PHYSICAL REVIEW A **71**, 053613 (2005)

Quantitative probe of pairing correlations in a cold fermionic-atom gas

R. A. Duine* and A. H. MacDonald†
Department of Physics, The University of Texas at Austin, 1 University Station C1600, Austin, Texas 78712-0264, USA
(Received 18 January 2005; published 20 May 2005)

A quantitative measure of the pairing correlations present in a cold gas of fermionic atoms can be obtained by studying the dependence of rf spectra on hyperfine-state populations. This proposal follows from a sum rule that relates the total interaction energy of the gas to rf spectrum line positions. We argue that this indicator of pairing correlations provides information comparable to that available from the spin-susceptibility and NMR measurements common in condensed-matter systems.

DOI: 10.1103/PhysRevA.71.053613 PACS number(s): 03.75.Kk, 32.80.Pj, 67.40.−w

I. INTRODUCTION

The realization of degenerate atomic Fermi gases [1–7] has opened new opportunities for experimental discovery. The focus to date has mainly been on efforts to observe the condensation of atomic Cooper pairs to form a superfluid state similar to the BCS state of electrons in a superconductor [8]. Strategies for achieving observable pairing effects have so far hinged on the occurrence of strong attractive atom-atom interactions near a Feshbach resonance [9–11], in which a molecular bound state of one atom-atom scattering channel is close to the two-atom continuum threshold of another. The proximity of a Feshbach resonance can be adjusted by tuning a magnetic bias field, drastically altering the scattering behavior of atoms and allowing the *s*-wave scattering length to be varied over values corresponding to effective interactions that are weak or strong, and repulsive or attractive. The scattering length, henceforth denoted by a , completely characterizes the interaction properties of atoms at low temperatures and low densities. The Feshbach resonance makes it possible to study one of the paradigms [12–14] of fermion pairing theory, the BCS–Bose-Einstein Condensate(BEC) crossover, experimentally.

Experimental groups have already observed the formation of thermal gases [15,16] and Bose-Einstein condensates [17–19] of diatomic molecules. Condensation of fermionic-atom pairs on the attractive interaction side of the resonance, where there is no two-atom bound state and the analogy to the BCS-BEC crossover problem is closer, has also been reported [20,21]. Because the BCS transition does not manifest itself strongly [22] in the expanded density profile of the gas there is a need for quantitative and direct measurements of pairing correlations, one that has motivated a large number of proposals. Several works focused on the change of light scattering due to the transition from the normal to the superfluid phase as a detection method [23–26]. Later, resonant laser light was proposed to induce tunneling between the superfluid and normal states of the gas [27]. The experimental realization of Bragg spectroscopy in a Bose gas [28] has inspired theoretical work on pairing effects in the dynamic structure function [29,30]. Other interesting proposals include ones based on pairing-induced changes in collective-mode frequencies [31–36], rotational properties of the gas [37,38], the expansion of the gas [39], and atomic-density noise-correlation properties [40].

In this paper we propose a more direct probe of pairing correlations that is similar to the spin-magnetic-susceptibility and nuclear-spin-relaxation probes commonly used to detect electron pairing in condensed-matter systems, and is able to detect pairing even when it does not lead to long-range coherence. We suggest a measurement of the cost in interaction energy when the number of Cooper pairs in the system is reduced by making the hyperfine-state populations unequal. As we show in Sec. II, this energy change can be extracted from data obtained using the rf spectroscopy techniques that have already been developed by several experimental groups [41–43]. In Sec. III we illustrate the direct relationship between the hyperfine population dependence of the interaction energy and pairing correlations, and compare the predictions of BCS theory for this quantity with its predictions for the more familiar magnetic-susceptibility probe, which measures instead the dependence of total (interaction plus kinetic) energy on the same variable. We end in Sec. IV with a discussion and our conclusions.

