Electromagnetically induced transparency in *V*-, Λ-, and cascade-type schemes beyond steady-state analysis

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We analyze the electromagnetically induced transparency (EIT) in *V*-, Λ-, and cascade-type schemes in a time-dependent way via the Schrödinger-Maxwell formalism. We derive explicitly the analytical expressions of the space-time dependent probe field, the corresponding phase shift, absorption or amplification, group velocity, and group velocity dispersion for all the three schemes. These simple analytical expressions not only demonstrate explicitly the similarities and essential differences of the three schemes but also provide a convenient basis for investigating how the many-body effects in solids modify the magnitude, spectral shape, and space and time dependence of EIT and EIT-related quantum coherence phenomena.

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I. INTRODUCTION

There has been recently considerable interest in multiwave mixing processes relying on the three-state electromagnetically induced transparency (EIT) [1–6] in both cold atom media [7–41] and semiconductors [42–45]. The motivation of such studies lies in the potential wide range of applications in diverse fields such as high-efficiency generation of short-wave length coherent radiation at pump intensities approaching the single-photon level, nonlinear spectroscopy at very low light intensity, quantum single-photon nonlinear optics and quantum information science [1–45].

In this paper, we shall investigate the three-state EIT in *V*-, Λ-, and cascade-type schemes beyond steady-state analysis. We explicitly provide the *appropriate* transformations of these three schemes to make the Schrödinger-Maxwell formalism suitable for dealing with EIT. This formalism is obviously much simpler than the usually adapted formalism using master equation for density matrix and Maxwell equation(s) and hence we are able to obtain the explicit analytical expression of the pulsed probe field in all the three schemes, which describes its space-time dependence including the corresponding phase shift, absorption or amplification, group velocity and group velocity dispersion.

Our motivations are twofold. First, the previous studies on the three-state EIT usually adapt the steady-state treatment while we want to go one step further here by a time-dependent analysis. Such analysis beyond the steady-state treatment is indispensable for completely describing the situations of the ultrafast probe pulse such as those in semiconductors in which the probe pulse duration τ is usually of the order 10^{-8} s or even much shorter than that [42–45]. As a matter of fact, even in the cold atom gases, the probe pulse duration τ in free space for the three-state EIT in the *V*-type scheme should at least be of the order 10^{-7} s or shorter. Second, we shall derive simple analytical expressions of the atoms' responses and the space-time function of the pulsed probe field including its phase shift, absorption or amplification, group velocity and group velocity dispersion in all the three schemes. In this regard, we note that Boon *et al.* have recently considered numerically the steady-state description of all the three schemes based on density matrix formalism [49]. However, our purpose here is to derive simple analytical expressions which not only clearly demonstrate the similarities and essential differences of all the three schemes but also provide a convenient basis for investigating how the many-body effects [42] in semiconductors modify the magnitude, spectral shape, and space and time dependence of the EIT phenomenon. The paper is organized as follows. In Sec. II, we provide the appropriate transformation for each of the three schemes to derive the explicit analytical expressions of the atom's response to the fields. In Sec. III, we solve the wave equation to obtain the analytical expressions of the pulsed probe field, its phase shift and amplification, the group velocity and the group velocity dispersion. We also discuss the similarities and essential differences of all the three schemes according to these simple analytical expressions. Section IV concludes the paper with some discussions.

II. ATOM RESPONSE

In this section, we provide the appropriate transformation (i.e., the appropriate Hamiltonian H_0 or H_{int}) for each of the *V*-, Λ-, and cascade-type EIT schemes (Fig. 1) to derive the corresponding explicit analytical expressions of the atom response to the fields. The Hamiltonian for these three schemes is ($\hbar = 1$)

$$H = \sum_{j=1}^{3} \epsilon_j |j\rangle\langle j| - (\Omega_p e^{i\theta_p}|3\rangle\langle 1| + \Omega_c e^{i\theta_c}|2\rangle\langle 1| + \text{H.c.}) \quad (V), \tag{1a}$$

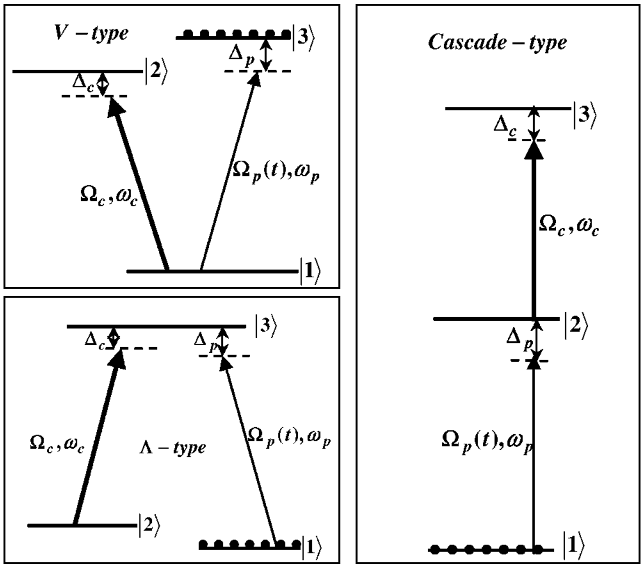

FIG. 1. Schematic of EIT in V-, Λ-, and cascade-type schemes. The strong continuous wave (cw) laser field with frequency ω_c and Rabi frequency $2\Omega_c$, and the weak pulsed probe field has the frequency ω_p and the time-dependent Rabi frequency $2\Omega_p(t)$, respectively. The detunings Δ_p and Δ_c in all the schemes are defined as the corresponding transition frequency minus the corresponding laser frequency; see Eq. (4) for their explicit forms.

