Measurement of the trap properties of a magneto-optical trap by a transient oscillation method

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(Received 27 December 2004; published 18 May 2005)

We have measured the trap frequency as well as the damping coefficient of a magneto-optical trap by using a transient oscillation method. The dependence of such trap properties on the various experimental parameters such as the cooling laser intensity, detuning, and magnetic field gradient is investigated. We find that the measured trap frequency is in excellent agreement with the simple rate-equation analysis based on the Doppler cooling theory. In contrast, the damping coefficient is about twice as large as the calculated one, which is attributed to the existence of the sub-Doppler trap near the trap center.

DOI: 10.1103/PhysRevA.71.053406 PACS number(s): 32.80.Pj, 42.50.Vk

I. INTRODUCTION

One of the simplest methods to collect cold atoms is the magneto-optical trap (MOT), which was proposed and experimentally realized in 1987 [1]. It has been, since then, popularly employed in the first stage of atom optics or atom trap experiments [2] such as Bose-Einstein condensation, atom chip, and optical dipole trap. In addition, there have been a number of experimental and theoretical reports on the MOT itself: for example, observation of the sub-Doppler force [3], cold collisions [4], nonlinear spectroscopy [5], and nonlinear dynamical study [6–9].

In particular, the characteristic trap parameters such as the trap frequency and the damping constant were measured to demonstrate the existence of sub-Doppler cooling inside the MOT or in the atom molasses. For the trap-frequency (or spring-constant) measurement, several methods have been developed, such as the equipartition theorem associated with the atomic spatial profile and temperature [10,11], beam-imbalance method [11,12], oscillatory magnetic-field method [13,14], and method of free oscillation of pushed atoms [15]. Note that most studies have been applied to justify the sub-Doppler cooling or the one-dimensional Doppler cooling of two-level atoms. However, few studies have been reported for the multilevel atoms in a realistic three-dimensional trap.

Recently, we have demonstrated a novel method to measure the trap frequency by using the parametric resonance of the driven MOT [16] and have studied the Doppler-cooling theory of multilevel atoms in three dimension. In particular, we have found that the simple Doppler-cooling force for two-level atoms was not applicable to the multilevel atoms in a three-dimensional trap, despite the fact that the rate equation model was adequate to account for the trap-frequency data. We also have observed that the trap frequency was dependent on the intensity ratios of the six laser beams in the MOT.

In this paper, we make a quantitative investigation of the Doppler-cooling theory for multilevel atoms in a three-dimensional MOT by measuring the trap frequency as well as the damping coefficient. The measurement method is to detect the temporal oscillatory behavior of the pushed atomic cloud [15] as the magnetic field gradient or the laser detuning is varied. The atomic motion in the MOT is simply given by a damped harmonic oscillator model with the damping coefficient β and the trap frequency f_0. When a uniform magnetic field (B_z) is applied to the MOT, the position of the trap center is shifted by B_z/b, where b is the magnetic field gradient in the z axis of the MOT.

When the uniform magnetic field is suddenly turned off, the atomic cloud returns to the original trap center. In case of the underdamped motion, one can extract the trap parameters by measuring the trajectory of the released atomic cloud. In particular, it is found that the damping coefficient is increased due to the existence of the sub-Doppler trap near the magnetic field center, which is explained by the Monte Carlo simulation. Finally we also compare the results of the free oscillation method with those of the parametric resonance method [16]. Note that the atomic number dependence of the trap parameters is not considered because the experiment was performed at low saturation conditions and for the small number of atoms in the MOT.

