Energy-level displacement of excited *np* states of kaonic hydrogen

A. N. Ivanov,[1,2,3,*] M. Cargnelli,[2] M. Faber,[1] H. Fuhrmann,[2] V. A. Ivanova,[3] J. Marton,[2] N. I. Troitskaya,[3] and J. Zmeskal[2]
[1]*Atominstitut der Österreichischen Universitäten, Arbeitsbereich Kernphysik und Nukleare Astrophysik, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Wien, Austria*
[2]*Stefan Meyer Institut für Subatomare Physik der Österreichische Akademie der Wissenschaften, Boltzmanngasse 3, A-1090, Wien, Austria*
[3]*Department of Nuclear Physics, State Polytechnic University, 195251 St. Petersburg, Russian Federation*
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We compute the energy-level displacement of the excited *np* states of kaonic hydrogen within the quantum field theoretic and relativistic covariant model of strong low-energy $\bar{K}N$ interactions suggested by [Ivanov *et al.*Eur. Phys. J. A **21**, 11 (2004)]. For the width of the energy-level of the excited 2*p* state of kaonic hydrogen, caused by strong low-energy interactions, we find $\Gamma_{2p}=2$ meV$=3\times 10^{12}$ s^{-1}. This result is important for the theoretical analysis of the x-ray yields in kaonic hydrogen.

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I. INTRODUCTION

Recently [1] we have computed the energy-level displacement of the ground state of kaonic hydrogen,

$$-\epsilon_{1s}^{(\text{theor})} + i\frac{\Gamma_{1s}^{(\text{theor})}}{2} = (-203 \pm 15) + i(113 \pm 14)\ \text{eV}. \tag{1.1}$$

This result has been obtained within a quantum field theoretic and relativistic covariant model of strong low-energy $\bar{K}N$ interactions near threshold of K^-p scattering, based on the dominant role of strange resonances $\Lambda(1405)$ and $\Sigma(1750)$ in the *s* channel of low-energy elastic and inelastic K^-p scattering and the exotic four-quark (or $K\bar{K}$ molecules) scalar states $a_0(980)$ and $f_0(980)$ in the *t* channel of low-energy elastic K^-p scattering.

The theoretical result (1.1) agrees well with recent experimental data obtained by the DEAR Collaboration [2],

$$-\epsilon_{1s}^{(\text{expt})} + i\frac{\Gamma_{1s}^{(\text{expt})}}{2} = (-194 \pm 41) + i(125 \pm 59)\ \text{eV}. \tag{1.2}$$

A systematic analysis of corrections, caused by electromagnetic and QCD isospin-breaking interactions, to the energy-level displacements of the *ns* states of kaonic hydrogen, where *n* is the principal quantum number, has been recently carried out by Meißner, Raha, and Rusetsky [3] within effective field theory by using the nonrelativistic effective Lagrangian approach based on chiral perturbation theory (ChPT) by Gasser and Leutwyler [4,5]. For the *s*-wave amplitude of K^-N scattering near threshold, computed in [1,6], the energy-level displacement of the ground state of kaonic hydrogen obtained by Meißner *et al.* [3] is equal to

$$-\epsilon_{1s}^{(\text{theor})} + i\frac{\Gamma_{1s}^{(\text{theor})}}{2} = (-266 \pm 17) + i(177 \pm 16)\ \text{eV}.$$

This agrees well with both our theoretical result (1.1) and experimental data (1.2) within 1.5 standard deviations.

In this paper, we compute the energy-level displacement of the excited *np* states of kaonic hydrogen, where *n* is the principal quantum number and *p* corresponds to the excited state with $\ell=1$. The knowledge of the energy-level displacement of the excited *np* states of kaonic hydrogen is very important for the understanding of the accuracy of experimental measurements of the energy-level displacement of the ground state of kaonic hydrogen and the theoretical analysis of the *x*-ray yields in kaonic hydrogen [2,7–13].

The paper is organized as follows. In Sec. II we extend our approach to the description of low-energy K^-p interaction in the *s*-wave state to the analysis of the low-energy K^-p interaction in the *p*-wave state with a total angular moment $J=3/2$ and $J=1/2$, respectively. We compute the *p*-wave scattering lengths of elastic K^-p scattering and the energy-level shift of the *np* excited state of kaonic hydrogen. In Sec. III, we compute the *p*-wave scattering lengths of inelastic reactions $K^-p \to Y\pi$, where $Y\pi=\Sigma^-\pi^+$, $\Sigma^+\pi^-$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$. We compute the energy-level width of the *np* excited state of kaonic hydrogen. For the 2*p* state of kaonic hydrogen, we get $\Gamma_{2p}=2$ meV$=3\times 10^{12}$ s^{-1}. The rate of the hadronic decays of kaonic hydrogen from the *np* excited state is important for the theoretical analysis of the x-ray yields in kaonic hydrogen, which are the main experimental tool for the measurement of the energy-level displacement of the ground state of kaonic hydrogen [2]. In the Conclusion, we discuss the obtained results.