$$H = \sum_{j=1}^{3} \epsilon_j |j\rangle\langle j| - (\Omega_p e^{i\theta_p}|3\rangle\langle 1| + \Omega_c e^{i\theta_c}|3\rangle\langle 2| + \text{H.c.}) \quad (\Lambda), \tag{1b}$$

$$H = \sum_{j=1}^{3} \epsilon_j |j\rangle\langle j| - (\Omega_p e^{i\theta_p}|2\rangle\langle 1| + \Omega_c e^{i\theta_c}|3\rangle\langle 2| + \text{H.c.}) \quad (C), \tag{1c}$$

where (C) stands for (cascade), ϵ_j is the energy of state $|j\rangle$, $\theta_n = k_n z - \omega_n t$ $(n=p,c)$, and ω_n, k_n and $2\Omega_n$ are the frequency, wave number and Rabi frequency of the pulsed probe $(n=p)$ or CW control $(n=c)$ laser field, respectively.

Taking $(\hbar=1)$

$$H_0 = (\epsilon_3 - \omega_p)|1\rangle\langle 1| + [\epsilon_3 - (\omega_p - \omega_c)]|2\rangle\langle 2| + \epsilon_3|3\rangle\langle 3| \quad (V), \tag{2a}$$

$$H_0 = \epsilon_1|1\rangle\langle 1| + [\epsilon_1 + (\omega_p - \omega_c)]|2\rangle\langle 2| + (\epsilon_1 + \omega_p)|3\rangle\langle 3| \quad (\Lambda), \tag{2b}$$

$$H_0 = \epsilon_1|1\rangle\langle 1| + (\epsilon_1 + \omega_p)|2\rangle\langle 2| + [\epsilon_1 + (\omega_p + \omega_c)]|3\rangle\langle 3| \quad (C), \tag{2c}$$

we have the interaction Hamiltonian H_{int} in the interaction picture as follows $(\hbar=1)$,

$$H_{int} = -\Delta_p|1\rangle\langle 1| + (\Delta_c - \Delta_p)|2\rangle\langle 2| - (\Omega_p e^{ik_p z}|3\rangle\langle 1| + \Omega_c e^{ik_c z}|2\rangle\langle 1| + \text{H.c.}) \quad (V), \tag{3a}$$

$$H_{int} = (\Delta_p - \Delta_c)|2\rangle\langle 2| + \Delta_p|3\rangle\langle 3| - (\Omega_p e^{ik_p z}|3\rangle\langle 1| + \Omega_c e^{ik_c z}|3\rangle\langle 2| + \text{H.c.}) \quad (\Lambda), \tag{3b}$$

$$H_{int} = \Delta_p|2\rangle\langle 2| + (\Delta_p + \Delta_c)|3\rangle\langle 3| - (\Omega_p e^{ik_p z}|2\rangle\langle 1| + \Omega_c e^{ik_c z}|3\rangle\langle 2| + \text{H.c.}) \quad (C), \tag{3c}$$

where detunings $\Delta_{p,c}$ in all the three schemes are defined as the corresponding transition frequencies minus the corresponding laser frequencies. Specifically, (V-type), or (cascade-type), respectively,

$$\Delta_p = (\epsilon_3 - \epsilon_1)/\hbar - \omega_p \; \Delta_c = (\epsilon_2 - \epsilon_1)/\hbar - \omega_c \quad (V), \tag{4a}$$

$$\Delta_p = (\epsilon_3 - \epsilon_1)/\hbar - \omega_p \; \Delta_c = (\epsilon_3 - \epsilon_2)/\hbar - \omega_c \quad (\Lambda), \tag{4b}$$

$$\Delta_p = (\epsilon_2 - \epsilon_1)/\hbar - \omega_p \; \Delta_c = (\epsilon_3 - \epsilon_2)/\hbar - \omega_c \quad (C). \tag{4c}$$

Defining the atomic state as

$$|\Psi\rangle = e^{-ik_p z}B_1|1\rangle + e^{i(k_c - k_p)z}B_2|2\rangle + B_3|3\rangle \quad (V) \tag{5a}$$

$$= B_1|1\rangle + e^{i(k_p - k_c)z}B_2|2\rangle + e^{ik_p z}B_3|3\rangle \quad (\Lambda) \tag{5b}$$

$$= B_1|1\rangle + e^{ik_p z}B_2|2\rangle + e^{i(k_p + k_c)z}B_3|3\rangle \quad (C), \tag{5c}$$

we then have the atomic equations of motion as

$$\left[\frac{\partial}{\partial t} - i(\Delta_p + i\gamma_1)\right]B_1 = i\Omega_p^* B_3 + i\Omega_c^* B_2 \quad (V),$$

$$\left[\frac{\partial}{\partial t} - i(\Delta_p - \Delta_c + i\gamma_2)\right]B_2 = i\Omega_c B_1 \quad (V),$$

$$\left(\frac{\partial}{\partial t} + \gamma_3\right)B_3 = i\Omega_p B_1 \quad (V), \tag{6a}$$

$$\left(\frac{\partial}{\partial t} + \gamma_1\right)B_1 = i\Omega_p^* B_3 \quad (\Lambda),$$