II. INTERACTION ENERGY SUM RULE

We consider a gas of fermionic atoms that consists of a mixture of two hyperfine states denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$. We assume the temperatures to be low enough so that only *s*-channel interactions, forbidden between atoms in the same hyperfine state by the Pauli exclusion principle, are significant. The Hamiltonian of the gas is therefore $H=H_0+H_{\text{int}}$, where the noninteracting part is

$$H_0 = \int d\mathbf{x} \sum_{\alpha=\{\uparrow,\downarrow\}} \psi_\alpha^\dagger(\mathbf{x})\left(-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m}+\epsilon_\alpha\right)\psi_\alpha(\mathbf{x}), \qquad (1)$$

and ψ_α is the fermionic annihilation operator for hyperfine state $|\alpha\rangle$. The internal Zeeman energy of a hyperfine state is denoted by ϵ_α, and for simplicity we have neglected any inhomogeneity of the magnetic field. In particular, this im-

*Electronic address: duine@physics.utexas.edu; URL: http://www.ph.utexas.edu/~duine
†Electronic address: macd@physics.utexas.edu; URL: http://www.ph.utexas.edu/~macdgrp

plies that we neglect the effects of the magnetic trapping potential. As we will see later on, this does not impose important restrictions on the applicability of our theory. We take the interaction between unlike hyperfine states to be a contact interaction with strength $V_{\uparrow\downarrow}$, which should be chosen to produce the correct two-body scattering amplitude [44]. With these assumptions, the interaction part of the Hamiltonian is

$$H_{\text{int}} = V_{\uparrow\downarrow} \int d\mathbf{x}\, \psi_\uparrow^\dagger(\mathbf{x})\psi_\downarrow^\dagger(\mathbf{x})\psi_\downarrow(\mathbf{x})\psi_\uparrow(\mathbf{x}). \tag{2}$$

In the rf experiments one of the system hyperfine species (say $|\downarrow\rangle$) is coupled to a spectator hyperfine state ($|s\rangle$), and the number of atoms in the spectator state, N_s, is detected as a function of the frequency of the rf field. In the linear response limit N_s is proportional to the rate of $|\downarrow\rangle \to |s\rangle$, transitions which we denote by $I(\omega)$. We define the position of the associated rf spectrum absorption line as

$$\hbar\overline{\omega} \equiv \frac{\int d\omega\, \hbar\omega I(\omega)}{\int d\omega\, I(\omega)}. \tag{3}$$

Using a formal golden-rule expression, $I(\omega)$ can be expressed in terms of a two-particle correlation function of fermion fields. It then follows from the fermion analog of sum rules derived in Refs. [45,46] that

$$\hbar\overline{\omega} = \hbar\omega_0 + \frac{1}{n_\uparrow}(V_{\downarrow\uparrow} - V_{s\uparrow})\langle\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow\rangle, \tag{4}$$

where $V_{s\uparrow}$ denotes the strength of the contact interaction between the spectator and $|\uparrow\rangle$ hyperfine states, and n_α denotes the average density in spin state $|\alpha\rangle$. The shift of $\hbar\overline{\omega}$ from the bare line position $\hbar\omega_0 = \epsilon_\downarrow - \epsilon_s$ differs from the interaction energy per volume by a factor $V_{\downarrow\uparrow}n_\uparrow/(V_{\downarrow\uparrow} - V_{s\uparrow})$, which can be held constant through the experiments and if necessary can be accurately determined by separate measurements. We conclude that the rf spectra enable a direct measurement of the interaction energy density

$$e_{\text{int}}(n_\uparrow, n_\downarrow) \equiv V_{\uparrow_\downarrow}\langle\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow\rangle. \tag{5}$$

III. PAIRING AND INTERACTION ENERGY IN BCS THEORY

The inverse spin susceptibility of an unpolarized system of spin-1/2 particles may be expressed in terms of the dependence of free energy on spin polarization:

$$\chi_s^{-1} = \left.\frac{\partial^2 f_{\text{tot}}}{\partial\,\delta n^2}\right|_n = \frac{1}{2}\left[\frac{\partial^2 f_{\text{tot}}(n_\uparrow, n_\downarrow)}{\partial n_\uparrow^2} - \frac{\partial^2 f_{\text{tot}}(n_\uparrow, n_\downarrow)}{\partial n_\uparrow\,\partial n_\downarrow}\right], \tag{6}$$

where f_{tot} is the total free energy per unit volume of the gas, n is the total density, and $\delta n \equiv n_\uparrow - n_\downarrow$ is the spin density. It is well known that χ_s is strongly suppressed when atoms can gain energy by pairing. (χ_s vanishes as $T \to 0$ in the BCS state.) For attractive atom-atom interactions, the energy cost of finite-spin polarization has positive contributions from both interaction and kinetic energy. In the following we compare the spin susceptibility with an alternate quantity that is defined in terms of the interaction energy alone and can be extracted from rf spectroscopy experiments performed for a series of hyperfine-state populations:

$$\chi_{\text{int},s}^{-1} = \left.\frac{\partial^2 e_{\text{int}}(n_\uparrow, n_\downarrow)}{\partial\,\delta n^2}\right|_n. \tag{7}$$