II. ENERGY-LEVEL DISPLACEMENT OF THE $n\ell$ EXCITED STATES OF KAONIC HYDROGEN: GENERAL FORMULAS

According to [14], the energy-level displacement of the excited $n\ell$ states of kaonic hydrogen can be defined by

*Electronic address: ivanov@kph.tuwien.ac.at

$$-\epsilon_{n\ell}+i\frac{\Gamma_{n\ell}}{2}=\frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}\int\frac{d^3k}{(2\pi)^3}\frac{\Phi^{\dagger}_{n\ell}(k)}{\sqrt{2E_{K^-}(k)2E_p(k)}}\int\frac{d^3q}{(2\pi)^3}\frac{\Phi_{n\ell}(q)}{\sqrt{2E_{K^-}(q)2E_p(q)}}$$
$$\times\int\int\frac{d\Omega_{\vec{k}}}{\sqrt{4\pi}}\frac{d\Omega_{\vec{q}}}{\sqrt{4\pi}}Y^*_{\ell m}(\vartheta_{\vec{k}},\varphi_{\vec{k}})M[K^-(\vec{q})p(-\vec{q},\sigma_p)\to K^-(\vec{k})p(-\vec{k},\sigma_p)]Y_{\ell m}(\vartheta_{\vec{q}},\varphi_{\vec{q}}),\qquad(2.1)$$

where $M[K^-(\vec{q})p(-\vec{q},\sigma_p)\to K^-(\vec{k})p(-\vec{k},\sigma_p)]$ is the amplitude of elastic K^-p scattering and $\Phi_{n\ell}(k)$ is a radial wave function of kaonic hydrogen in the $n\ell$ excited state in momentum representation. It is defined by [14]

$$\Phi_{n\ell}(k)=\sqrt{4\pi}\int_0^{\infty}j_{\ell}(kr)R_{n\ell}(r)r^2dr,\qquad(2.2)$$

where $j_\ell(kr)$ are spherical Bessel functions [15] and $R_{n\ell}(r)$ is a radial wave function of kaonic hydrogen in the coordinate representation [16],

$$R_{n\ell}(r)=-\frac{2}{n^2}\sqrt{\frac{(n-\ell-1)!}{[(n+\ell)!]^3a_B^3}}\left(\frac{2}{n}\frac{r}{a_B}\right)^{\ell}e^{-r/na_B}L^{2\ell+1}_{n+\ell}\left(\frac{2}{n}\frac{r}{a_B}\right).\qquad(2.3)$$

Here $L^{2\ell+1}_{n+\ell}(\rho)$ are the generalized Laguerre polynomials given by [16]

$$L^{2\ell+1}_{n+\ell}(\rho)=(-1)^{2\ell+1}\frac{(n+\ell)!}{(n-\ell-1)!}\rho^{-(2\ell+1)}\times e^{\rho}\frac{d^{n-\ell-1}}{d\rho^{n-\ell-1}}(\rho^{n+\ell}e^{-\rho}),\qquad(2.4)$$

where $\rho=r/na_B$ and $a_B=1/\alpha\mu=83$ fm is the Bohr radius of kaonic hydrogen with $\mu=m_Km_N/(m_K+m_N)=324$ MeV, and $\alpha=1/137.036$ are the reduced mass of the K^-p pair, computed for $m_K=494$ MeV and $m_N=940$ MeV, and the fine-structure constant, respectively. Spherical harmonics $Y_{\ell m}(\vartheta,\varphi)$ are normalized by

$$\int d\Omega Y^*_{\ell' m'}(\vartheta,\varphi)Y_{\ell m}(\vartheta,\varphi)=\delta_{\ell'\ell}\delta_{m'm},\qquad(2.5)$$

where $d\Omega=\sin\vartheta d\vartheta d\varphi$ is a volume element of solid angle.

In Eq. (2.1), due to the wave functions $\Phi^*_{n\ell}(k)$ and $\Phi_{n\ell}(q)$ the integrand of the momentum integrals is concentrated at momenta of order of $k\sim q\sim 1/na_B=\alpha\mu/n=2.4/n$ MeV. Therefore, the amplitude of elastic K^-p scattering can be defined in the low-energy limit at $k,q\to 0$. Since in the low-energy limit there is no spin flip in the transition $K^-+p\to K^-+p$, the amplitude of low-energy elastic K^-p scattering can be determined by [17–19] (see also [14]),

$$M[K^-(\vec{q})p(-\vec{q},\sigma_p)\to K^-(\vec{k})p(-\vec{k},\sigma_p)]$$
$$=8\pi\sqrt{s}\sum_{\ell'=0}^{\infty}[(\ell'+1)f_{\ell'+}(\sqrt{kq})+\ell' f_{\ell'-}(\sqrt{kq})]P_{\ell'}(\cos\vartheta)$$
$$=8\pi\sqrt{s}\sum_{\ell'=0}^{\infty}[(\ell'+1)f_{\ell'+}(\sqrt{kq})+\ell' f_{\ell'-}(\sqrt{kq})]\times\sum_{m'=-\ell'}^{\ell'}\frac{4\pi}{2\ell'+1}Y^*_{\ell'm'}(\vartheta_{\vec{q}},\varphi_{\vec{q}})Y_{\ell'm'}(\vartheta_{\vec{k}},\varphi_{\vec{k}}),\qquad(2.6)$$

where $\sqrt{s}$ is the total energy in the s channel of K^-p scattering, $P_{\ell'}(\cos\vartheta)$ are Legendre polynomials [15], and ϑ is the angle between the relative momenta $\vec{k}$ and $\vec{q}$. The amplitudes $f_{\ell'+}(\sqrt{kq})$ and $f_{\ell'-}(\sqrt{kq})$ describe elastic K^-p scattering in the states with a total angular momentum $J=\ell'+1/2$ and $J=\ell'-1/2$, respectively. They are defined by

$$f_{\ell'+}(\sqrt{kq})=\frac{1}{2i\sqrt{kq}}[\eta_{\ell'+}(\sqrt{kq})e^{+2i\delta_{\ell'+}(\sqrt{kq})}-1],$$
$$f_{\ell'-}(\sqrt{kq})=\frac{1}{2i\sqrt{kq}}[\eta_{\ell'-}(\sqrt{kq})e^{+2i\delta_{\ell'-}(\sqrt{kq})}-1],\qquad(2.7)$$

where $\eta_{\ell'\pm}(\sqrt{kq})$ and $\delta_{\ell'\pm}(\sqrt{kq})$ are inelasticities and phase shifts of elastic K^-p scattering [17–19].