$$\left[\frac{\partial}{\partial t} - i(\Delta_c - \Delta_p + i\gamma_2)\right]B_2 = i\Omega_c^* B_3 \quad (\Lambda),$$

$$\left[\frac{\partial}{\partial t} + i(\Delta_p - i\gamma_3)\right]B_3 = i\Omega_p B_1 + i\Omega_c B_2 \quad (\Lambda), \tag{6b}$$

$$\left(\frac{\partial}{\partial t} + \gamma_1\right)B_1 = i\Omega_p^* B_2 \quad (C),$$

$$\left[\frac{\partial}{\partial t} + i(\Delta_p - i\gamma_2)\right]B_2 = i\Omega_p B_1 + i\Omega_c^* B_3 \quad (C),$$

$$\left[\frac{\partial}{\partial t} + i(\Delta_c + \Delta_p - i\gamma_3)\right]B_3 = i\Omega_c B_2 \quad (C), \tag{6c}$$

where γ_j is the decay rate of the state $|j\rangle$. Notice that these decay rates sometimes include (or are) the decoherence or relaxation rates.

Just as shown in Fig. 1, we assume that atoms are initially in the excited (ground) state $|3\rangle$ ($|1\rangle$), i.e., taking $B_3(t=0)=1$ $[B_1(t=0)=1]$ for the V-type (for both the Λ- and cascade-type schemes). We further assume the strong pump approximation $|\Omega_c| \gg |\Omega_p|$ and the duration τ of the probe pulse in free space satisfying $\tau\gamma_3 \ll 1$ ($\tau\gamma_1 \ll 1$) for the V-type scheme (for both the Λ- and cascade-type schemes). Consequently, $B_3(t) \simeq \exp(-\gamma_3 t) \simeq 1$ $[B_1(t) \simeq \exp(-\gamma_1 t) \simeq 1]$ for the V-type scheme (for both the Λ- and the cascade-type schemes) under the strong pump approximation within the duration τ. It is pointed out that the constraint $\tau\gamma_3 \ll 1$ is much more stringent than the constraint $\tau\gamma_1 \ll 1$ because the ground state relaxation rate γ_1 is much smaller than the excited state decay rate γ_3 (see Fig. 1). In other words, an ultra fast probe pulse is usually needed to realize EIT in the V-type scheme while it is not necessarily so in the other two schemes. We only need to solve Eq. (6a) [Eqs. (6b) and (6c)] with $B_3 \simeq 1$ ($B_1 \simeq 1$) for the V-type scheme (for both the Λ- and cascade-type schemes). The solutions to the Fourier transforms of Eq. (6a) with $B_3 \simeq 1$ and Eqs. (6b) and (6c) with $B_1 \simeq 1$ are

$$\beta_1 = \frac{(\Delta_t + \omega + i\gamma_2)\Lambda_p^*}{|\Omega_c|^2 - (\Delta_p + \omega + i\gamma_1)(\Delta_t + \omega + i\gamma_2)} \quad (V),$$

$$\beta_2 = -\frac{\Omega_c \Lambda_p^*}{|\Omega_c|^2 - (\Delta_p + \omega + i\gamma_1)(\Delta_t + \omega + i\gamma_2)} \quad (V), \quad (7a)$$

$$\beta_2 = -\frac{\Omega_c^* \Lambda_p}{|\Omega_c|^2 - (\omega - \Delta_p + i\gamma_3)(\omega - \Delta_t + i\gamma_2)} \quad (\Lambda),$$

$$\beta_3 = \frac{(\omega - \Delta_t + i\gamma_2)\Lambda_p}{|\Omega_c|^2 - (\omega - \Delta_p + i\gamma_3)(\omega - \Delta_t + i\gamma_2)} \quad (\Lambda), \quad (7b)$$

$$\beta_2 = \frac{(\omega - \Delta_t + i\gamma_3)\Lambda_p}{|\Omega_c|^2 - (\omega - \Delta_p + i\gamma_2)(\omega - \Delta_t + i\gamma_3)} \quad (C),$$

$$\beta_3 = -\frac{\Omega_c \Lambda_p}{|\Omega_c|^2 - (\omega - \Delta_p + i\gamma_2)(\omega - \Delta_t + i\gamma_3)} \quad (C), \quad (7c)$$

where Δ_t is the two photon or Raman detuning (see Fig. 1) and is defined as $\Delta_t = \Delta_p - \Delta_c$ for both V- and Λ-type schemes, and $\Delta_t = \Delta_p + \Delta_c$ for the cascade-type scheme, β_j and Λ_p are the Fourier transforms of B_j and Ω_p respectively, and ω is the Fourier transform variable.

III. PROPAGATION CHARACTERISTICS OF PULSED PROBE FIELD

With the explicit analytical expressions (9) known for the atom response to the strong control field and weak pulsed probe field, we consider how atom response affects the propagation characteristics of the pulsed probe field in this section.