As we show below, this quantity and the inverse spin susceptibility provide similar probes of pairing correlations.

We evaluate this *interaction susceptibility* using BCS theory from which it follows that

$$e_{\text{int}}(n_\uparrow, n_\downarrow) = \frac{|\Delta|^2}{V_{\uparrow\downarrow}} + \frac{4\pi a\hbar^2 n_\uparrow n_\downarrow}{m}, \tag{8}$$

where the dependence of the gap $\Delta \equiv V_{\uparrow\downarrow}\langle\psi_\downarrow\psi_\uparrow\rangle$ on temperature and hyperfine densities can be determined by solving the self-consistent mean-field equations. The mean-field Hamiltonian is [22]

$$\begin{aligned} H_{\text{MF}} = \int d\mathbf{x}\Bigg\{ & \psi_\uparrow^\dagger(\mathbf{x})\left(-\frac{\hbar^2\nabla^2}{2m} + \frac{4\pi a\hbar^2 n_\downarrow}{m} - \mu_\uparrow\right)\psi_\uparrow(\mathbf{x}) \\ & + \psi_\downarrow^\dagger(\mathbf{x})\left(-\frac{\hbar^2\nabla^2}{2m} + \frac{4\pi a\hbar^2 n_\uparrow}{m} - \mu_\downarrow\right)\psi_\downarrow(\mathbf{x}) \\ & + \Delta\psi_\uparrow^\dagger(\mathbf{x})\psi_\downarrow^\dagger(\mathbf{x}) + \Delta^*\psi_\downarrow(\mathbf{x})\psi_\uparrow(\mathbf{x}) - \frac{|\Delta|^2}{V_{\uparrow\downarrow}} - \frac{4\pi a\hbar^2 n_\uparrow n_\downarrow}{m}\Bigg\}. \end{aligned} \tag{9}$$

Note that the renormalization $V_{\uparrow\downarrow} \to 4\pi a\hbar^2/m$ can be made at this stage in the Hartree mean-field potential. However, we should not make this replacement in the part of the Hamiltonian that is quadratic in the gap parameter. As we will see below, this is because the BCS theory automatically incorporates this renormalization, and we would otherwise double count the effects of the interactions.

The chemical potentials of the two hyperfine states are denoted by μ_α, and are not necessarily equal, thus allowing for a density difference between the two hyperfine states.

The partial densities are given by

$$n_\alpha = \int \frac{d\mathbf{k}}{(2\pi)^3}\{|u_\mathbf{k}|^2 N(\hbar\omega_{\mathbf{k},\alpha}) + |v_\mathbf{k}|^2[1 - N(\hbar\omega_{\mathbf{k},-\alpha})]\}, \tag{10}$$

where $u_\mathbf{k}$ and $v_\mathbf{k}$ are the Bogoliubov coherence factors, $N(x) = [e^{\beta x} + 1]^{-1}$ is the Fermi distribution function, and the $|\uparrow\rangle$ quasiparticle dispersion is given by

$$\hbar\omega_{\mathbf{k},\uparrow} = \frac{\mu_\downarrow' - \mu_\uparrow'}{2} + \sqrt{[\epsilon_\mathbf{k} - (\mu_\uparrow' + \mu_\downarrow')/2]^2 + |\Delta|^2}. \tag{11}$$

An identical expression, with the hyperfine labels interchanged, applies for $\hbar\omega_{\mathbf{k},\downarrow}$. The Hartree-Fock mean-field shift is absorbed in the chemical potential via $\mu_\alpha' = \mu_\alpha - 4\pi a\hbar^2 n_{-\alpha}/m$. These equations for the densities need to be solved together with the BCS gap equation

$$\int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1 - N(\hbar\omega_{\mathbf{k},\uparrow}) - N(\hbar\omega_{\mathbf{k},\downarrow})}{2\sqrt{[\epsilon_{\mathbf{k}} - (\mu_\uparrow + \mu_\downarrow)/2]^2 + |\Delta|^2}} = -\frac{1}{V_{\uparrow\downarrow}}. \quad (12)$$