Near threshold, the amplitudes $f_{\ell'+}(\sqrt{kq})$ and $f_{\ell'-}(\sqrt{kq})$ possess the real and imaginary parts. The real parts of the amplitudes $f_{\ell'+}(\sqrt{kq})$ and $f_{\ell'-}(\sqrt{kq})$ are defined by the ℓ'-wave scattering lengths of K^-p scattering [14,18],

$$\mathrm{Re}\,f_{\ell'+}(\sqrt{kq})=a^{K^-p}_{\ell'+}(kq)^{\ell'},$$
$$\mathrm{Re}\,f_{\ell'-}(\sqrt{kq})=a^{K^-p}_{\ell'-}(kq)^{\ell'}.\qquad(2.8)$$

Using Eq. (2.8) for the shift of the energy-level of the $n\ell$ excited state of kaonic hydrogen, we obtain [14]

$$\epsilon_{n\ell}=-\frac{2\pi}{\mu}\frac{(\ell+1)a^{K^-p}_{\ell+}+\ell a^{K^-p}_{\ell-}}{2\ell+1}\times\left|\int\frac{d^3k}{(2\pi)^3}\sqrt{\frac{m_Km_N}{E_{K^-}(k)E_p(k)}}k^{\ell}\Phi_{n\ell}(k)\right|^2.\qquad(2.9)$$

The imaginary parts of the amplitudes $f_{\ell'+}(\sqrt{kq})$ and $f_{\ell'-}(\sqrt{kq})$ are defined by inelastic channels $K^-p\to\Sigma^-\pi^+$, $K^-p\to\Sigma^+\pi^-$, $K^-p\to\Sigma^0\pi^0$, and $K^-p\to\Lambda^0\pi^0$. According to

[14], the width $\Gamma_{n\ell}$ of the energy-level of the $n\ell$ excited state of kaonic hydrogen is given by

$$\Gamma_{n\ell} = \frac{4\pi}{\mu}\sum_{Y\pi}\left[\frac{(\ell+1)a^{Y\pi}_{\ell+} + \ell a^{Y\pi}_{\ell-}}{2\ell+1}\right]^2 [k_{Y\pi}(W_{n\ell})]^{2\ell+1} \times \left|\int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{m_K m_N}{E_{K^-}(k)E_p(k)}} k^\ell \Phi_{n\ell}(k)\right|^2, \quad (2.10)$$

where we sum over all $Y\pi$ pairs $Y\pi = \Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$; $k_{Y\pi}(W_{n\ell})$ is a relative momentum of the $Y\pi$ pair,

$$k_{Y\pi}(W_{n\ell}) = \frac{\sqrt{[W^2_{n\ell} - (m_Y + m_\pi)^2][W^2_{n\ell} - (m_Y - m_\pi)^2]}}{2W_{n\ell}} \quad (2.11)$$

with $W_{n\ell} = m_K + m_N + E_{n\ell}$ and $E_{n\ell}$ is the binding energy of kaonic hydrogen in the $n\ell$ excited state [19].

The analysis of experimental data obtained by the DEAR Collaboration [2] requires knowledge of the energy-level displacement of the excited *np* states. For $\ell = 1$, the formulas and (2.9) and (2.10) read

$$\epsilon_{np} = -\frac{2\pi}{3}\frac{1}{\mu}(2a^{K^-p}_{3/2} + a^{K^-p}_{1/2}) \times \left|\int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{m_K m_N}{E_{K^-}(k)E_p(k)}} k\Phi_{np}(k)\right|^2,$$

$$\Gamma_{np} = \frac{4\pi}{9}\frac{1}{\mu}\sum_{Y\pi}(2a^{Y\pi}_{3/2} + a^{Y\pi}_{1/2})^2 k^3_{Y\pi} \times \left|\int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{m_K m_N}{E_{K^-}(k)E_p(k)}} k\Phi_{np}(k)\right|^2, \quad (2.12)$$

where $\Phi_{np}(k)$ is the radial wave function of kaonic hydrogen in the *np* excited state in the momentum representation, and the indices $3/2$ and $1/2$ denote the *p*-wave amplitudes of the reactions $K^-p \to K^-p$ and $K^-p \to Y\pi$ with total angular momentum $J = 3/2$ and $J = 1/2$, respectively [17–19].

