Generation of entangled lights with temporally reversed photon wave packets

S. A. Moiseev[1,2] and B. S. Ham[1]
[1]*The Graduate School of Information and Communications, Inha University, Incheon 402-751, Republic of Korea*
[2]*Kazan Physical-Technical Institute of Russian Academy of Sciences, Kazan 420029, Russia*
(Received 25 November 2004; published 3 May 2005)

We theoretically study the control of a weak quantum light in a resonant three-level condensed medium derived by a pair of resonant counterpropagating laser fields. We analytically demonstrate that the adiabatic switching on of one of the control fields generates an entangled state of photon wave packets moving together at subnormal slow velocity in the medium with relative amplitudes determined by the control laser fields. An abrupt switching-off the control field makes the entangled wave packets propagate in the opposite direction with temporally reversed profiles relative to each other. We have analyzed the quantum control of the entanglement generation in terms of the adiabatic and nonadiabatic switching operations of the coupling laser pulse.

DOI: 10.1103/PhysRevA.71.053802 PACS number(s): 42.50.Dv, 42.50.Gy, 03.67.Mn

Control of light velocity promises unique possibilities in quantum manipulations of a weak quantum light, especially for quantum information science [1]. Using electromagnetically induced transparency (EIT) [2], extremely slow light was observed experimentally in different resonant media [3]. Usually such a slow light has only been realized experimentally using macroscopic coherence between long-lived quantum states [4], which provides a spectrally sensitive refractive index leading to a slow group velocity of the light. The EIT effect has been successfully applied for giant Kerr nonlinearity [5], quantum switching [6], and quantum memory [7,8]. Recently the slow-light scheme has been suggested for the effective generation of quantum entangled state of single photon fields [9,10]. Such possibilities of controlling a weak light interacting with medium are especially interesting for applications in quantum computing and communications [11–13]. In this paper we propose a scheme of entanglement generation with photons using the properties of almost stationary slow light in a condensed medium. Our intention is to study nonlinear perspectives of the photon entanglement generation due to a lengthened interaction time using standing-wave grating. The slow-light standing scheme has been suggested [14] and experimentally demonstrated recently in a three-level atomic system associated with the Doppler effect [15]. We propose that this technique can be used to generate an entangled state of two-photon wave packets with temporally reversed profiles due to the enhanced nondegenerated four-wave mixing processes based on almost stationary light condition.

For quantum entanglement generation we use slow-light dynamics in a three-level Λ-type system of a condensed medium derived by a pair of strong counterpropagating control laser fields (see Fig. 1). In Fig. 1 the interaction scheme corresponds to Ref. [15], where long-lived coherence is generated in the form of multiple spatial gratings with different periods. The multiple gratings induce variable dynamics into the slow-light and provide the split of the original pulse into two spatially separated entangled quantum fields. We assume that initially one weak probe quantum field E_+ enters the three-level (condensed) medium with a resonant frequency to the transition $|1\rangle$-$|3\rangle$, $\omega_+=\omega_{31}$, and that the all atoms are in the ground state $|1\rangle$. The probe field propagates in slow group velocity through the medium under the influence of one strong copropagating control laser field Ω_+, which is resonant to the transition $|2\rangle$-$|3\rangle$: ($\omega_c=\omega_{32}$) [3]. After the probe pulse completely enters the medium, we adiabatically switch on the second control field $\Omega_-(t)$, which propagates in a backward ($-z$) direction with respect to the probe. The counterpropagating control fields Rabi frequencies are Ω_+ and $\Omega_-(t)$, respectively. The probe pulse will be almost stationary due to the intensive interference of the counterpropagating control fields with different Rabi frequency ratio. Below we study analytically the evolution of the stationary light at the manipulation by the second control field intensity.

For analytical purposes we use two quantum operators $\hat{E}_\sigma=\sqrt{\hbar\omega/(2\varepsilon_0 V)}A_\sigma e^{-i\omega_{31}(t-\sigma z/c)}+\text{H.c.}$ for the weak fields (where $\hat{A}_\sigma$ are slowly varying field operators [4], $\sigma=+,-$ correspond to the forward and backward waves, the quantization volume being the $V=1$ value below). Ignoring the inhomogeneous broadening we write the following Hamiltonian for the quantum field and atoms in the interaction picture:

$$H=\hbar g\sum_{j=1}[\hat{A}_+(t,z_j)e^{ikz_j}+\hat{A}_-(t,z_j)e^{-ikz_j}]P^j_{31} - \hbar\sum_{j=1}\{(\Omega_+e^{i(Kz_j+\varphi_+)}+\Omega_-e^{-i(Kz_j+\varphi_-)})P^j_{32}\}+\text{H.c.}, \quad (1)$$

where P^j_{nm} are the atomic operators, $n=1,2$; $g=\mathcal{P}_{31}\sqrt{\omega_{31}/(2\varepsilon_0\hbar V)}$ $(V=1)$ is a coupling constant of photons

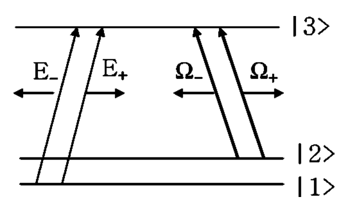

FIG. 1. The energy level diagram of quantum entanglement generation. $E_\pm$ are weak fields resonant to the transition $|1\rangle$-$|3\rangle$. Two control fields $\Omega_\pm$ resonant to the transition $|2\rangle$-$|3\rangle$. The "+" ("−") field propagates in the $+z$ ($-z$) direction.

with atoms and $\mathcal{P}_{31}$ is a dipole moment for the transition $|1\rangle$-$|3\rangle$ and $K=\omega_{32}/c, \varphi_{+,-}$ are the phases of the laser pulses.