Equation (12) contains an ultraviolet divergence that is renormalized by introducing the *s*-wave scattering length (a) between $|\uparrow\rangle$ and $|\downarrow\rangle$ hyperfine states by means of the Lippmann-Schwinger equation

$$\frac{m}{4\pi a\hbar^2} = \frac{1}{V_{\uparrow\downarrow}} + \int_{\mathbf{k}\le k_\Lambda} \frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{2\epsilon_{\mathbf{k}}}, \quad (13)$$

where we have introduced an ultraviolet cutoff k_Λ. It follows [22] from Eq. (13) that

$$V_{\uparrow\downarrow} = \frac{4\pi a\hbar^2}{m}\frac{1}{1 - 2ak_\Lambda/\pi}. \quad (14)$$

Since a typical range of the interatomic potential is $\sim 100a_0$, we take $k_\Lambda \simeq (100a_0)^{-1}$ and $a = -2000a_0$, and find that $V_{\uparrow\downarrow} \simeq 0.07(4\pi a\hbar^2/m)$. Note that although the spin densities and the BCS gap parameter are independent of the short-range properties of the interatomic potential, the interaction energy in Eq. (8) is not [22]. In principle, we could improve upon this theory by generalizing the BCS theory to incorporate the full interatomic interaction potential between the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. Although this is in principle straightforward to do, it would be numerically cumbersome, since the BCS gap would not be a constant any more, and the gap equation would become an integral equation. We believe that the above estimate of the interaction strength $V_{\uparrow\downarrow}$ is sufficient for demonstrating the use of the probe of pairing correlations presented in this paper.

Above the critical temperature $T_{\mathrm{BCS}} \simeq 0.6T_F e^{-\pi/2k_F|a|}$, $\Delta \to 0$ so that

$$e_{\mathrm{int}}(n_\uparrow, n_\downarrow) = \frac{4\pi a\hbar^2 n_\uparrow n_\downarrow}{m}, \quad (15)$$

and $\chi^{-1}_{\mathrm{int},s} = 2\pi|a|\hbar^2/m$ is temperature independent.

For $T < T_{\mathrm{BCS}}$ the interaction energy is given by Eq. (8). In Fig. 1 we show the interaction energy density as a function of temperature, normalized to its normal-state value given in Eq. (15). The calculations are made for the case of $n_\uparrow = n_\downarrow = n/2$, scattering length $a = -2000a_0$ where a_0 is the Bohr radius, and density $n = 10^{12}\ \mathrm{cm}^{-3}$. For these parameters $T_{\mathrm{BCS}} \simeq 0.005T_{\mathrm{F}}$. To find a nonzero solution for the BCS gap parameter, we obviously need a negative value of the scattering length, as this corresponds to effectively attractive interactions which favor pairing. The case of repulsive interactions is nonetheless very interesting, since for sufficiently large interactions one might observe a transition to a ferromagnetic phase [22]. Note that in the absence of pairing the interaction energy is for low densities equal to the Hartree shift given in Eq. (15) and independent of temperature. As is suggested by Fig. 1, a possible signature for the occurence of the BCS transition is a measurement of the change in the interaction energy as a function of temperature. We show now that a more sensitive probe is the interaction energy susceptibility defined in Eq. (7).