The wave equation for the probe field is

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = i\kappa_{13} B_3 B_1^* \simeq i\kappa_{13} B_1^* \Rightarrow \frac{\partial \Lambda_p}{\partial z} - i\frac{\omega}{c}\Lambda_p \simeq i\kappa_{13}\beta_1^* \quad (V), \quad (8a)$$

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = i\kappa_{13} B_3 B_1^* \simeq i\kappa_{13} B_3 \Rightarrow \frac{\partial \Lambda_p}{\partial z} - i\frac{\omega}{c}\Lambda_p \simeq i\kappa_{13}\beta_3 \quad (\Lambda), \quad (8b)$$

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = i\kappa_{12} B_2 B_1^* \simeq i\kappa_{12} B_2 \Rightarrow \frac{\partial \Lambda_p}{\partial z} - i\frac{\omega}{c}\Lambda_p \simeq i\kappa_{12}\beta_2 \quad (C), \quad (8c)$$

where $\kappa_{1k} = 2N\omega_p |D_{1k}|^2/(\hbar c)$ ($k=2,3$), N is atomic concentration and D_{1k} is the dipole moment for the transitions between states $|1\rangle$ and $|k\rangle$.

Using Eqs. (7) and (8), we readily obtain

$$\Lambda_p(z,\omega) = \Lambda_p(0,\omega)\exp(iKz), \quad (9)$$

where $\Lambda_p(0,\omega)$ is the Fourier transform of the probe field at the entrance $z=0$, and

$$K = \frac{\omega}{c} + \frac{\kappa_{13}(\Delta_t + \omega - i\gamma_2)}{|\Omega_c|^2 - (\Delta_p + \omega - i\gamma_1)(\Delta_t + \omega - i\gamma_2)} \quad (V), \quad (10a)$$

$$K = \frac{\omega}{c} + \frac{\kappa_{13}(\omega - \Delta_t + i\gamma_2)}{|\Omega_c|^2 - (\omega - \Delta_p + i\gamma_3)(\omega - \Delta_t + i\gamma_2)} \quad (\Lambda), \quad (10b)$$

$$K = \frac{\omega}{c} + \frac{\kappa_{12}(\omega - \Delta_t + i\gamma_3)}{|\Omega_c|^2 - (\omega - \Delta_p + i\gamma_2)(\omega - \Delta_t + i\gamma_3)} \quad (C). \quad (10c)$$

Expanding K into the Taylor series of variable ω: $K = K_0 + \omega/V_g + K_2\omega^2 + O(\omega^3)$, and neglecting the terms of the order $O(\omega^3)$, and further assuming that the probe field at $z=0$ is a Gaussian pulse of duration τ or $\Omega_p(z=0,t) = \Omega_{p0}\exp(-t^2/\tau^2) \Rightarrow \Lambda_p(0,\omega) = \Omega_{p0}\tau\sqrt{\pi}\exp[-(\omega\tau)^2/4]$, we then readily obtain from Eq. (9) the probe field [14]

$$\Omega_p(z,t) = \frac{\Omega_{p0}}{\sqrt{b_1 - ib_2}}\exp\left[iK_0 z - \frac{(t - z/V_g)^2}{\tau^2(b_1 - ib_2)}\right], \quad (11)$$

where $b_1 \equiv b_1(z) = 1 + 4z\,\mathrm{Re}(K_2)/\tau^2$ and $b_2 \equiv b_2(z) = 4z\,\mathrm{Im}(K_2)/\tau^2$, $K_0 = \phi + i\alpha/2$, and

$$\phi + i\frac{\alpha}{2} = \frac{\kappa_{13}(\Delta_t - i\gamma_2)}{|\Omega_c|^2 - (\Delta_p - i\gamma_1)(\Delta_t - i\gamma_2)}, \quad \frac{1}{V_g} \simeq \frac{1}{c} + \mathrm{Re}\left\{\frac{\kappa_{13}[|\Omega_c|^2 + (\Delta_t - i\gamma_2)^2]}{[|\Omega_c|^2 - (\Delta_p - i\gamma_1)(\Delta_t - i\gamma_2)]^2}\right\} \quad (V),$$

$$K_2 = \frac{\kappa_{13}\{(\Delta_t - i\gamma_2) + [\Delta_t + \Delta_p - i(\gamma_1 + \gamma_2)][(\Delta_t - i\gamma_2)^2 + |\Omega_c|^2]\}}{[|\Omega_c|^2 - (\Delta_p - i\gamma_1)(\Delta_t - i\gamma_2)]^3}, \quad \Delta_t = \Delta_p - \Delta_c \quad (V), \tag{12a}$$

$$\phi + i\frac{\alpha}{2} = -\frac{\kappa_{13}(\Delta_t - i\gamma_2)}{|\Omega_c|^2 - (\Delta_p - i\gamma_3)(\Delta_t - i\gamma_2)}, \quad \frac{1}{V_g} \simeq \frac{1}{c} + \mathrm{Re}\left\{\frac{\kappa_{13}[|\Omega_c|^2 + (\Delta_t - i\gamma_2)^2]}{[|\Omega_c|^2 - (\Delta_p - i\gamma_3)(\Delta_t - i\gamma_2)]^2}\right\} \quad (\Lambda),$$

$$K_2 = -\frac{\kappa_{13}\{(\Delta_t - i\gamma_2) + [\Delta_t + \Delta_p - i(\gamma_2 + \gamma_3)][(\Delta_t - i\gamma_2)^2 + |\Omega_c|^2]\}}{[|\Omega_c|^2 - (\Delta_p - i\gamma_3)(\Delta_t - i\gamma_2)]^3}, \quad \Delta_t = \Delta_p - \Delta_c \quad (\Lambda), \tag{12b}$$