Using the Hamiltonian we obtain the usual Heisenberg equations for the atomic operators P_{nm}^{j},

$$\frac{\partial}{\partial t}P_{13} = -\gamma P_{13} - ig(\hat{A}_{+}e^{ikz} + \hat{A}_{-}e^{-ikz}) + i(\Omega_{+}e^{i(Kz+\varphi_{+})} + \Omega_{-}e^{-i(Kz+\varphi_{-})})P_{12}, \tag{2}$$

$$\frac{\partial}{\partial t}P_{12} = -\gamma_2 P_{12} + i(\Omega_{+}e^{-i(Kz+\varphi_{+})} + \Omega_{-}e^{i(Kz+\varphi_{-})})P_{13}, \tag{3}$$

and the following equations for the field operators:

$$\left(\frac{\partial}{c\partial t} + \frac{\partial}{\partial z}\right)\hat{A}_{+}(t,z) = -i(N_0 g/c)P_{+}, \tag{4}$$

$$\left(\frac{\partial}{c\partial t} - \frac{\partial}{\partial z}\right)\hat{A}_{-}(t,z) = -i(N_0 g/c)P_{-}, \tag{5}$$

where N_0 is the atomic density; we have used the phase matching condition in Eqs. (4) and (5) with the following decomposition rates of the atomic operator: $P_{13}(t,z) = P_{+}(t,z)e^{ikz} + P_{-}(t,z)e^{-ikz} + \ldots$, where the operators $P_{+}(t,z)$ and $P_{-}(t,z)$ slowly vary in space and time. Equations for the atomic operators (2) and (3) take into account the fact that the population of the excited state is negligible during the interaction with a weak quantum probe field, so $P_{11}-P_{33} \cong P_{11} \cong 1$ and $P_{23} \ll P_{13}, P_{12}$ [4]. We have also introduced one decay constant γ for both transitions: $|3\rangle$-$|1\rangle$ and $|3\rangle$-$|2\rangle$. We note that the coherent atomic dynamics and the evolution of the slow light are not affected by the typical Langeven forces at the adiabatical interactions [16], where large temporal duration δt of the pulses are $\gamma\delta t \gg 1$. In this case the optical atomic coherence P_{13} follows the pattern set by the long-lived coherence P_{12} and external optical fields:

$$P_{13} \cong -i\gamma^{-1}\{g(\hat{A}_{+}e^{ikz} + \hat{A}_{-}e^{-ikz}) - (\Omega_{+}e^{i(Kz+\varphi_{+})} + \Omega_{-}e^{-i(Kz+\varphi_{-})})P_{12}\}. \tag{6}$$

Substituting Eq. (6) into Eq. (3), we find the formal solution for the long-lived coherence P_{12}:

$$\begin{aligned} P_{12}(\Delta,t,z) &= P_{12}(\Delta,t_0,z)\exp\left(-\int_{t_0}^{t} dt'\,\Gamma(\Delta,t',z)\right) + \int_{t_0}^{t} dt' \exp\left\{-\int_{t'}^{t} dt''\Gamma(\Delta,t'',z)\right\}F_p(t',z) \\ &= P_{12}(\Delta,t_0,z)\exp\{-f_1(\Delta,t,t_0)\}\sum_{n=-\infty}^{\infty}(-1)^n I_{|n|} \times\{[2f_2(\Delta,t,t_0)]\}\cos n[2(k-k_0)z+\phi] \\ &\quad + \int_{t_0}^{t} dt' F_p(t',z)\exp\{-f_1(\Delta,t,t')\}\sum_{n=-\infty}^{\infty}(-1)^n I_{|n|} \times\{[2f_2(\Delta,t,t')]\}\cos n[2(k-k_0)z+\phi], \end{aligned} \tag{7}$$

where $I_{n=0,1,2,\ldots}[x]$ are the first-class Bessel functions of the imaginary argument [17], $P_{12}(\Delta,t_0,z)$ is an initial coherence at $t=t_0$, below $t_0 \to -\infty$, and we assume that initially $P_{12}(\Delta,t_0,z)=0$:

$$\begin{aligned} F_p(t,z) &= g(\gamma+i\Delta)^{-1}\{\hat{A}_{+}[\Omega_{+}(t)e^{i(k_0z-\varphi_{+})} + \Omega_{-}(t)e^{2ikz}e^{-i(k_0z-\varphi_{-})}] + \hat{A}_{-}[\Omega_{-}(t)e^{-i(k_0z-\varphi_{-})} + \Omega_{+}(t)e^{-2ikz}e^{i(k_0z-\varphi_{+})}]\}, \\ \Gamma(\Delta,t,z) &= \Gamma_1(\Delta,t) + \Gamma_2(\Delta,t)[e^{i[2(k-k_0)z+\phi]} + e^{-i[2(k-k_0)z+\phi]}], \\ \Gamma_1(\Delta,t) &= \gamma_2 + (\gamma+i\Delta)^{-1}\Omega_{\Sigma}^2(t), \\ \Gamma_2(\Delta,t) &= (\gamma+i\Delta)^{-1}\Omega_{+}(t)\Omega_{-}(t), \\ f_{1,2}(\Delta,t,t') &= \int_{t'}^{t} dt''\Gamma_{1,2}(\Delta,t''), \quad \Omega_{\Sigma}^2(t) = \Omega_{+}^2(t) + \Omega_{-}^2(t), \\ \phi &= \varphi_{+} + \varphi_{-}. \end{aligned} \tag{8}$$

Thus the long-lived atomic coherence P_{12} between the levels $|1\rangle$-$|2\rangle$ includes the superposition of a number of the spatial gratings

$$P_{12}(t,z) = \sum_{n=0}^{\infty}\hat{\beta}_n(t,z)\cos n[2(k-k_0)z+\phi],$$

where $\hat{\beta}_n(t,z)$ are the operators slowly varying in space and time.