Linearization of the gap equation implies that for $T \uparrow T_{\mathrm{BCS}}$,

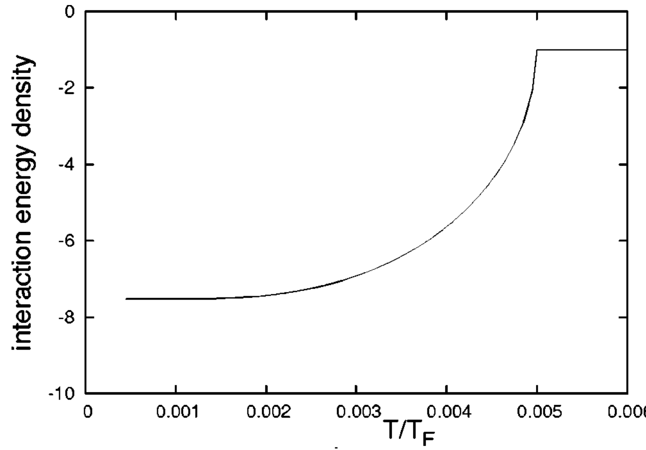

FIG. 1. Plot of $e_{\mathrm{int}}(n_\uparrow, n_\downarrow)$ *vs.* temperature, in units of $4\pi|a|\hbar^2 n_\uparrow n_\downarrow/m$. In these units, the interactions energy density goes to minus one in the normal state

$$\chi^{-1}_{\mathrm{int},s} = \frac{(\pi + 2|a|k_\Lambda)}{k_{\mathrm{F}}|a|}\left(\frac{\partial n}{\partial \mu}\right)^{-1} + 2\pi|a|\hbar^2/m. \quad (16)$$

In Fig. 2 we plot $\chi^{-1}_{\mathrm{int},s}$ and the inverse susceptibility vs temperature. The parameters are the same as in Fig. 1. The inverse interaction susceptibility is greatly enhanced for temperatures below the BCS transition temperature, thus providing a clear signature of atomic Cooper pairs.

IV. DISCUSSION AND CONCLUSIONS

Although we have so far assumed a homogeneous Fermi gas, we believe that our results also apply to the case of a trapped, and therefore inhomogeneous, Fermi gas. There are two main reasons for this. First, because the inverse Fermi wave vector is much smaller than the harmonic oscillator length for the experimental systems of interest, the system may be treated within a local density approximation [22].

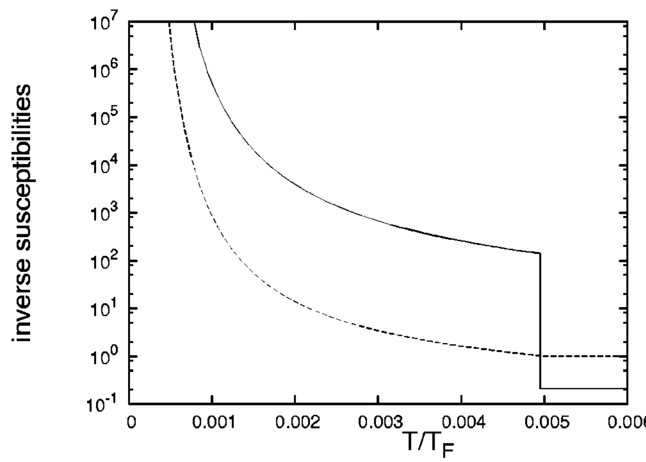

FIG. 2. Plot of $\chi^{-1}_{\mathrm{int},s}$ (solid line) and χ^{-1}_s (dashed line) vs temperature. Both susceptibilities are plotted in units of the quasiparticle density of states $(\partial n/\partial\mu)$ at the Fermi level. Note that the mean-field-theory interaction susceptibility is discontinuous at the critical temperature.

Since the BCS gap parameter Δ will be largest in the center of the trap, the result in Eq. (16) should be evaluated at the center of the trap. Moreover, the density is largest in this region, and therefore most uniform close to the center. Second, as demonstrated by Gupta *et al.* [42], one is able to experimentally resolve the number of atoms that are transferred to the state $|s\rangle$ in a small region near the center of the trap. Therefore, the results presented in this paper should be applicable to such experiments.

For strong attractive interactions, $k_F|a|\sim 1$, mean-field theory is not expected to be accurate. In particular, the superfluid transition temperature is expected to be limited by the loss of long-range coherence rather than by the thermodynamics of pair formation. The thermodynamic probe we discuss here is sensitive to the occurrence of pair correlations and not particularly sensitive to the establishment of long-range coherence. It should therefore be able to detect the gradual development of pairing correlations with increasing interaction strength as the superfluid state is approached. We note that Bourdel *et al.* [47] have measured the ratio of the interaction energy and kinetic energy of a Fermi gas by comparing expanded density profiles of an interacting gas of atoms with expansion profiles of a gas at zero scattering length. In the weak-coupling limit such a measurement would provide direct information on the temperature dependence of the interaction energy, since the kinetic energy is almost independent of temperature in this case. In the strong-coupling limit it is, however, not clear how the kinetic energy depends on the density difference, and it is, therefore, not obvious that a measurement of the ratio of interaction and kinetic energy for different hyperfine-state populations would provide a sensitive probe of pairing correlations in the gas in this limit.