II. TRANSIENT OSCILLATION METHOD

We have employed a similar experimental scheme that was well described in the previous reports [9,17]. Here we mention some additional procedures used in the experiment. We captured about 2×10^8 atoms in a standard vapor-cell MOT by using six counterpropagating laser beams with a $e^{-1/2}$ width of 1.5 cm. Addition of a uniform magnetic field (B_z) along the anti-Helmholtz coils axis (z axis) resulted in movement of the center of the atomic cloud by B_z/b. In the experiment B_z=8.8 G, so that the initial displacement of the center of the MOT is 5.4 mm at b=12 G/cm (Fig. 1). Note that we have found that the trap parameters are independent of the changes of the displacement at various values of b. Each part of the Helmholtz coils to produce a uniform magnetic field is composed of 5 turns, which has a 50 μs switching time by a FET switch. This response time is quite shorter than the characteristic time of the atomic motion (20–50 ms).

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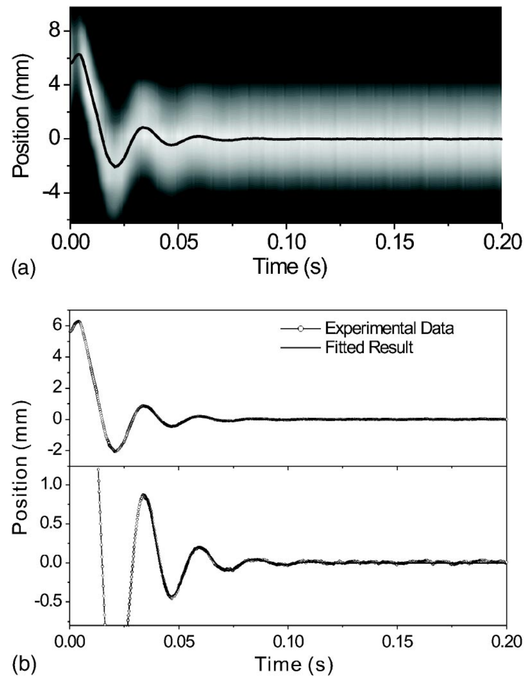

FIG. 1. (a) The contour plot of the signals recorded by the photodiode array shows a typical oscillating motion of atomic cloud. The curve presenting the maximum brightness of the absorption signal is superposed on the contour plot. (b) The same curve as in (a) and the fitted curve obtained by Eq. (1). The detailed plot after one period is shown in the lower panel.

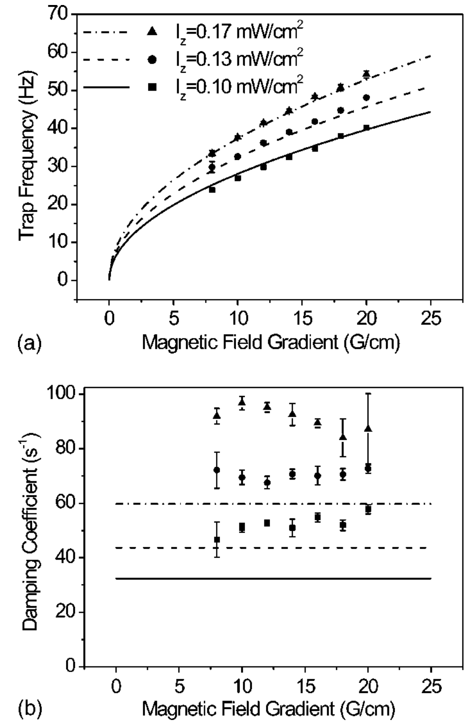

FIG. 2. The dependence of the trap frequency (a) and damping coefficient (b) on the magnetic field gradient. Here the trapping beam intensities are $I_z = 0.10\ \mathrm{mW/cm^2}$ (filled square), $0.13\ \mathrm{mW/cm^2}$ (filled circle), and $0.17\ \mathrm{mW/cm^2}$ (filled triangle). The solid, dashed, and dashed-dotted lines are derived from theoretical model.