Using the decomposition rate (7) of the long-lived coherence in Eq. (6), we find the values $P_{+}(t,z)$ and $P_{-}(t,z)$ in Eqs. (4) and (5), and then introducing the new field operators $\hat{a}_{+}(t,z)=\hat{A}_{+}(t,z)\exp\{-i\varphi_{+}+ik_0z\}$, $\hat{a}_{-}(t,z)=\hat{A}_{-}(t,z)\exp\{i\varphi_{-}-ik_0z\}$ we obtain the following integral-differential equations for these field operators:

$$\left(\frac{\partial}{c\partial t} - ik_0 + \frac{\partial}{\partial z}\right)\hat{a}_{+}(t,z) = -\xi_0\hat{a}_{+}(t,z) + (\xi_0/\gamma)\int_{-\infty}^{t} dt'\{G_1(t,t')\hat{a}_{+}(t',z) + G_{2,-}(t,t')\hat{a}_{-}(t',z)\}, \tag{9}$$

$$\left(\frac{\partial}{c\partial t} - ik_0 - \frac{\partial}{\partial z}\right)\hat{a}_{-}(t,z) = -\xi_0\hat{a}_{-}(t,z) + (\xi_0/\gamma) \times\int_{-\infty}^{t} dt'\{G_1(t,t')\hat{a}_{-}(t',z) + G_{2,+}(t,t')\hat{a}_{+}(t',z)\}, \tag{10}$$

where

$$G_1(t,t') = \exp[-f_1(t,t')]\{\Omega_{\Sigma}^2(t,t')I_0[2f_2(t,t')] - [\Omega_{+}(t)\Omega_{-}(t') + \Omega_{-}(t)\Omega_{+}(t')]I_1[2f_2(t,t')], \tag{11}$$

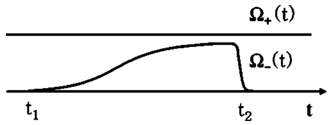

FIG. 2. Temporal scheme of the adiabatic (nonadiabatic) switching of the second control field $\Omega_-(t)$ at $t=t_1$ $(t=t_2)$, while keeping the first control field $\Omega_+(t)$ constant.

$$G_{2,\pm}(t,t') = \exp[-f_1(t,t')]\{(\Omega_\mp(t)\Omega_\pm(t')I_0[2f_2(t,t')] + \Omega_\pm(t)\Omega_\mp(t')I_2[2f_2(t,t')] - \Omega_\Sigma^2(t,t')I_1[2f_2(t,t')]\}, \quad (12)$$

where $f_1(t,t')=\gamma^{-1}\int_{t'}^{t}\Omega_\Sigma^2(t'')dt''$, $f_2(t,t')=\gamma^{-1}\int_{t'}^{t}\Omega_+(t'')\times\Omega_-(t'')dt''$, $\Omega_\Sigma^2(t,t')=\Omega_+(t)\Omega_+(t')+\Omega_-(t)\Omega_-(t')$, $\Omega_\Sigma^2(t)=\Omega_+^2(t)+\Omega_-^2(t)$, $k_0=\omega_{21}/c$, and $\xi_0=(N_0g^2/c\gamma)$ is the absorption coefficient.

Equations (9) and (10) can be used for either the adiabatic or nonadiabatic regime of the control laser fields' switching operations. The system includes all the orders of the nonlinear interaction with the two strong laser fields and demonstrates that the two weak fields $\hat{a}_+(t,z)$ and $\hat{a}_-(t,z)$ are coupled with each other due to the terms proportional to $G_{2,\pm}(t,t')\neq 0$ only if the two control laser fields are simultaneously applied. The slow-light dynamics in Eqs. (9) and (10) depends on the parameters of the four gratings in the long-lived atomic coherence $P_{12}(t,z)$ $[\sim\exp\{\pm i(k\pm K)z\}]$ with the largest spatial periods in accordance with the phase-matching conditions. We note that all these atomic gratings are excited directly by the weak quantum fields in the presence of the two control laser fields. For comparison we note that the two of these coherence gratings $[\sim\exp\{\pm i(k+K)z\}]$ with spatial periods $(\sim\lambda/2)$ have a short lifetime and should be negligible in hot gaseous systems due to fast Doppler dephasing. In condensed media, however, we show that these two gratings cause new properties of the coupled fields propagation. Mathematically the grating amplitudes in Eqs. (9) and (10) are expressed nonlinearly with the control field in forms of Bessel functions $I_{n=0,1,2}(f_2(t,t'))$, whereas the Bessel functions with $n>2$ reflect the excitation of other gratings $[\sim\exp\{\pm i(k\pm nK)z\}]$ with smaller spatial periods due to the higher nonlinearity of the atomic response to the action of the two interfered control waves. We note that the grating amplitudes β_n decrease with increasing of the number n and the influence of the gratings with $n\gg 2$ becomes negligible in the slow-light dynamics.

We study the adiabatic switching on the second control laser field with the condition $(\partial/\partial t)\Omega_-\ll(\Omega_\Sigma^2/\gamma)\Omega_-$ (see Fig. 2). Given this condition we can assume $\Omega_-(t')\cong\Omega_-(t)-\varsigma(t)(t-t')$, where $\varsigma(t)=(\partial/\partial t)\Omega_-(t)$ is a small parameter in the functions $G_m(t,t')$ $(m=1;2+;2-)$, which play a role of the memory functions in the Eqs. (9) and (10), have a sharp temporal behavior with a maximum at $t=t'$ compared with the long weak pulses $a_\pm(t,z)$ $(\Omega_\Sigma^2\delta t/\gamma\gg 1)$. Taking into account the influence of the switching operation in the first order of the term $\zeta(t)$ in the function $G_m(t,t')$ we can find the decomposition rate

$$G_m(t,t')\cong G_m(t,t')\big|_{\varsigma(t)=0}+\varsigma(t)\left[\frac{\partial}{\partial\varsigma(t)}G_m(t,t')\right]\Bigg|_{\varsigma(t)=0}.$$