Finally we remark on the possibility of realizing inhomogeneous pair-condensate states [48] in cold atom systems with unbalanced hyperfine state populations [49,50]. The appearance of such a state slightly limits the probe for pairing correlations presented in this paper. In particular, these so-called Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) states are expected to occur for temperatures below $T\simeq 0.55T_{\rm BCS}$ [51] and for a hyperfine density difference that is of the order of the gap divided by the Fermi energy [50], i.e.,

$$\frac{\delta n}{n}\sim\frac{\Delta}{\epsilon_F}, \tag{17}$$

corresponding to roughly 10% for the parameters used in this paper. We note, however, that the FFLO state only occurs over a narrow range of population imbalance, and, in particular, does not occur for an imbalance that is too large [52]. An interesting feature of these states, if they can be realized, is their unusual [53] vortex-lattice structures, which could be detected by bringing the system to equilibrium in a rotating reference frame.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation under Grant No. DMR-0115947 and by the Welch Foundation. We are grateful for informative conversations with D. S. Jin and R. G. Hulet.

[1] B. DeMarco, S. B. Papp, and D. S. Jin, Phys. Rev. Lett. **86**, 5409 (2001).
[2] A. G. Truscott, K. E. Strecker, W. I. McAlexander, G. Partridge, and R. G. Hulet, Science **291**, 2570 (2001).
[3] F. Schreck, L. Khaykovich, K. L. Corwin, G. Ferrari, T. Bourdel, J. Cubizolles, and C. Salomon, Phys. Rev. Lett. **87**, 080403 (2001).
[4] S. R. Granade, M. E. Gehm, K. M. O'Hara, and J. E. Thomas, Phys. Rev. Lett. **88**, 120405 (2002).
[5] S. Jochim, M. Bartenstein, G. Hendl, J. H. Denschlag, R. Grimm, A. Mosk, and M. Weidemüller, Phys. Rev. Lett. **89**, 273202 (2002).
[6] G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Phys. Rev. Lett. **89**, 150403 (2002).
[7] Z. Hadzibabic, S. Gupta, C. A. Stan, C. H. Schunck, M. W. Zwierlein, K. Dieckmann, and W. Ketterle, Phys. Rev. Lett. **91**, 160401 (2003).
[8] H. T. C. Stoof, M. Houbiers, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **76**, 10 (1996).
[9] W. C. Stwalley, Phys. Rev. Lett. **37**, 1628 (1976).
[10] E. Tiesinga, B. J. Verhaar, and H. T. C. Stoof, Phys. Rev. A **47**, 4114 (1993).
[11] P. Courteille, R. S. Freeland, D. J. Heinzen, F. A. van Abeelen, and B. J. Verhaar, Phys. Rev. Lett. **81**, 69 (1998).
[12] A. J. Leggett, J. Phys. (Paris), Colloq. **41**, 7 (1980).
[13] P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
[14] There are, however, some subtle differences between the single-channel BCS-BEC crossover discussed in Refs. [12,13] and the BCS-BEC crossover in the multichannel case of an atomic Fermi gas near a Feshbach resonance.
[15] K. E. Strecker, G. B. Partridge, and R. G. Hulet, Phys. Rev. Lett. **91**, 080406 (2003).
[16] J. Cubizolles, T. Bourdel, S. J. J. M. F. Kokkelmans, G. V. Shlyapnikov, and C. Salomon, Phys. Rev. Lett. **91**, 240401 (2003).
[17] S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J. Hecker Denschlag, and R. Grimm, Science **302**, 2101 (2003).
[18] M. Greiner, C. A. Regal, and D. S. Jin, Nature (London) **426**, 537 (2003).
[19] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, S. Gupta, Z. Hadzibabic, and W. Ketterle, Phys. Rev. Lett. **91**, 250401 (2003).
[20] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).
[21] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).
[22] M. Houbiers, R. Ferwerda, H. T. C. Stoof, W. I. McAlexander,