As soon as the uniform magnetic field is turned off, the atomic cloud starts to return to the original trap center. The trajectory of the atomic cloud center is simply given by

$$z(t) = z_0 + A\exp(-\beta t/2)\left(\cos 2\pi f_0 t + \frac{\beta}{4\pi f_0}\sin 2\pi f_0 t\right), \tag{1}$$

where f_0 is the trap frequency, β is the damping coefficient, z_0 is the equilibrium position, and A is the initial displacement from equilibrium. Due to the nonlinearity of MOT, the trajectory is not perfectly described by Eq. (1) for large displacement. Therefore, as presented in Fig. 1, we have fitted the data after one period of the atomic motion. Triggered by the turn off of the B_z field, the motion of atomic cloud is recorded at the 16-element photodiode array of 1 mm width. Each channel detects the absorption change of a resonant probe laser (1.6 cm wide and 0.5 cm high) that is illuminated perpendicular to the atomic oscillation direction.

Figure 1(a) shows the contour plot of the typical absorption signals of the 16-channel photodiode array versus time (taken at 1/5000 s time interval), superposed by a curve corresponding to the maximum brightness. Here the vertical axis shows the position of the photodiode array and horizontal axis represents the time elapsed after switch off of the magnetic field. The bright region indicates large absorption due to a large number of atoms. Figure 1(b) shows the same curve as in Fig. 1(a) and its fitted result using Eq. (1) for the region after elapse of one period, which is shown in the lower panel. From the fit presented in Fig. 1(b) one can obtain the trap frequency and the damping coefficient. The experimental conditions and the fitted results are presented in Fig. 2 for the magnetic field gradient of 12 G/cm.

We have measured the trap parameters by varying the laser intensity, detuning, and magnetic field gradient. The results are presented in Figs. 2 and 3. Figure 2 shows the dependence of trap frequency (a) and damping coefficient (b) on the magnetic field gradient. Here the frequency detuning is $\Delta = -2.71\ \Gamma$ ($\Delta = \omega_L - \omega_A$, for the laser frequency ω_L, the atomic resonance frequency ω_A, and the natural linewidth $\Gamma = 2\pi \times 5.9$ MHz), and the trapping beam intensities are $I_z = 0.10\ \mathrm{mW/cm^2}$ (filled square), $0.13\ \mathrm{mW/cm^2}$ (filled circle) and $0.17\ \mathrm{mW/cm^2}$ (filled triangle), respectively. Note that the laser intensities in the transverse directions ($I_x = I_y$) are $0.62\ \mathrm{mW/cm^2}$. The solid, dashed, and dashed-dotted lines in the figure are the calculated results from a theoretical model explained in the next section.

The dependence of the trap frequency (a) and the damping coefficient (b) on the detuning is shown in Fig. 3, where $I_z = 0.17\ \mathrm{mW/cm^2}$, $I_x = I_y = 0.62\ \mathrm{mW/cm^2}$, and $b = 10$ G/cm. Note that the solid line is obtained from the rate equation for the Doppler theory, whereas the dashed line is from the simple Doppler theory for two level atom where the saturation intensity is substituted by the averaged value $(I_s)_{av} = 3.78\ \mathrm{mW/cm^2}$. As shown in Figs. 2 and 3, the trap frequencies are in good agreement with the theoretical values. The damping coefficients, on the other hand, are about twice

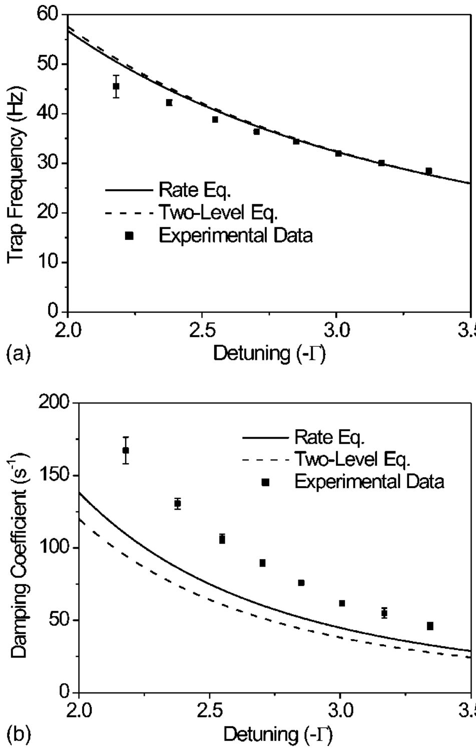

FIG. 3. The dependence of the trap frequency (a) and damping coefficient (b) on the laser detuning is presented, where $I_z = 0.17\ \mathrm{mW/cm^2}$, $I_x = I_y = 0.62\ \mathrm{mW/cm^2}$, and $b = 10$ G/cm.