Herewith together with the substitution $a_\pm(t',z)\cong\{1-(t-t')(\partial/\partial t)\}a_\pm(t,z)$ we obtain the formula for the integrals in Eqs. (9) and (10):

$$\int_{t_1}^{t}dt'G_m(t,t')\hat{a}_\pm(t',z) \cong \hat{a}_\pm(t,z)\left\{\int_{t_1}^{t}dt'\{G_m(t,t')\Big|_{\varsigma=0}+\varsigma\frac{\partial}{\partial\varsigma}G_m(t,t')\Big|_{\varsigma=0}\right\} - \left[\frac{\partial}{\partial t}\hat{a}_\pm(t,z)\right]\int_{t_1}^{t}dt'(t-t')G_m(t,t')\big|_{\varsigma=0}. \quad (13)$$

We note that the numerical calculations show the increasing of the temporal intervals where the integrals in Eq. (13) are converged if the ratio $\chi=\Omega_-/\Omega_+$ becomes larger then 0.8. The value 0.8 determines approximately the interval $(0<\chi<0.8)$, where the memory functions in the integrals of Eqs. (9) and (10) are at their maximum within the temporal interval $t-t'\leqslant 50\gamma\Omega_+^{-2}$. Substituting Eq. (13) into Eqs. (9) and (10) we obtain the differential equations

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}-ik_0\right)\hat{a}_+(t,z)=-\dot{\xi}_-(t)\{[\beta_1(\chi)\hat{a}_+-\beta_{2,-}(\chi)]\hat{a}_-\} - \frac{1}{\nu_1(\chi)}\frac{\partial}{\partial t}\hat{a}_+(t,z)+\frac{1}{\nu_2(\chi)}\frac{\partial}{\partial t}\hat{a}_-(t,z), \quad (14)$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial z}-ik_0\right)\hat{a}_-(t,z)=-\dot{\xi}_-(t)\{\beta_1(\chi)\hat{a}_- - \beta_{2,+}(\chi)\hat{a}_+)\} - \frac{1}{\nu_1(\chi)}\frac{\partial}{\partial t}\hat{a}_-(t,z)+\frac{1}{\nu_2(\chi)}\frac{\partial}{\partial t}\hat{a}_+(t,z), \quad (15)$$

where the integrals from the exponent and Bessel functions equal to the simple algebraic formulas

$$\frac{\Omega_+^3}{\gamma^2}\int_{-\infty}^{t}dt'\left\{\frac{\partial}{\partial\varsigma}\tilde{G}_1(t,t')\right\}\Bigg|_{\varsigma=0}=-\beta_1(\chi)=-\frac{\chi}{(1-\chi^2)^2}, \quad (16)$$

$$\frac{\Omega_+^3}{\gamma^2}\int_{-\infty}^{t}dt'\left\{\frac{\partial}{\partial\varsigma}\tilde{G}_{m=2+,2-}(t,t')\right\}\Bigg|_{\varsigma=0}=\beta_m(\chi), \quad (17)$$

where $\beta_{2+}=\beta_{2-}+(1-\chi^2)^{-1}$, $\beta_{2-}=\chi^2/(1-\chi^2)^2$, $\dot{\xi}_-(t)=\xi_0(\gamma\varsigma/\Omega_+^3)=\xi_0(\gamma\dot{\Omega}_-/\Omega_+^3)=\dot{\chi}\nu_1^{-1}(0)$, and $\nu_1^{-1}(0)=\xi_0\gamma/\Omega_+^2=N_0g^2/(c\Omega_+^2)$ is the initial velocity of the slow light before the moment of switching on the second control field. We have also found the relation $\gamma^{-1}\int_{-\infty}^{t}dt'G_{2,\pm}(t,t')\big|_{\varsigma=0}=1-\gamma^{-1}\int_{-\infty}^{t}dt'G_1(t,t')\big|_{\varsigma=0}=\alpha(\chi(t))=0$, where the absorption coefficients $\xi_0\alpha(\chi)$ equals zero for both weak lights, which means the existence of the windows transparency for the weak fields; we have also introduced the new velocities $\nu_1(\chi)$ and $\nu_2(\chi)$:

$$(\xi_0/\gamma) \int_{-\infty}^{t} dt'(t-t')G_1(t,t')\bigg|_{s=0} = \frac{1}{\nu_1(0)(1-\chi^2)} = \nu_1^{-1}(\chi),$$

$$(\xi_0/\gamma) \int_{-\infty}^{t} dt'(t-t')G_{2,\pm}(t,t')\bigg|_{s=0} = -\frac{\chi}{\nu_1(0)(1-\chi^2)} = -\nu_2^{-1}(\chi).$$

The ratio between the introduced velocities coincides with the ratio between the Rabi frequencies of the control fields $\nu_1(\chi)/\nu_2(\chi)=\Omega_-/\Omega_+=\chi$. This interesting consequence of the integrals with Bessel functions leads to a simple structure for the system equations (14) and (15). We note that Eqs. (14) and (15) stay in a symmetrical form for the fields $\hat{a}_+(t,z)$ and $\hat{a}_-(t,z)$ with the constant control fields, whereas the switching procedure with $\dot{\xi}_-(t)\neq 0$ breaks the symmetry because of the nonsymmetrical switching operation. If the switching is slow enough, $\dot{\xi}_-(t)\ll 1$, the relative field parameters can be found using Eqs. (14) and (15) ignoring the terms proportional to $\dot{\xi}_-(t)$ and herewith we find the dispersion relation for the fields to be $\omega_{1,2}(k)=\nu_1(0)\{k_0\pm\sqrt{(\chi k_0)^2+(1-\chi^2)k^2}\}$. Let us consider the case of a sufficiently small splitting between the two ground levels:

$$\omega_{21} \ll \frac{c}{\nu_1(0)\chi}(1-\chi^2)^{1/2}\delta\omega_f \approx \frac{c}{\nu_1(0)}\delta\omega_f.$$