$$\phi + i\frac{\alpha}{2} = -\frac{\kappa_{12}(\Delta_t - i\gamma_3)}{|\Omega_c|^2 - (\Delta_p - i\gamma_2)(\Delta_t - i\gamma_3)}, \quad \frac{1}{V_g} \simeq \frac{1}{c} + \mathrm{Re}\left\{\frac{\kappa_{12}[|\Omega_c|^2 + (\Delta_t - i\gamma_3)^2]}{[|\Omega_c|^2 - (\Delta_p - i\gamma_2)(\Delta_t - i\gamma_3)]^2}\right\} \quad (C),$$

$$K_2 = -\frac{\kappa_{12}\{(\Delta_t - i\gamma_3) + [\Delta_t + \Delta_p - i(\gamma_2 + \gamma_3)][(\Delta_t - i\gamma_3)^2 + |\Omega_c|^2]\}}{[|\Omega_c|^2 - (\Delta_p - i\gamma_2)(\Delta_t - i\gamma_3)]^3}, \quad \Delta_t = \Delta_p + \Delta_c \quad (C), \tag{12c}$$

where Δ_t is the two photon detuning.

Equation (11) explicitly gives the space-time dependent analytical expression of the probe field while Eq. (12) gives the explicit analytical expressions of the corresponding phase shift per unit length ϕ and (energy) absorption coefficient α (a negative α denotes in fact the amplification instead of the absorption), group velocity V_g, and group velocity dispersion K_2. We note that the group velocity dispersion K_2 term changes the pulse's width ($\tau \Rightarrow \tau\sqrt{b_1 + b_2^2/b_1}$) and magnitude ($|\Omega_{p0}| \Rightarrow |\Omega_{p0}|/\sqrt{b_1^2 + b_2^2}$).

The simple analytical expressions (11) and (12) demonstrate in an explicit and clear way the similarities and essential differences of the V-, Λ-, and cascade-type EIT schemes. It is pointed out that the analytical expressions of the quantity $K_0 = \phi + i\alpha/2$ in Eq. (12) are the steady-state results in both the density matrix and the Schrödinger formalisms. In particular, $K_0 = \phi + i\alpha/2$ in Eq. (12b) for the Λ-type EIT scheme has already been obtained previously [41]. Figures 2 and 3 demonstrate the typical features of the phase shift per unit length ϕ and absorption coefficient α versus the detuning Δ_p for all the three-state EIT schemes according to the analytical results in Eq. (12). From Eq. (12), we see that under the usual EIT on-resonance condition $\Delta_p = \Delta_c = 0$, and the condition $|\Omega_c|^2 \gg \gamma_1\gamma_2$, γ_2^2 (V-type) or $|\Omega_c|^2 \gg \gamma_2\gamma_3$, γ_3^2 (Λ-type) or $|\Omega_c|^2 \gg \gamma_1\gamma_2$, γ_2^2 (cascade-type), the group velocity V_g has the simple form

$$\frac{1}{V_g} \simeq \frac{1}{c} + \frac{\kappa}{|\Omega_c|^2}, \tag{13}$$

where $\kappa = \kappa_{13}$ for both V- and Λ-type schemes and $\kappa = \kappa_{12}$ for the cascade-type scheme.

We would like to emphasize that although there exist many (mathematical) similarities about the three schemes as explicitly demonstrated in Eqs. (11) and (12) (see also Refs. [46–49]), there are a number of important and essential physical differences about these schemes due to different populated states, different expressions of the two-photon detuning, and different relative magnitudes of various decay or relaxation rates. Notice that even the same symbol may have greatly different values in different schemes. For instance, the symbol γ_2 in V- and cascade-type schemes describes the decay of the state that has dipole allowed transition channel while in the Λ-type scheme it describes the decoherence rate (between $|2\rangle$ and the ground state $|1\rangle$) of the state $|2\rangle$ that does not have dipole allowed transition channel. Therefore γ_2 in V- and cascade-type schemes is usually much greater than γ_2 in the Λ-type scheme in cold atom media. However, γ_2 and γ_3 of the Λ-type scheme in typical solid EIT studies have comparable magnitudes [42,45].

Just as shown in Fig. 1, the heavily populated state in V-type scheme is an excited state $|3\rangle$ (it connects to the state

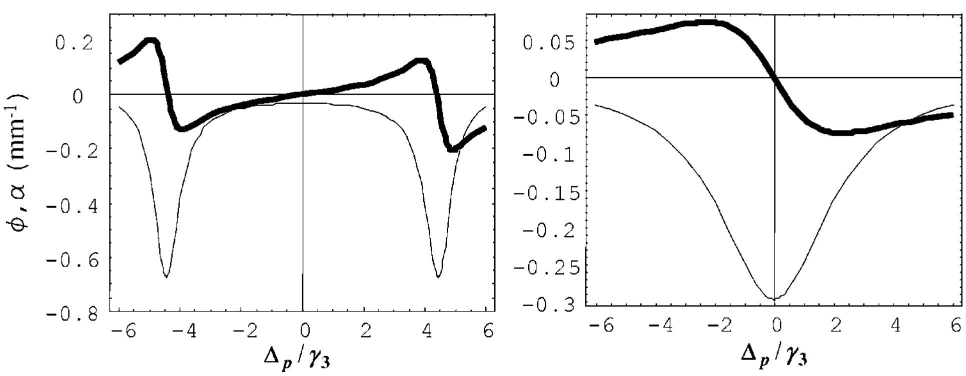

FIG. 2. Phase shift per unit length ϕ (thick line) and absorption coefficient α (thin line) versus dimensionless detuning Δ_p/γ_3 in V-type (left panel) and Λ-type (right panel) schemes according to Eq. (12). $\Delta_c = 0$ and $\kappa_{13} = \gamma_3/(3\text{ mm})$ for both panels while $\gamma_1 = 10^{-3}\gamma_3$, $\gamma_2 = \gamma_3$ ($\gamma_2 = 10^{-3}\gamma_3$), and $\Omega_c = 4.5\gamma_3$ ($\Omega_c = 1.5\gamma_3$) for the left (right) panel.