The momentum integral on the r.h.s. of Eq. (2.12) has been computed in [14]. Using this result, the energy-level displacement of the *np* excited states reads

$$\epsilon_{np} = -\frac{2}{3}\frac{\alpha^5}{n^3}\left(1 - \frac{1}{n^2}\right)\left(\frac{m_K m_N}{m_K + m_N}\right)^4 (2a^{K^-p}_{3/2} + a^{K^-p}_{1/2}),$$

$$\Gamma_{np} = \frac{4}{9}\frac{\alpha^5}{n^3}\left(1 - \frac{1}{n^2}\right)\left(\frac{m_K m_N}{m_K + m_N}\right)^4 \sum_{Y\pi}(2a^{Y\pi}_{3/2} + a^{Y\pi}_{1/2})^2 k^3_{Y\pi}. \quad (2.13)$$

Thus, the problem of the calculation of the energy-level displacement of the *np* excited states of kaonic hydrogen reduces to the problem of the calculation of the *p*-wave scattering lengths $a^{K^-p}_{1/2}$ and $a^{K^-p}_{3/2}$ of elastic K^-p scattering and *p*-wave scattering lengths $a^{Y\pi}_{1/2}$ and $a^{Y\pi}_{3/2}$ of inelastic reactions $K^-p \to Y\pi$ with $Y\pi = \Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$.

III. MODEL FOR LOW-ENERGY K^-p SCATTERING IN THE *p*-WAVE STATE

For the description of the *p*-wave amplitude of low-energy K^-p scattering, we follow [1,6] and assume the following.

(i) The amplitudes with total angular momentum $J = 1/2$ are defined by the contributions of the elastic background and the octets of baryon resonances with spin $1/2$ and positive parity such as $(N(1440), \Lambda^0(1600), \Sigma(1660)) = B_1(\mathbf{8})$ and $(N(1710), \Lambda^0(1810), \Sigma(1880)) = B_2(\mathbf{8})$.

(ii) The amplitudes with total angular momentum $J = 3/2$ are defined by the contributions of the elastic background and the baryon resonances with spin $3/2$ and positive parity from decuplet $(\Delta(1232), \Sigma(1385)) = B_3(\mathbf{10})$ and octet $(N(1720), \Lambda^0(1890), \Sigma(1840)) = B_4(\mathbf{8})$ [20]. We would like to emphasize that the baryon resonances we will treat as elementary particles defined by local fields and local phenomenological Lagrangians with phenomenological coupling constants [1,6] (see also [21]). We include the contribution of the octet of low-lying baryons with spin $1/2$ and positive parity $(N(940), \Lambda^0(1116), \Sigma(1193)) = B(\mathbf{8})$ in the elastic background.

A. *p*-wave scattering lengths of elastic K^-p scattering

The *p*-wave amplitude of elastic K^-p scattering at threshold is defined by two *p*-wave scattering lengths $a^{K^-p}_{1/2}$ and $a^{K^-p}_{3/2}$ caused by the interactions of the K^-p pair in the states with a total angular momentum $J = 1/2$ and $J = 3/2$, respectively.

1. *p*-wave scattering length $a^{K^-p}_{1/2}$

According to our approach to the description of the low-energy K^-p interaction in the *s*-wave state extended to the low-energy K^-p interaction in the *p*-wave state, the amplitude $a^{K^-p}_{1/2}$ has the following form:

$$a^{K^-p}_{1/2} = (a^{K^-p}_{1/2})_B + \sum_R (a^{K^-p}_{1/2})_R, \quad (3.1)$$

where $(a^{K^-p}_{1/2})_B$ is the contribution of an elastic background and $(a^{K^-p}_{1/2})_R$ is the contribution of the baryon resonance $R = \Lambda^0_1$, Σ^0_1, Λ^0_2, and Σ^0_2.

2. Resonance contribution to *p*-wave scattering length $a^{K^-p}_{1/2}$

The phenomenological low-energy interactions $B_1(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$ and $B_2(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$, necessary for the calculation of the contribution of the baryon resonances to the *p*-wave amplitude $a^{K^-p}_{1/2}$, can be defined using the results obtained in [6]. As a result, for the sum of the baryon resonance contributions, we obtain

$$\sum_R (a_{1/2}^{K^-p})_R = -\frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_1)g_{\pi NN_1}\right]^2\frac{1}{2m_N}\frac{1}{m_{\Lambda_1^0}-m_N-m_K} - \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha_1-1)g_{\pi NN_1}]^2\frac{1}{2m_N}\frac{1}{m_{\Sigma_1^0}-m_N-m_K} - \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_2)g_{\pi NN_2}\right]^2\frac{1}{2m_N}\frac{1}{m_{\Lambda_2^0}-m_N-m_K} - \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha_2-1)g_{\pi NN_2}]^2\frac{1}{2m_N}\frac{1}{m_{\Sigma_2^0}-m_N-m_K}. \quad (3.2)$$

The coupling constants of the interactions $K^-pB_1(\mathbf{8})$ and $K^-pB_2(\mathbf{8})$ are equal to $g_{\pi NN_1}=6.28$, $\alpha_1=0.85$, $g_{\pi NN_2}=1.20$, and $\alpha_2=-1.55$. These numerical values of the coupling constants one can obtain by using the phenomenological SU(3)-invariant interactions $B_1(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$ and $B_2(\mathbf{8})B(\mathbf{8})P(\mathbf{8})$ (see [6]), where $P(\mathbf{8})$ is the octet of low-lying pseudoscalar mesons and experimental data on the partial widths of the resonances $B_1(\mathbf{8})$ and $B_2(\mathbf{8})$ [20]. Using the recommended masses for the resonances $m_{\Lambda_1^0}=1600$ MeV, $m_{\Sigma_1}=1660$ MeV, $m_{\Lambda_2^0}=1810$ MeV, and $m_{\Sigma_2}=1880$ MeV, we compute

$$\sum_R (a_{1/2}^{K^-p})_R = -0.013 \text{ m}_\pi^{-3}. \quad (3.3)$$

Now we proceed to computing the contribution of the elastic background $(a_{1/2}^{K^-p})_B$.