($\delta\omega_f\approx\delta t^{-1}$ is a spectrum width of the slow light.) This condition can be easily realized for typical experimental conditions $\delta\omega_f\approx 10^6 c^{-1}$, $c/\nu_1(0)\approx 10^6$, $\omega_{21}\ll 10^{12}c^{-1}$, which was observed in Ref. [3]. Therefore, we can ignore the band gap, $\Delta\omega_b=2\chi\nu_1(0)k_0=2\omega_{21}\Omega_-\Omega_+/(Ng^2)\ll\delta\omega_f$, so the field equations [Eqs. (14) and (15)] transform into the following:

$$\frac{1}{\nu_1}\frac{\partial}{\partial t}\hat{a}_+(t,z)+\frac{\partial}{\partial z}\hat{a}_+(t,z)=\frac{1}{\nu_2}\frac{\partial}{\partial t}\hat{a}_-(t,z), \tag{18}$$

$$\frac{1}{\nu_1}\frac{\partial}{\partial t}\hat{a}_-(t,z)-\frac{\partial}{\partial z}\hat{a}_-(t,z)=\frac{1}{\nu_2}\frac{\partial}{\partial t}\hat{a}_+(t,z), \tag{19}$$

with a linear dispersion relation for the coupled fields $\omega_{1,2}(k)=\pm k\nu_g$, where $\nu_g=\nu_1\sqrt{1-\chi^2}$ is a their common group velocity. Equations (18) and (19) have the following two pairs of the fundamental solutions ($a_\pm^{(1)}$ and $\hat{a}_\pm^{(2)}$) for the two waves propagated in the forward direction: $\hat{a}_+^{(1)}=\hat{a}_1[\int^t\nu_g(\chi)dt-z]$ and $\hat{a}_-^{(1)}=\mu\hat{a}_1[\int^t\nu_g(\chi)dt-z]$, and for the modes in the backward direction, $\hat{a}_+^{(2)}=\mu\hat{a}_2[\int_0^t\nu_g(\chi)dt+z]$ and $\hat{a}_-^{(2)}=\hat{a}_2[\int_0^t\nu_g(\chi)dt+z]$ [where $\mu(\chi)=\chi/(1+\sqrt{1-\chi^2})$]. The first pair of the coupled fields $\hat{a}_\pm^{(1)}$ corresponds to our initial condition for a nonzero probe field $\hat{a}_+(t<t_1,z)\neq 0$. Thus the adiabatic switching on of the second control field leads to the excitation of the new counterpropagating slow light $\hat{a}_-^{(1)}$, which propagates together with the original forward field $\hat{a}_+^{(1)}$, and the relative amplitude increases, $\hat{a}_-^{(1)}=\mu\hat{a}_+^{(1)}$, as $\Omega_-(t)$ increases. The amplitudes of both fields become mutually comparable (asymptotically in our approach) for $\chi\to 1$ where $\hat{a}_-^{(1)}{}_{\chi\to 1}\to\hat{a}_+^{(1)}$. In order to find the absolute amplitude of the forward field $\hat{a}_+^{(1)}$ (and $\hat{a}_-^{(1)}$, respectively) we have to solve the complicated equations (14) and (15). However, taking into account that our field parameters must be close to the adiabatic limit at very slow switching speeds we can find the approximate solution using the following anzatz for the fields in Eq. (14): $\hat{a}_+(t,z)\cong f(\chi)\hat{A}_{+,in}[\int_0^t\nu_g(t')dt'-z]$, $\hat{a}_-(t,z)\cong\mu(\chi)\hat{a}_+(t,z)$. After substitution of these values into Eq. (14) we find

$$df/f=\frac{\left\{-\frac{\nu_1(\chi)}{\nu_1(0)}(\beta_1-\mu\beta_{2,-})\sqrt{1-\chi^2}+\mu\right\}}{(1-\chi^2)}d\chi=\frac{(-\chi+\mu)}{(1-\chi^2)}d\chi = -\frac{(1-\sqrt{1-\chi^2})}{\chi\sqrt{1-\chi^2}}d\chi, \tag{20}$$

with the solution

$$f(\chi)=\tfrac{1}{2}(1+\sqrt{1-\chi^2}), \tag{21}$$

and the second field amplitude $\mu(\chi)f(\chi)=\chi/2$. Using the solution $f(\chi)$ we can evaluate the total electromagnetic energy of these two fields:

$$W=W_++W_-=\int_0^L dz\{\langle a_+^\dagger a_+(t,z;\chi)\rangle+\langle a_-^\dagger a_-(t,z;\chi)\rangle\} \cong \tfrac{1}{2}(1+\sqrt{1-\chi^2})W_{+,in},$$

which does not equal zero in the point of the complete stoppage of light, $W(\chi\to 1)=\frac{1}{2}W_{+,in}$. This nonvanishing field can stay macroscopically in the medium ($\chi=1$) or move along with the ultraslow velocity $\nu_g=\nu_1(0)\sqrt{1-\chi^2}$ without absorption. So we can manipulate with the ultraslow group velocity within the spin decoherence time γ_2^{-1} of the second level. For example, if we adiabatically switch off the second control field [$\Omega_-(t)\to 0$], the nearly stationary weak field resumes to move as it was before without dissipative losses: $\hat{a}=\hat{a}_{+,in}(t\nu_1-z+\delta z)$. At a certain time delay $\tau=\delta z/\nu_1=\int_{t_1}^{t_2}(\nu_1-\nu_g(\chi(t))dt\cong\int_{t_1}^{t_2}dt\chi^2(t)/[1+\chi^2(t)]$, the backward field disappears: $\hat{a}_-(t,z)=\mu(\chi)\hat{a}_+(t,z)|_{\chi\to 0}\to 0$. The advantage of this light stoppage with respect to the velocity control of the usual slow light propagation is the possibility of controlling the interaction time of this nonvanishinig stopped light with chosen spatially localized atoms in the medium [14,15] by the variation of the light group velocity ν_g. The two fields $\hat{a}_\pm(t,z)$ will be coupled indissociably to each other in this evolution and the total state of these fields are expressed through an entangled wave function. Using the obtained solution for the field operators we can find the wave function for the field and medium in the adiabatic limit where the original probe pulse was a single-photon wave packet:

$$|\Psi_{a+f}\rangle \cong \left(\frac{\nu_g}{c}\right)^{1/2} f(\chi)|1\rangle_a\{|\Psi_+(\nu_g t - z)\rangle_f + \mu(\chi)e^{-i\phi} \times |\Psi_-(\nu_g t - z)\rangle_f\} + \sum_j P_{21}^j(\nu_g t - z;\chi)|1\rangle_a|0\rangle_f,$$

where $|1\rangle_a=\Pi_{j=1}^N|1_j\rangle$ and $|0\rangle_f$ are the ground states of the atoms, and the field, $|\Psi_\pm(\nu_g t-z)\rangle_f$ is the single-photon wave packet of the fields $a_\pm(t,z)$ [where $\langle\Psi_\pm(\nu_g t-z)|\Psi_\pm(\nu_g t-z)\rangle_f=1$ and $P_{21}^j(t)=(P_{12}^j(-t))^+$ [see comment to Eq. (8) about the properties of the operator $P_{21}^j(t)$]. Excitation of the photon component of the state $|\Psi_{a+f}\rangle$ is much smaller than the component with atomic excitation due to the small group velocity ν_g, which takes place just before its emission from the medium. We found that the fields $a_\pm(t,z)$ can be spatially separated from each other before emission from the medium by the nonadiabatic switching off of the second control field (see Fig. 2). This nonadiabatic process influences the photon component of the state $|\Psi_{a+f}\rangle$ and the entanglement between the two generated pulses, which begin to propagate in opposite directions to each other.

We study this process using general Eqs. (9) and (10) and assuming the control field $\Omega_-(t>t_2)=0$. In this case the memory functions become proportional to the same exponentional function $G_{1;2,\pm}(t,t'>t_2)=C_{1;2,\pm}\exp[-(1/\gamma)\Omega_+^2(t-t')]$, where $C_{1;2,\pm}$ are constants with $C_{2,\pm}=0$ for $t'>t_2$. Therefore we obtain

$$(\xi_0/\gamma)\int_{-\infty}^{t} dt'\{G_1(t,t')\hat{a}_\pm(t',z) + G_{2,\mp}(t,t')\hat{a}_\pm(t',z)\} = -\exp\left\{-\frac{1}{\gamma}\Omega_+^2(t-t_2)\right\}\hat{\rho}_\pm(t_2,z) + (\xi_0\Omega_+^2/\gamma) \times \int_{t_2}^{t} dt' \exp\left\{-\frac{1}{\gamma}\Omega_+^2(t-t')\right\}\hat{a}_\pm(t',z), \tag{22}$$

where

$$\hat{\rho}_\pm(t_2,z) = (\xi_0\Omega_+^2/\gamma)\int_{-\infty}^{t_2} dt'\{G_{1,0}(t_2,t')\hat{a}_\pm(t',z) + G_{2,0,\mp}(t_2,t')\hat{a}_\mp(t',z)\}, \tag{23}$$

$$G_{1,0}(t_2-t') = \exp\left\{-\frac{1}{\gamma}\Omega_\Sigma^2(t_2)(t_2-t')\right\}\{I_0[2f_2(t_2,t')] - [\Omega_-(t_2)/\Omega_+]I_1[2f_2(t_2,t')]\}, \tag{24}$$

$$G_{2,0,+}(t_2-t') = -\exp\left\{-\frac{1}{\gamma}\Omega_\Sigma^2(t_2)(t_2-t')\right\}\{I_1[2f_2(t_2,t')] - [\Omega_-(t_2)/\Omega_+]I_2[2f_2(t_2,t')]\}, \tag{25}$$

$$G_{2,0,-}(t_2-t') = \exp\left\{-\frac{1}{\gamma}\Omega_\Sigma^2(t_2-t')\right\} \times\{[\Omega_-(t_2)/\Omega_+]I_0[2f_2(t_2,t')] - I_1[2f_2(t_2,t')]\}. \tag{26}$$

Using Eq. (22) we transform Eqs. (9) and (10) into the equations for the counterpropagated field operators $\hat{A}_\sigma(t,z)=\hat{a}_\sigma^{(1)}(t,z)\exp\{\sigma i\varphi_\sigma)\}$ (the index $\sigma=\pm$), which become decoupled from each other and have the form

$$\left(\frac{\partial}{c\partial t} + \sigma\frac{\partial}{\partial z}\right)\hat{A}_\sigma(t,z) = -[\xi_0\hat{A}_\sigma(t,z) + \hat{\rho}_{\sigma,0}(t,z)], \tag{27}$$

where the new atomic operators

$$\hat{\rho}_\sigma(t,z) = \hat{\rho}_{\sigma,0}(t,z)\exp\{-i\sigma\varphi_\sigma)\}$$

satisfy the equations

$$\frac{\partial}{\partial t}\hat{\rho}_{\sigma,0}(t,z) = -\frac{1}{\gamma}\Omega_+^2\{\hat{\rho}_{\sigma,0}(t,z) + \xi_0\hat{A}_\sigma(t,z)\}. \tag{28}$$

Using the decomposition, Eq. (13), for Eq. (23) and the condition for slow-light fields,

$$\xi_0\hat{a}_\pm^{(1)} \gg \frac{1}{\nu_{1,2}}\frac{\partial}{\partial t_2}a_\pm^{(1)}(t_2,z),$$

which takes place in the high-optical-density medium, and taking into account Eqs. (24)–(26) we find

$$\hat{\rho}_+(t_2,z) = -\xi_0\hat{a}_-^{(1)}(t_2,z)\hat{\rho}_-(t_2,z) = -\{\xi_0[\hat{a}_-^{(1)}(t_2,z) - \chi(t_2)\hat{a}_+^{(1)}(t_2,z)]\ .$$