C. A. Sackett, and R. G. Hulet, Phys. Rev. A **56**, 4864 (1997).
[23] Weiping Zhang, C. A. Sackett, and R. G. Hulet, Phys. Rev. A **60**, 504 (1999).
[24] J. Ruostekoski, Phys. Rev. A **60**, R1775 (1999).
[25] F. Weig and W. Zwerger, Europhys. Lett. **49**, 282 (2000).
[26] G. M. Bruun and Gordon Baym, Phys. Rev. Lett. **93**, 150403 (2004).
[27] P. Törmä and P. Zoller, Phys. Rev. Lett. **85**, 487 (2000).
[28] D. M. Stamper-Kurn, A. P. Chikkatur, A. Görlitz, S. Inouye, S. Gupta, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **83**, 2876 (1999).
[29] A. Minguzzi, G. Ferrari, and Y. Castin, Eur. Phys. J. D **17**, 49 (2001).
[30] H. P. Büchler, P. Zoller, and W. Zwerger, Phys. Rev. Lett. **93**, 080401 (2004).
[31] M. A. Baranov and D. S. Petrov, Phys. Rev. A **62**, 041601(R) (2000).
[32] F. Zambelli and S. Stringari, Phys. Rev. A **63**, 033602 (2001).
[33] G. M. Bruun and B. R. Mottelson, Phys. Rev. Lett. **87**, 270403 (2001).
[34] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).
[35] J. Kinast, S. L. Hemmer, M. E. Gehm, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. **92**, 150402 (2004).
[36] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. H. Denschlag, and R. Grimm, Phys. Rev. Lett. **92**, 203201 (2004).
[37] M. Farine, P. Schuck, and X. Viñas, Phys. Rev. A **62**, 013608 (2000).
[38] M. Cozzini and S. Stringari, Phys. Rev. Lett. **91**, 070401 (2003).
[39] C. Menotti, P. Pedri, and S. Stringari, Phys. Rev. Lett. **89**, 250402 (2002).
[40] Ehud Altman, Eugene Demler, and Mikhail D. Lukin, Phys. Rev. A **70**, 013603 (2004).
[41] C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. Hecker Denschlag, and R. Grimm, Science **305**, 1128 (2004).
[42] S. Gupta, Z. Hadzibabic, M. W. Zwierlein, C. A. Stan, K. Dieckmann, C. H. Schunck, E. G. M. van Kempen, B. J. Verhaar, and W. Ketterle, Science **300**, 1723 (2003).
[43] C. A. Regal and D. S. Jin, Phys. Rev. Lett. **90**, 230404 (2003).
[44] Since we do not include a molecular field that is coupled to the atomic field in the description of the interaction properties of the gas, we do not explicitly consider a Feshbach resonance. However, our results do apply to the case of a Feshbach resonance, provided we are far enough from the resonance.
[45] C. J. Pethick and H. T. C. Stoof, Phys. Rev. A **64**, 013618 (2001).
[46] M. Ö. Oktel, T. C. Killian, D. Kleppner, and L. S. Levitov, Phys. Rev. A **65**, 033617 (2002).
[47] T. Bourdel, J. Cubizolles, L. Khaykovich, K. M. F. Magalhães, S. J. J. M. F. Kokkelmans, G. V. Shlyapnikov, and C. Salomon, Phys. Rev. Lett. **91**, 020402 (2003).
[48] P. Fulde and A. Ferrell, Phys. Rev. **135**, A550 (1964); A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965).
[49] R. Combescot, Europhys. Lett. **55**, 150 (2001).
[50] T. Mizushima, K. Machida, and M. Ichioka, Phys. Rev. Lett. **94**, 060404 (2005).
[51] L. W. Gruenberg and Leon Gunther, Phys. Rev. Lett. **16**, 996 (1966).
[52] See R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. **76**, 263 (2004), and references therein.
[53] H. Shimahara and D. Rainer, J. Phys. Soc. Jpn. **66**, 3591 (1997); U. Klein, Phys. Rev. B **69**, 134518 (2004); Kun Yang and A. H. MacDonald, *ibid.* **70**, 094512 (2004).