3. Elastic background contribution to the *p*-wave scattering length $a_{1/2}^{K^-p}$

According to [1], the contribution of the elastic background $(a_{1/2}^{K^-p})_B$ to the p-wave scattering length $a_{1/2}^{K^-p}$ should be defined by the contribution of all low-energy interactions $(a_{1/2}^{K^-p})_{\rm CA}$, which can be described within the effective chiral lagrangian (the ECL) approach [22] or that is equivalent within current algebra (CA) [23–30], supplemented by soft-kaon theorems (SKT) [26–30], and the contribution $(a_{1/2}^{K^-p})_{K\bar{K}}$ of low-energy exchanges with the exotic scalar mesons $a_0(980)$ and $f_0(980)$, which are four-quark states [31,32] or $K\bar{K}$ molecules [32,33]. The description of strong low-energy interactions of these mesons goes beyond the ECL approach, describing strong low-energy interactions of mesons with $q\bar{q}$ and baryons with qqq quark structures. Recent experimental confirmation of the exotic structure of the scalar mesons $a_0(980)$ and $f_0(980)$ has been obtained by the DEAR Collaboration at DAPHNE [34].

Thus, the p-wave scattering length $(a_{1/2}^{K^-p})_B$ is defined by

$$(a_{1/2}^{K^-p})_B = (a_{1/2}^{K^-p})_{\rm CA} + (a_{1/2}^{K^-p})_{K\bar{K}}. \quad (3.4)$$

Using the results obtained in [1], we compute the contribution of the exotic scalar mesons,

$$a_{1/2,K\bar{K}}^{K^-p} = -\frac{1}{\pi}\frac{m_N}{m_K+m_N}\frac{g_D g_0}{m_{a_0}^4}\left(1-\frac{1}{8}\frac{m_{a_0}^2}{m_N^2}\right) = -0.018 \text{ m}_\pi^{-3}, \quad (3.5)$$

where $m_{f_0}=m_{a0}=980$ MeV, $g_0=g_{a_0K^+K^-}=g_{f_0K^+K^-}=2746$ MeV [32], and $g_D=\xi g_{\pi NN}/g_A=0.95 g_{\pi NN}$. For the calculation of g_D, we have used $\xi=1.2$ [1] and $g_A=1.267$ [20]. The coupling constant $g_{\pi NN}$ of the πNN interaction is equal to $g_{\pi NN}=13.21$ [35] (see also [36] by Ericson, Loiseau, and Wycech, where the authors have obtained $g_{\pi NN}=13.28\pm0.08$).

The contribution to the p-wave amplitude, caused by the ECL interactions, we represent in the form of the superposition of the contributions of the $\Lambda^0(1116)$ and $\Sigma^0(1193)$ hyperon exchanges and the term $(a_{1/2}^{K^-p})_{\rm SKT}$, which can be computed applying the soft-kaon technique [26–30]. Thus, we get

$$(a_{1/2}^{K^-p})_{\rm CA} = (a_{1/2}^{K^-p})_{\rm SKT} - \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha)g_{\pi NN}\right]^2 \times\frac{1}{2m_N}\frac{1}{m_{\Lambda^0}-m_N-m_K} - \frac{1}{8\pi}\frac{1}{m_K+m_N} \times[(2\alpha-1)g_{\pi NN}]^2\frac{1}{2m_N}\frac{1}{m_{\Sigma^0}-m_N+m_K}. \quad (3.6)$$

For $m_{\Lambda^0}=1116$ MeV, $\Sigma^0=1193$ MeV, $g_{\pi NN}=13.21$, and $\alpha=0.64$ [6], we obtain

$$(a_{1/2}^{K^-p})_{\rm CA} = (a_{1/2}^{K^-p})_{\rm SKT} + 0.024 \text{ m}_\pi^{-3}. \quad (3.7)$$

Summing up the contributions, for the p-wave scatting length $a_{1/2}^{K^-p}$ of K^-p scattering with a total angular momentum $J=1/2$, we get

$$a_{1/2}^{K^-p} = (a_{1/2}^{K^-p})_{\rm SKT} - 0.007 \text{ m}_\pi^{-3}. \quad (3.8)$$

We suggest to compute the quantity $(a_{1/2}^{K^-p})_{\rm SKT}$ together with $(a_{3/2}^{K^-p})_{\rm SKT}$, the contribution of the elastic background to the p-wave scattering length $a_{3/2}^{K^-p}$ of K^-p scattering with a total angular momentum $J=3/2$.

4. *p*-wave scattering length $a_{3/2}^{K^-p}$

The p-wave scattering length $a_{3/2}^{K^-p}$ we represent by

$$a_{3/2}^{K^-p} = (a_{3/2}^{K^-p})_B + \sum_R (a_{3/2}^{K^-p})_R, \quad (3.9)$$

where $(a_{3/2}^{K^-p})_B$ is the contribution of an elastic background and $(a_{3/2}^{K^-p})_R$ is the contribution of the baryon resonances $R=\Sigma_3^0$, Λ_4^0, and Σ_4^0. The elastic background $(a_{3/2}^{K^-p})_B$ does not contain rapidly changing contributions, therefore below we assume that $(a_{3/2}^{K^-p})_B=(a_{3/2}^{K^-p})_{\rm SKT}$.