Equations (27) and (28) reproduce the final stage of the field reconstruction in the quantum memory technique studied recently in [16]. The main difference between the present article and Ref. [16] is that the initial condition for fields is $\hat{A}_\sigma(t_2,z)\neq 0$ and the specific character of atomic excitation is determined by the atomic excitations $\hat{\rho}_\sigma(t_2,z)$. Due to these initial conditions, the solution of Eqs. (27) and (28) includes both the forward and backward slow-light fields, which are expressed via the usual Fourier transformation in k space:

$$\hat{\rho}_{\sigma,0}(t,z) = \sum_{m=1}^{2}\int_{-\infty}^{\infty} dk\{\hat{\rho}_{\sigma,k}^{(m)}\exp\{i[\omega_{m,k}^{(\sigma)}(t-t_2) + kz]\}, \tag{29}$$

$$\hat{A}_\sigma(t,z) = \sum_{m=1}^{2}\int_{-\infty}^{\infty} dk\{\hat{A}_{\sigma,k}^{(m)}\exp\{i[\omega_{m,k}^{(\sigma)}(t-t_2) + kz]\}, \tag{30}$$

where we have found the four eigenvalues of Eqs. (27) and (28) in k space $(m=1,2)$: $\omega_{1,2;k}^\sigma=\frac{1}{2}\Gamma_{\sigma,k}\{1\pm S_{\sigma,k}\}$ with $\Gamma_{\sigma,k}=[i(c\xi_0+\Omega_+^2/\gamma)-\sigma ck]$, $S_{\sigma,k}=\sqrt{1+4i\sigma ck\Omega_+^2/(\gamma\Gamma_{\sigma,k}^2)}$; the values $\hat{\rho}_{\sigma,k}^{(m)}$, $\hat{A}_{\sigma,k}^{(m)}$ are found using the spatial Fourier transformation for the initial conditions for the fields $\hat{A}_\sigma(t_2,z)$ and atomic operators $\hat{\rho}_\sigma(t_2,z)$. Thus we have

$$\hat{A}_{+,k}^{(1)} \cong \frac{1}{\xi_0}\{\hat{\rho}_{+,k}^{(0)} + \xi_0\hat{A}_{+,k}^{(0)}\} = 0,$$

$$\hat{A}_{-,k}^{(1)} \cong \frac{1}{\xi_0}\{-\xi_0[\mu-\chi]\hat{A}_{+,k}^{(0)} + \xi_0\mu\hat{A}_{+,k}^{(0)}\} = \chi\hat{A}_{+,k}^{(0)},$$

$$\hat{A}^{(2)}_{+,k} \cong -\frac{1}{\xi_0}\hat{\rho}^{(0)}_{+,k} \cong \hat{A}^{(0)}_{+,k}(t_2),$$

$$\hat{A}^{(2)}_{-,k} \cong -\frac{1}{\xi_0}\hat{\rho}^{(0)}_{-,k} \cong (\mu-\chi)\hat{A}^{(0)}_{+,k}(t_2). \quad (31)$$

After substitution of these values and eigenvalues into Eqs. (29) and (30) we have found finally the following solution for the field operator:

$$\hat{A}(t,z) = \hat{A}_+(t,z) + \hat{A}_-(t,z) = f(\chi)\{\hat{A}_{+,in}[\nu_1(t-\tau)-z] - (\chi-\mu)e^{-i\varphi_1} \times \hat{A}_{+,in}[\nu_1(t-\tau)+z]\}, \quad (32)$$

where τ is the temporal delay due to the staying of the slow-light field. This solution describes that the two pulses move opposite with the same velocity. We note that the nonadiabatic switching off of the second control field changes the backward field amplitude from $\hat{A}_- = \mu\hat{A}_+$ to the value $\hat{A}_- = (\mu-\chi)\hat{A}_+$. The additional component $-\chi\hat{A}_+$ was generated due to the atomic coherence gratings at $t=t_2$. The field $\hat{A}_-$ has the maximum amplitude for $\chi=2^{-1/2}\approx 0.7$ where we get $\hat{A}_- = \frac{1}{4}\hat{A}_{+,in}$, where $\hat{A}_+ = \frac{1}{4}(2+\sqrt{2})\hat{A}_{+,in}$; the amplitude ratio is $\hat{A}_-/\hat{A}_+ \cong 0.3$, and the total energy of both fields is $W = \frac{1}{2}(1+\sqrt{1-\chi^2} - \frac{1}{2}\chi^4)W_{+,in}|_{\chi=2^{-1/2}} \cong (\pi/4)W_{+,in} \cong 0.79W_{+,in}$. It is possible to assume that this loss of the total energy is connected with the decreasing of the four long-lived coherence grating amplitudes, which lead to the small value of the factor $f(\chi)$. Decreasing the four grating amplitudes correlates with an existence of other gratings $(n>2)$ after the abrupt nonadiabatical switching off the second control field. The present analytical approximation gives large transfer between the forward and backward fields, to be comparable with the recent copropagated scheme [14]. An interesting new aspect of the proposed technique is that the temporal profiles of the fields $\hat{A}_-$ and $\hat{A}_+$ are reversed with respect to each other, which is a consequence of the FWM processes and cannot be realized by using the usual mirrors. We have found the solution for $f(\chi)$ in Eq. (14) without using the second equation of Eqs. (15), which points out the possibility of a higher value for $f(\chi)$. Therefore, the possibility of a higher transformation for the total field energy is interesting to study [which can be done numerically elsewhere using the general equations (9) and (10)] generation of two temporally reversed pulses whose stoppage should be controlled. Ignoring the small decrease in total energy of $0.79W_{+,in}$ in the above analytical results, we can analyze the specific quantum properties of the generated pulse. If the initial probe field is an ideal single-photon wave packet after emission, we get the following state:

$$|\Psi_{a+f}(t,z')\rangle \cong f|\Psi_+[c(t-\tau-z/\nu_1)-z']\rangle - (\chi-\mu)fe^{-i\varphi_1} \times |\Psi_-\{c[t-\tau-(L-z)/\nu_1]-z'\}\rangle, \quad (33)$$

where z is the position of the ultraslow light in the medium at the point of switching off the second control field. The generated field is in the entangled state although maximum entanglement is not achieved. It is principally possible to increase the entanglement by repetition of this procedure using only the new forward field. After two such procedures we get the state

$$|\Psi_{a+f}(t,z')\rangle \approx f^2|\Psi_+[c(t-\tau_1)-z]\rangle - (\chi-\mu)f\{e^{-i\phi}|\Psi_-[c(t-\tau_2)+z']\rangle + fe^{-i\varphi_2}|\Psi_-[c(t-\tau_3)+z']\rangle\}, \quad (34)$$

where $\tau_{1,2,3}$ are the relative time delays and φ_2 is the additional controllable phase. Performing such a procedure several times we can change the spectral properties of the final state and increase its entanglement. At the same time it should be noted that the presented analytical approach predicts lower-energy transfer efficiency for the entangled states than takes place mentioned above for the one procedure. Principally the entangled states of the two temporally reversed fields can also be generated from the two-photon quantum states as well as from the squeezed states of the probe pulse. At the same time, if the initial quantum state is the coherent state $|A_{in}\rangle = \exp\{-\frac{1}{2}|A_{in}|^2 + A_{in}a^+_{k_1}\}|0\rangle$ **(**where $a^+_{k_1} = \int_{-\infty}^{\infty}d\omega\alpha_n(\omega-\omega_{31})a^+_{k_1}(\omega)[\int_{-\infty}^{\infty}d\omega|\alpha_n(\omega-\omega_{21})|^2=1]$**)**, we can lead finally to the state, $|A_{in}\rangle \cong \exp\{-\frac{1}{2}|A_{in}|^2 + fA_{in}[a^+_{k_1} - (\chi-\mu)\tilde{a}^+_{-k_1}]\}|0\rangle$, which is the product of the two coherent states of the fields with temporally and spectrally reversed profiles, where $\tilde{a}^+_{-k_2} = e^{i\varphi}\int_{-\infty}^{\infty}d\omega\alpha_n(\omega_{31}-\omega)e^{i\omega(\tau_2-\tau_1)}a^+_{-k_2}(\omega)$, and φ is a controllable constant phase. The proposed scheme plays a role of the semitransparent four-wave mirror with reversed temporal profile, and entanglement of the coherent states needs additional nonlinear interaction similar to the recent work [10,12], where π phase shift can be easily achieved due to lengthened interaction time between the coherent pulses. From this point of view the presented scheme introduces a new possibility of using standing or almost stationary light in the media with four and more large number of active atomic levels.

In conclusion we have demonstrated quantum control of a weak quantum field for ultraslow and almost stationary light in a three-level condensed medium not limited by Doppler broadening. We have proposed a scheme of quantum entangled photon generation based on a lengthened interaction time and nonadiabatic manipulation with almost stationary light phenomenon. Such a scheme can be realized in particularly in a rare-earth-doped solids demonstrated already for an ultraslow group velocity. We note that, owing to the temporal reversibility of the entangled single-photon wave packets, the proposed scheme can be similar to entanglement schemes based on photon polarization. The quantum manipulations and processing with the temporally reversed entangled states represent an interesting subject for further research especially for the spectral engineering of the quantum states.

The authors acknowledge that this work was supported by Korea Research Foundation Grant No. KRF-2003-070-C00024 and the Quantum Cryptography Project by Korean Ministry of Science and Technology.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).

[2] S. E. Harris, Phys. Today **50**(7), 36 (1997).

[3] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999); A. V. Turukhin, V. S. Sudarshanam, M. S. Shahriar, J. A. Musser, B. S. Ham, and P. R. Hemmer, Phys. Rev. Lett. **88**, 023602 (2002).

[4] M. O. Scully and S. M. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, UK, 1997).

[5] H. Schmidt and A. Imamoglu, Opt. Lett. **21**, 1936 (1996).

[6] S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. **81**, 3611 (1998); B. S. Ham and P. R. Hemmer, *ibid.* **84**, 4080 (2000); B. Ham, Appl. Phys. Lett. **85**, 893 (2004).

[7] B. S. Ham, M. S. Shahriar, M. K. Kim, and P. R. Hemmer, Opt. Lett. **22**, 1849 (1997).

[8] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001); D. F. Philips *et al.*, Phys. Rev. Lett. **86**, 783 (2001); A. S. Zibrov, A. B. Matsko, O. Kocharovskaya, Y. V. Rostovtsev, G. R. Welch, and M. O. Scully, *ibid.* **88**, 103601 (2002).

[9] M. D. Lukin and A. Imamoglu, Phys. Rev. Lett. **84**, 1419 (2000).

[10] M. G. Payne and L. Deng, Phys. Rev. Lett. **91**, 123602 (2003).

[11] M. Saffman and T. G. Walker, Phys. Rev. A **66**, 065403 (2002).

[12] D. Petrosyan and G. Kurizki, Phys. Rev. A **65**, 033833 (2002); M. Paternostro, M. S. Kim, and B. S. Ham, *ibid.* **67**, 023811 (2003), and references for the Schrodinger's cat therein.

[13] L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).

[14] A. Andre' and M. D. Lukin, Phys. Rev. Lett. **89**, 143602 (2002).

[15] M. Bajcsy, A. S. Zibrov, and M. D. Lukin, Nature (London) **426**, 638 (2003).

[16] A. B. Matsko, Y. V. Rostovtsev, O. Kocharovskaya, A. S. Zibrov, and M. O. Scully, Phys. Rev. A **64**, 043809 (2001).

[17] A. Erdte'lyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, *Higher Transcental Functions* (McGraw-Hill, New York, 1975).