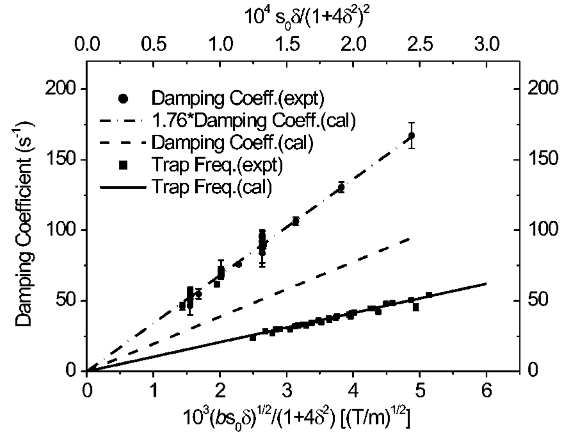

FIG. 4. The damping coefficient versus $s_0\delta/(1+4\delta^2)^2$ [filled circles, experimental data; dashed line, calculated results; dashed-dotted line, calculated results multiplied by 1.76] and the trap frequency versus $\sqrt{bs_0\delta}/(1+4\delta^2)$ [filled squares, experimental data; solid line, calculated results].

larger than the simple theoretical predictions. In the following section, we provide a quantitative description of the theoretical model and present a plausible explanation of the discrepancy found in the damping coefficient.

III. DAMPING COEFFICIENT

In the Doppler cooling theory for two-level atoms, the damping coefficient (β) and the trap frequency (f_0) are given by

$$\beta = \frac{8\hbar k^2}{m}\frac{s_0\delta}{(1+4\delta^2)^2}, \tag{2}$$

$$f_0 = \sqrt{\frac{2k\mu_B}{\pi^2 m}}\frac{\sqrt{bs_0\delta}}{(1+4\delta^2)}, \tag{3}$$

respectively, where k is the wave number, μ_B is Bohr magneton, m is the mass of a ^{85}Rb atom, $\delta(=-\Delta/\Gamma)$ is the normalized detuning, and $s_0(=I/I_s)$ is the normalized laser intensity with I_s being the saturation intensity. Due to the three-dimensional trap geometry and the multilevel property of atoms, Eq. (3) may not describe the experimental results well.

In order to investigate the Doppler part of the MOT, we have developed a model based on the rate equations, as presented in detail in Ref. [16]. We now compare the results of trap frequency and damping coefficient thereby derived with the experimental results. To our knowledge, the Doppler theory of the three-dimensional MOT with multilevel atoms has not been quantitatively compared with the experiment. We have found that Eqs. (2) and (3) could be still properly employed by using the averaged saturation intensity instead of the simple saturation intensity for two-level atoms. Note that for the wide range of the laser intensities along the transverse axes, Eqs. (2) and (3) provide the results quite similar to those obtained by the rate equation model.

We have summarized the data of Figs. 2 and 3 and presented them together in Fig. 4. The damping coefficient and the trap frequency are presented as a function of $s_0\delta/(1+4\delta^2)^2$ and $\sqrt{bs_0\delta}/(1+4\delta^2)$, respectively. One can observe that the measured trap frequencies are in excellence agreement with the calculated results. On the other hand, one has to multiply the simply calculated damping coefficients by a factor 1.76 to fit the experimental data. We find that the discrepancy in the damping coefficients results from the existence of the sub-Doppler contribution to the MOT. As is well known, the trap potential in the MOT is composed of two parts: the usual broad Doppler cooling part and the sub-Doppler part in the vicinity of the trap center. Direct observation of the sub-Doppler trap in the MOT has been reported in our previous work [17].