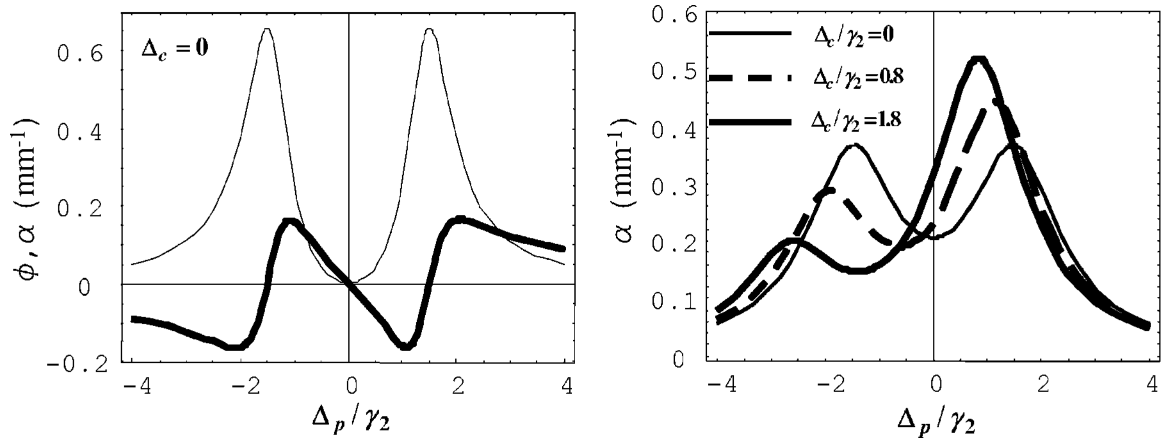

FIG. 3. Left and right panels show phase shift per unit length ϕ (thick line) and absorption coefficient α (thin line), and absorption coefficient α versus dimensionless detuning Δ_p/γ_2 in cascade-type scheme according to Eq. (12c). $\Omega_c=1.5\gamma_2$ and $\kappa_{12}=\gamma_3/(3\text{ mm})$ for both panels while $\gamma_3=0.01\gamma_2$ ($\gamma_3=\gamma_2$) for the left (right) panel. The detuning Δ_c is shown in the panels.

$|1\rangle$ by dipole transitions) with the decay rate γ_3 while the heavily populated state in the other two schemes is the ground state $|1\rangle$ with much smaller decay or relaxation rate γ_1 (usually $\gamma_3/\gamma_1>10^3$). Just as we have mentioned in the paragraph in between Eqs. (6) and (7), to achieve better EIT phenomenon, the duration τ of the probe pulse in free space should satisfy $\tau\gamma_3\ll 1$ ($\tau\gamma_1\ll 1$) for the V-type scheme (for both the Λ- and cascade-type schemes). Consequently, ultrafast probe pulse ($\tau<10^{-7}$ s because the typical γ_3 values in atom media is of the order 10 MHz) is usually needed in the V-type scheme while it is not necessarily so in the other two schemes. Besides, it is noted from Figs. 2 and 3 or Eq. (12) that the absorption coefficient α in the Λ- and cascade-type schemes is positive (denoting probe's absorption) while in the V-type scheme, it is negative (denoting probe's gain instead of absorption). This fact originates obviously from the different heavily populated states in these schemes.

The effects of Doppler broadening due to the atom's thermal velocity $\mathbf{v}$ can readily be included in Eq. (12) by two steps. (1) The detunings in Eq. (12) are replaced by the corresponding velocity dependent detunings by the rules $\Delta_{p,c}\Rightarrow\Delta_{p,c}+\mathbf{k}_{p,c}\cdot\mathbf{v}=\Delta_{p,c}+k_{p,c}v_z$ with positive (minus) wave number k denoting the propagating along the positive (negative) z direction. (2) The v_z-dependent quantities thus obtained are then averaged over a given velocity distribution $f(v_z)$. Here we do not perform such average operation but merely give a qualitative analysis through replacing the detunings in Eq. (12) by $\Delta_{p,c}\Rightarrow\Delta_{p,c}+k_{p,c}v$ with $v\propto\sqrt{\langle v_z^2\rangle}$ and $\langle v_z^2\rangle=\int v_z^2 f(v_z)dv_z$. From the expressions of the two-photon detuning Δ_t in Eq. (12), it is readily seen that the control and probe fields should propagate in the same (opposite) direction for both V- and Λ-type schemes (cascade scheme) in order to make the two-photon detuning Δ_t nearly independent of the average velocity v, and such choices can (sometimes greatly) decrease the effects of Doppler broadening.