5. Resonance contribution to the p-wave scattering length $a_{3/2}^{K^-p}$

The phenomenological low-energy interaction of the resonance Σ_3^0 with octets low-lying baryons $B(\mathbf{8})$ and pseudoscalar mesons $P(\mathbf{8})$ is defined by [18,19,37] (see also [20])

$$\begin{aligned}\mathcal{L}_{\Sigma_3^0 BP}(x) &= \frac{g_{\pi NN}}{\sqrt{6}m_N}\bar{\Sigma}^0_{3\mu}(x)[\Sigma^+(x)\partial^\mu\pi^-(x) - \Sigma^-(x)\partial^\mu\pi^+(x) \\ &+ p(x)\partial^\mu K^-(x) + \sqrt{3}\Lambda^0(x)\partial^\mu\pi^0(x)] \\ &+ \frac{g_{\pi NN}}{\sqrt{6}m_N}[\overline{\Sigma^+}(x)\partial^\mu\pi^+(x) - \overline{\Sigma^-}(x)\partial^\mu\pi^-(x) \\ &- \overline{p}(x)\partial^\mu K^+(x) + \sqrt{3}\overline{\Lambda^0}(x)\partial^\mu\pi^0(x)]\Sigma^0_{3\mu}(x),\end{aligned} \quad (3.10)$$

where we have written down only those interactions which contribute to the p-wave amplitude of low-energy K^-p scattering.

Using Eq. (3.10), we compute the contribution of the resonance $\Sigma(1385)$ to the p-wave scattering length $a_{3/2}^{K^-p}$,

$$\begin{aligned}(a_{3/2}^{K^-p})_{\Sigma_3^0} &= -\frac{g^2_{\pi NN}}{36\pi m_N}\frac{1}{m_K+m_N}\frac{m_{\Sigma_3^0}}{m^2_{\Sigma_3^0}-(m_K+m_N)^2}\left\{\left[1-\frac{1}{2}\frac{m_K}{m_N}\right.\right. \\ &\left.-\frac{1}{4}\frac{m_K^2}{m_N^2}\right] + \frac{(m_K+m_N)}{m_{\Sigma_3^0}}\left[1+\frac{1}{2}\frac{m_K}{m_N}\frac{(m_K+m_N)}{m_{\Sigma_3^0}}\right. \\ &\left.\left.-\frac{1}{4}\frac{m_K^2}{m_N^2}\left(1+\frac{(m_K+m_N)}{m_{\Sigma_3^0}}-\frac{(m_K+m_N)^2}{m^2_{\Sigma_3^0}}\right)\right]\right\} \\ &= 0.060\ \mathrm{m}_\pi^{-3}.\end{aligned} \quad (3.11)$$

The contribution of the resonances Λ_4^0 and Σ_4^0 to $a_{3/2}^{K^-p}$ is equal to

$$\begin{aligned}\sum_{R=\Lambda_4^0,\Sigma_4^0}(a_{3/2}^{K^-p})_R &= -\frac{1}{6\pi m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_4)g_{\pi NN_4}\right]^2\frac{1}{m_K+m_N}\frac{m_{\Lambda_4^0}}{m^2_{\Lambda_4^0}-(m_K+m_N)^2}\left\{\left[1-\frac{1}{2}\frac{m_K}{m_N}-\frac{1}{4}\frac{m_K^2}{m_N^2}\right]+\frac{(m_K+m_N)}{m_{\Lambda_4^0}}\right. \\ &\left.\times\left[1+\frac{1}{2}\frac{m_K}{m_N}\frac{(m_K+m_N)}{m_{\Lambda_4^0}}-\frac{1}{4}\frac{m_K^2}{m_N^2}\left(1+\frac{(m_K+m_N)}{m_{\Lambda_4^0}}-\frac{(m_K+m_N)^2}{m^2_{\Lambda_4^0}}\right)\right]\right\}-\frac{1}{6\pi m_N}[(2\alpha_4-1)g_{\pi NN_4}]^2 \\ &\times\frac{1}{m_K+m_N}\frac{m_{\Sigma_4^0}}{m^2_{\Sigma_4^0}-(m_K+m_N)^2}\left\{\left[1-\frac{1}{2}\frac{m_K}{m_N}-\frac{1}{4}\frac{m_K^2}{m_N^2}\right]+\frac{(m_K+m_N)}{m_{\Sigma_4^0}}\left[1+\frac{1}{2}\frac{m_K}{m_N}\frac{(m_K+m_N)}{m_{\Sigma_4^0}}\right.\right. \\ &\left.\left.-\frac{1}{4}\frac{m_K^2}{m_N^2}\left(1+\frac{(m_K+m_N)}{m_{\Sigma_4^0}}-\frac{(m_K+m_N)^2}{m^2_{\Sigma_4^0}}\right)\right]\right\}.\end{aligned} \quad (3.12)$$

Using the experimental data on the resonances from the octet $B_4(\mathbf{8})$ [20], we compute the coupling constants $g_{\pi NN_4}=1.16$ and $\alpha_4=0.32$. For $m_{\Lambda_4^0}=1890$ MeV and $m_{\Sigma_4^0}=1840$ MeV, the numerical value of the contribution of the resonances Λ_4^0 and Σ_4^0 to the p-wave scattering length $a_{3/2}^{K^-p}$ reads

$$\sum_{R=\Lambda_4^0,\Sigma_4^0}(a_{3/2}^{K^-p})_R = -0.001\ \mathrm{m}_\pi^{-3}. \quad (3.13)$$