In order to show that the existence of the sub-Doppler force affects the Doppler-cooling parameters, we have performed Monte Carlo simulation with 1000 atoms. We first solve the equation of motion $m\ddot{z}=F(z,\dot{z})$, where the force consists of three parts: the Doppler force, the sub-Doppler force, and the random force. Since the detailed description of the forces is given in Ref. [17], we mention only the sub-Doppler force here. The sub-Doppler force for the $F=1 \rightarrow F'=2$ atomic transition in the counterpropagating σ^+-σ^- laser configuration is given by

$$F_{\mathrm{sub}} = -\frac{\hbar k\Gamma s^2 x}{P_1 s^2 + P_2 x^2}, \tag{4}$$

where P_1 and P_2 are the detuning-dependent numerical factors, $x=(k/\Gamma)\dot{z}+(g_g\mu_B b/\hbar\Gamma)z$, $g_g=1$ is the g factor of ground

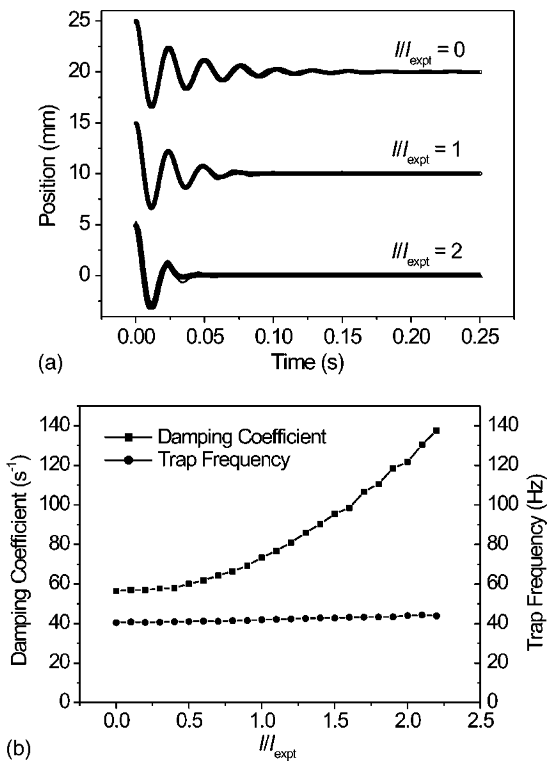

FIG. 5. The Monte Carlo simulation results. (a) The averaged trajectories for 1000 atoms together with the fitted curves obtained from Eq. (1), where the intensities are $I/I_{\mathrm{expt}}=0$, 1, and 2 from top to bottom. (b) The damping coefficient (filled square) and the trap frequency (filled circle) as a function of the laser intensity I/I_{expt}.

state, and $s=I/I_s$. For the simple $F=1\rightarrow F'=2$ transition, Eq. (4) can accurately describe the sub-Doppler force for any values of z and $\dot{z}$, because the g factors of the ground and excited states are equal. However, this is not the case for the $F=3\rightarrow F'=4$ transition of the multilevel ^{85}Rb atoms and thus it is not possible to obtain an analytic form of the force like Eq. (4). Therefore, we fit the calculated force for a given detuning with Eq. (4), which is then used in the calculation with $g_g=1/3$.

The results are presented in Fig. 5. Here we averaged the trajectories for 1000 atoms by using the same parameters as used in Fig. 1. We have varied the intensity (I) associated with F_{sub} without affecting the intensity for the Doppler force, and obtained the averaged trajectory, where $I_{\mathrm{expt}}=0.17\ \mathrm{mW/cm^2}$ is the laser intensity used in the experiment (Fig. 2). We then infer the damping coefficient and the trap frequency by fitting the averaged trajectory with Eq. (1). Note that we have neglected the sub-Doppler force for the zero intensity. As can be seen in Fig. 5(a), the trajectories decay more rapidly as the intensity is increased, which indicates the increase of the damping coefficient.

