certain parameter regimes.

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