The p-wave scattering length of K^-p scattering with total angular momentum $J=3/2$ is given by

$$a_{3/2}^{K^-p} = (a_{3/2}^{K^-p})_{\mathrm{SKT}} + 0.059\ \mathrm{m}_\pi^{-3}. \quad (3.14)$$

Summing up the contributions (3.8) and (3.14), we obtain the total p-wave scattering length of elastic K^-p scattering in the p-wave state,

$$2a_{3/2}^{K^-p} + a_{1/2}^{K^-p} = (2a_{3/2}^{K^-p} + a_{1/2}^{K^-p})_{\mathrm{SKT}} + 0.111\ \mathrm{m}_\pi^{-3}. \quad (3.15)$$

Now we turn to the calculation of the term $(2a_{3/2}^{K^-p} + a_{1/2}^{K^-p})_{\mathrm{SKT}}$.

B. Soft-kaon theorem for amplitude of elastic K^-p scattering and elastic p-wave background

Soft-kaon theorems, as a part of ChPT [4,5], define amplitudes of low-energy reactions with kaons as expansions in powers of 4-momenta of kaons k, with kaons treated off-mass shell $k^2\neq m_K^2$. Using the reduction technique and the PCAC hypothesis [23–30], the S-matrix element of elastic low-energy transition $K^-p\to K^-p$ can be defined by

$$\begin{aligned}&\langle \mathrm{out}; K^-(\vec{k})p(-\vec{k},\sigma_p)|K^-(\vec{q})p(-\vec{q},\sigma_p);\mathrm{in}\rangle \\ &= -\frac{(m_K^2-k^2)}{\sqrt{2}F_K m_K^2}\frac{(m_K^2-q^2)}{\sqrt{2}F_K m_K^2}\int d^4x d^4y e^{+ikx-iqy}\langle p(-\vec{k},\sigma_p)| \\ &\times \mathrm{T}[\partial^\mu J_{5\mu}^{4+i5}(x)\partial^\nu J_{5\nu}^{4-i5}(y)]|p(-\vec{q},\sigma_p)\rangle,\end{aligned} \quad (3.16)$$

where T is a time-ordering operator and $J_{5\mu}^{4+i5}(x)$ and $J_{5\nu}^{4-i5}(x)$ are axial-vector hadronic currents with quantum numbers of the K^- and K^+ mesons [23,25]; $F_K=113$ MeV is the PCAC constant of charged K mesons. For further reduction of the r.h.s. of Eq. (3.16), we use the relation [23]

$$\mathrm{T}[\partial^{\mu}J^{4+i5}_{5\mu}(x)\partial^{\nu}J^{4-i5}_{5\nu}(y)] = \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}\mathrm{T}[J^{4+i5}_{5\mu}(x)J^{4-i5}_{5\nu}(y)] - \frac{1}{2}\frac{\partial}{\partial x_\mu}\{\delta(x^0-y^0)[J^{4-i5}_{50}(y),J^{4+i5}_{5\mu}(x)]\} - \frac{1}{2}\frac{\partial}{\partial y_\nu}\{\delta(x^0-y^0)[J^{4+i5}_{50}(x),J^{4-i5}_{5\nu}(y)]\} - \frac{1}{2}\delta(x^0-y^0)[J^{4-i5}_{50}(y),\partial^{\mu}J^{4+i5}_{5\mu}(x)] - \frac{1}{2}\delta(x^0-y^0)[J^{4+i5}_{50}(x),\partial^{\nu}J^{4-i5}_{5\nu}(y)]. \quad (3.17)$$

Substituting (3.17) into (3.16) and making integration by parts and dropping surface terms, we arrive at the expression

$$\langle \mathrm{out}; K^-(\vec{k})p(-\vec{k},\sigma_p)|K^-(\vec{q})p(-\vec{q},\sigma_p);\mathrm{in}\rangle = -\frac{(m_K^2-k^2)}{\sqrt{2}F_K m_K^2}\frac{(m_K^2-q^2)}{\sqrt{2}F_K m_K^2}\int d^4x d^4y e^{+ikx-iqy}\Big\{k^{\mu}q^{\nu}\langle p(-\vec{k},\sigma_p)|\mathrm{T}[J^{4+i5}_{5\mu}(x)J^{4-i5}_{5\nu}(y)]|p(-\vec{q},\sigma_p)\rangle + \frac{1}{2}ik^{\mu}\delta(x^0-y^0)\langle p(-\vec{k},\sigma_p)| \times[J^{4-i5}_{50}(y),J^{4+i5}_{5\mu}(x)]|p(-\vec{q},\sigma_p)\rangle - \frac{1}{2}iq^{\nu}\delta(x^0-y^0)\langle p(-\vec{k},\sigma_p)|[J^{4+i5}_{50}(x),J^{4-i5}_{5\nu}(y)]|p(-\vec{q},\sigma_p)\rangle - \frac{1}{2}\delta(x^0-y^0)\langle p(-\vec{k},\sigma_p)|[J^{4-i5}_{50}(y),\partial^{\mu}J^{4+i5}_{5\mu}(x)]|p(-\vec{q},\sigma_p)\rangle - \frac{1}{2}\delta(x^0-y^0)\langle p(-\vec{k},\sigma_p)|[J^{4+i5}_{50}(x),\partial^{\nu}J^{4-i5}_{5\nu}(y)]|p(-\vec{q},\sigma_p)\rangle\Big\}. \quad (3.18)$$