The fitted results for the damping coefficient and the trap frequency are shown in Fig. 5(b). While the trap frequency remains nearly constant, the damping coefficient increases with the intensity. When there is no sub-Doppler force, given the experimental parameters, the damping coefficient for two-level model is just $51.1\ \mathrm{s^{-1}}$, whereas that for the rate-equation model is $59.8\ \mathrm{s^{-1}}$. When $I/I_{\mathrm{expt}}=1$, it is now $73.5\ \mathrm{s^{-1}}$, which is 1.23 times the value given by the rate-equation model. Note that to obtain an increase of factor 1.76 as shown in Fig. 4, one should use $I/I_{\mathrm{expt}}=1.6$. Although our simulation does not provide a quantitative result, the reason for the increase of the damping coefficient can be well explained qualitatively. If one takes into account in our simulation the fact that the real MOT is a three-dimensional, not a one-dimensional trap, one should get more quantitative results.

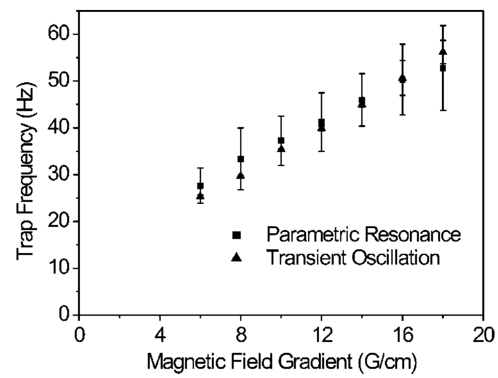

FIG. 6. Comparison of the measured trap frequencies, obtained by two different methods: parametric resonance (filled squares) and oscillating atomic cloud (filled triangles).

Finally we compare the transient oscillation method with the parametric resonance technique for experimental measurement of the trap frequency [16]. The measured trap frequencies are presented in Fig. 6 as the magnetic field gradient is changed at a given laser intensity ($I_z=0.10\ \mathrm{mW/cm^2}$) and detuning ($\Delta=-2.3\Gamma$). The data with filled squares are those obtained by the parametric resonance technique, whereas those with filled triangles are from the transient oscillation method. As shown in Fig. 6, the two methods give nearly the same values of the trap frequencies within experimental error.

IV. CONCLUSIONS

In conclusion, we have measured the trap parameters (damping coefficient and trap frequency) by a transient oscillation method and have made a quantitative study of the Doppler cooling theory in the MOT. The parameters were measured at various laser intensities, detunings, and magnetic field gradients. We have found that the simple rate-equation model can accurately describe the experimental data of trap frequencies. As for the damping coefficients, on the other hand, the measured data are about twice larger than the calculated ones, which is explained by the simulation that includes the sub-Doppler force.

Due to the sub-Doppler trap near the center of the magnetic field, one could not measure the damping coefficient of the Doppler part of the MOT. In order to measure the damping coefficient directly, one should separate spatially the sub-Doppler trap. Recently, we have experimentally demonstrated that when the detunings of the transverse lasers (in the x or y direction) are different from those in the longitudinal z direction, the sub-Doppler trap disappears. By using this method, one can measure the damping coefficient directly, which is currently in progress. The information of the Doppler trap parameters may be important in various experiments on parametric resonance of the MOT, such as the observation of transitions between two attractors under parametric excitation, where a quantitative comparison of the analytical calculations and the simulations is necessary [18].

ACKNOWLEDGMENTS

This work was supported from the Creative Research Initiative Project of the Korea Ministry of Science and Technology. H.R.N. was supported by Korea Research Foundation Grant (KRF-2004-041-C00149).

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