IV. CONCLUSIONS

In summary, we have presented a time-dependent analysis of the three-state EIT in V-, Λ-, and cascade-type schemes by means of the Schrödinger-Maxwell formalism. We have ex-

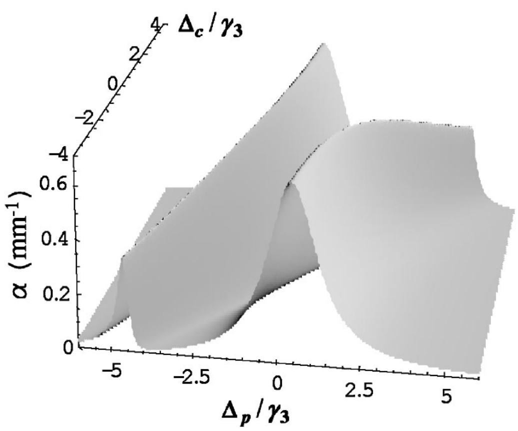

FIG. 4. Surface plot of absorption coefficient α versus dimensionless detunings Δ_p/γ_3 and Δ_c/γ_3 in Λ-type scheme according to Eq. (12b). The parameters are $\Omega_c=1.5\gamma_3$, $\kappa_{13}=\gamma_3/(3\text{ mm})$, and $\gamma_2=0.1\gamma_3$.

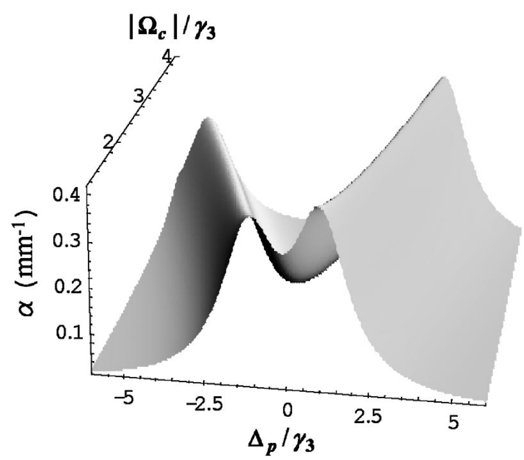

FIG. 5. Surface plot of absorption coefficient α versus Δ_p/γ_3 and $|\Omega_c|/\gamma_3$ in Λ-type scheme according to Eq. (12b) with $\Delta_c=-0.1|\Omega_c|^2$. The parameters are $\kappa_{13}=\gamma_3/(3\text{ mm})$ and $\gamma_2=\gamma_3$.

plicitly provided the *appropriate* transformations of these three schemes to make the Schrödinger-Maxwell formalism suitable for dealing with the three-state EIT. In this way, we have derived the explicit analytical expressions of the space-time dependent probe field, the corresponding phase shift, absorption or amplification, group velocity and group velocity dispersion. These analytical expressions demonstrate the similarities and essential differences of all the three schemes in a clear and simple way.

We note that except for a few recent numerical studies [49,50], the *V*-type scheme in cold atoms, compared with the other two schemes, has received little attention in the past due partly to the reason that the conditions for realizing EIT are rather demanding compared with other two schemes. However, there exist some indications that the stringent constraints to the *V*-type scheme may be loosed somewhat via electron spin coherence in semiconductors [45].

It is well known [42] that the many-body effects in solids lead to the renormalizations of decay or decoherence or relaxation rates, energy levels and the control field. The simple analytical results in Eqs. (10)–(12) express the propagation characteristics of the pulsed probe field for the three schemes explicitly in terms of these parameters (notice that the renormalization of energy levels can be accounted for by the detunings) and hence provide a convenient basis for investigating how the many-body effects in solids [42] modify the magnitude, spectral shape, and space-time dependence of the EIT and EIT related quantum coherence phenomena in solids [42–45]. In Figs. 3 (right panel) and 4, we have used Eq. (12) to show graphically how the off-resonance control field ($\Delta_c \neq 0$) modifies the profile of the absorption coefficient α with respect to another detuning Δ_p. Notice that these figures can also serve as demonstrating how the renormalization of energy levels affects the profile of the absorption coefficient α when one or both of the frequencies of the control and probe fields are fixed; see Refs. [42,44] for comparisons. It has been found that many-body effects in semiconductors mediated by the control field can lead to the renormalization of energy levels proportional to the intensity of the control field [44]. This phenomenon can also readily be described by Eq. (12) simply by taking $\Delta_c = \eta|\Omega_c|^2$ with η being a numerical factor. Figure 5 gives the surface plot of the absorption coefficient α versus detuning Δ_p and control field's amplitude $|\Omega_c|$.

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[1] S. E. Harris, Phys. Today **50**(7), 36 (1997).
[2] M. D. Lukin, P. R. Hemmer, and M. O. Scully, Adv. At., Mol., Opt. Phys. **42**, 347 (2000).
[3] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997), Chap. 7.
[4] J. H. Marangos, J. Mod. Opt. **45**, 471 (1998).
[5] Z. Ficek and S. Swain, J. Mod. Opt. **46**, 3 (2002).
[6] M. D. Lukin and A. Imamoglu, Nature (London) **413**, 273 (2001).
[7] E. Paspalakis and P. L. Knight, Phys. Rev. A **66**, 015802 (2002).
[8] D. McGloin, D. J. Fulton, and M. H. Dunn, Opt. Commun. **190**, 221 (2001).
[9] S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999).
[10] A. V. Sokolov, D. R. Walker, D. D. Yavuz, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **85**, 562 (2000).
[11] L. Deng, M. Kozuma, E. W. Hagley, and M. G. Payne, Phys. Rev. Lett. **88**, 143902 (2002).
[12] Y. Wu, J. Saldana, and Y. Zhu, Phys. Rev. A **67**, 013811 (2003).
[13] Y. Wu, L. Wen, and Y. Zhu, Opt. Lett. **28**, 631 (2003); Y. Niu, R. Li, and S. Gong, Phys. Rev. A **71**, 043819 (2005).
[14] L. Deng and M. G. Payne, Phys. Rev. A **68**, 051801(R) (2003).
[15] L. Deng and M. G. Payne, Phys. Rev. Lett. **91**, 243902 (2003).
[16] M. G. Payne and L. Deng, Phys. Rev. Lett. **91**, 123602 (2003).
[17] Y. Wu and L. Deng, Opt. Lett. **29**, 1144 (2004).
[18] M. Yan, E. G. Rickey, and Y. Zhu, Phys. Rev. A **64**, 043807 (2001); P. Zhang, Y. Li, C. P. Sun, and L. You, Phys. Rev. A **70**, 063804(R) (2004); H. Kang, G. Hernandez, and Y. Zhu, Phys. Rev. Lett. **93**, 073601 (2004).
[19] A. S. Zibrov, C. Y. Ye, Y. V. Rostovtsev, A. B. Matsko, and M. O. Scully, Phys. Rev. A **65**, 043817 (2002).
[20] A. B. Matsko, I. Novikova, G. R. Welch, and M. S. Zubairy, Opt. Lett. **28**, 96 (2003).
[21] A. K. Popov, V. V. Kimberg, and T. F. George, Phys. Rev. A **69**, 043816 (2004).
[22] Y. Wu, M. G. Payne, E. W. Hagley, and L. Deng, Phys. Rev. A **69**, 063803 (2004).
[23] Y. Wu, M. G. Payne, E. W. Hagley, and L. Deng, Phys. Rev. A **70**, 063812 (2004).
[24] Y. Wu and X. Yang, Phys. Rev. A **70**, 053818 (2004).
[25] X. Yang and Y. Wu, J. Opt. B: Quantum Semiclassical Opt. **7**, 54 (2005).
[26] D. McGloin, J. Phys. B **36**, 2861 (2003); D. McGloin, M. H. Dunn, and D. J. Fulton, Phys. Rev. A **62**, 053802 (2000); D. McGloin, D. J. Fulton, and M. H. Dunn, Opt. Commun. **190**, 221 (2001).
[27] A. Joshi and M. Xiao, Eur. Phys. J. D **30**, 431 (2004).
[28] A. Joshi and M. Xiao, Phys. Lett. A **317**, 370 (2003).
[29] M. Xiao, IEEE J. Sel. Top. Quantum Electron. **9**, 86 (2003).
[30] M. V. Jouravlev and G. Kurizki, Phys. Rev. A **70**, 053804 (2004).
[31] G. Kurizki *et al.*, New J. Phys. **2**, 281 (2000).
[32] A. B. Klimov *et al.*, Opt. Commun. **230**, 393 (2004).
[33] A. B. Klimov, R. Guzman, J. C. Retamal, and C. Saavedva, Phys. Rev. A **67**, 062313 (2003).
[34] L. M. Li, X. Peng, C. Liu, H. Guo, and X. Z. Chen, J. Phys. B

37, 1873 (2004).
[35] F. Xiao, H. Guo, L. M. Li, C. Liu, and X. Z. Chen, Phys. Lett. A **327**, 15 (2004).
[36] J. Wang *et al.*, Phys. Lett. A **328**, 437 (2004).
[37] C. P. Sun, Y. Li, and X. F. Liu, Phys. Rev. Lett. **91**, 147903 (2003).
[38] L. M. Kuang, G. H. Chen, and Y. S. Wu, J. Opt. B: Quantum Semiclassical Opt. **5**, 341 (2003).
[39] H. Schmidt and A. Imamoğlu, Opt. Lett. **21**, 1936 (1996).
[40] S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. **81**, 3611 (1998).
[41] M. M. Kash, V. A. Sautenkov, A. S. Zibrov, L. Hollberg, G. R. Welch, M. D. Lukin, Y. Rostovtsev, E. S. Fry, and M. O. Scully, Phys. Rev. Lett. **82**, 5229 (1999).
[42] W. W. Chow, H. C. Schneider, and M. C. Phillips, Phys. Rev. A **68**, 053802 (2003).
[43] M. C. Phillips and H. Wang, Phys. Rev. Lett. **89**, 186401 (2002).
[44] M. C. Phillips *et al.*, Phys. Rev. Lett. **91**, 183602 (2003).
[45] T. Li, H. Wang, N. H. Kwong, and R. Binder, Opt. Express **11**, 3298 (2003).
[46] Y. Wu, Phys. Rev. A **54**, 1586 (1996); Y. Wu and X. Yang, Phys. Rev. A **56**, 2443 (1997).
[47] M. B. Plenio, Phys. Rev. A **62**, 015802 (2000).
[48] A. B. Klimov, H. de Guise, and L. L. Sanchez-Soto, Phys. Rev. A **68**, 065801 (2003).
[49] J. R. Boon, E. Zekou, D. McGloin, and M. H. Dunn, Phys. Rev. A **59**, 4675 (1999).
[50] M. A. Antón, O. G. Calderón, and F. Carreño, Phys. Rev. A **69**, 023801 (2004).