From Eq. (3.18), we obtain the amplitude of elastic low-energy K^-p scattering with K^- mesons off-mass shell. It reads

$$M[K^-(\vec{q})p(-\vec{q},\sigma_p)\to K^-(\vec{k})p(-\vec{k},\sigma_p)] = \frac{(m_K^2-k^2)}{\sqrt{2}F_K m_K^2}\frac{(m_K^2-q^2)}{\sqrt{2}F_K m_K^2}i\int d^4x e^{+ikx}\Big\{k^{\mu}q^{\nu}\langle p(-\vec{k},\sigma_p)|\mathrm{T}[J^{4+i5}_{5\mu}(x)J^{4-i5}_{5\nu}(0)]|p(-\vec{q},\sigma_p)\rangle + \frac{1}{2}ik^{\mu}\delta(x^0)\langle p(-\vec{k},\sigma_p)|[J^{4-i5}_{50}(0),J^{4+i5}_{5\mu}(x)]|p(-\vec{q},\sigma_p)\rangle - \frac{1}{2}iq^{\nu}\delta(x^0)\langle p(-\vec{k},\sigma_p)|[J^{4+i5}_{50}(x),J^{4-i5}_{5\nu}(0)]|p(-\vec{q},\sigma_p)\rangle - \frac{1}{2}\delta(x^0)\langle p(-\vec{k},\sigma_p)|[J^{4-i5}_{50}(0),\partial^{\mu}J^{4+i5}_{5\mu}(x)]|p(-\vec{q},\sigma_p)\rangle - \frac{1}{2}\delta(x^0)\langle p(-\vec{k},\sigma_p)|[J^{4+i5}_{50}(x),\partial^{\nu}J^{4-i5}_{5\nu}(0)]|p(-\vec{q},\sigma_p)\rangle\Big\}. \quad (3.19)$$

The equal-time commutators read [23,25]

$$\delta(x^0)[J^{4+i5}_{50}(x),J^{4-i5}_{5\nu}(0)] = [J^3_{\nu}(0)+\sqrt{3}J^8_{\nu}(0)]\delta^{(4)}(x),$$
$$\delta(x^0)[J^{4+i5}_{50}(x),\partial^{\nu}J^{4-i5}_{5\nu}(0)] = -i[\sigma_{44}(0)+\sigma_{55}(0)]\delta^{(4)}(x), \quad (3.20)$$

where $J^3_{\nu}(0)$ and $J^8_{\nu}(0)$ are vector hadronic currents, related to the electromagnetic $J^{(\mathrm{em})}_{\nu}(0)$ and hypercharge $Y_{\nu}(0)$ currents by

$$J^3_{\nu}(0)+\sqrt{3}J^8_{\nu}(0) = J^{(\mathrm{em})}_{nu}(0)+Y_{\nu}(0), \quad (3.21)$$

and $\sigma_{ab}(0)$ is a so-called σ-*term* operator. The σ-*term* operator $\sigma_{ab}(0)$ is related to the breaking of chiral symmetry. It can also be defined by the double commutator [26] $\sigma_{ab}(0) = [Q^a_5(0),[Q^b_5(0),H_{\chi SB}(0)]]$, where $Q^a_5(0)$ is the axial-vector charge operator and $H_{\chi SB}$ is the Hamiltonian of strong interactions breaking of chiral symmetry. In terms of current quark fields, it reads $H_{\chi SB}(0)=m_u\overline{u}(0)u(0)+m_d\overline{d}(0)d(0)+m_s\overline{s}(0)s(0)$, where $m_q(q=u,d,s)$ and $q(0)=u(0),d(0),s(0)$ are masses and interpolating fields of current quarks.

Substituting Eq. (3.20) into Eq. (3.19) and using Eq. (3.21), we get

$$M[K^-(\vec{q})p(-\vec{q},\sigma_p)\to K^-(\vec{k})p(-\vec{k},\sigma_p)] = \frac{(m_K^2-k^2)}{\sqrt{2}F_K m_K^2}\frac{(m_K^2-q^2)}{\sqrt{2}F_K m_K^2}\Big\{k^{\mu}q^{\nu}i\int d^4x e^{+ikx}\langle p(-\vec{k},\sigma_p) \times|\mathrm{T}[J^{4+i5}_{5\mu}(x)J^{4-i5}_{5\nu}(0)]|p(-\vec{q},\sigma_p)\rangle + \frac{1}{2}(k^{\mu}+q^{\mu}) \times\langle p(-\vec{k},\sigma_p)|J^{(\mathrm{em})}_{\mu}(0)+Y_{\mu}(0)|p(-\vec{q},\sigma_p)\rangle - \langle p(-\vec{k},\sigma_p)| \times\sigma_{44}(0)+\sigma_{55}(0)|p(-\vec{q},\sigma_p)\rangle\Big\}. \quad (3.22)$$

The matrix elements of the σ-*term* operator can be represented by [27–30,38]

$$\langle p(-\vec{k},\sigma_p)|\sigma_{44}(0)+\sigma_{55}(0)|p(-\vec{q},\sigma_p)\rangle = 2\sigma^{(I=1)}_{\overline{K}N}(t)\overline{u}(-\vec{k},\sigma_p)u(-\vec{q},\sigma_p), \quad (3.23)$$

where $\sigma^{(I=1)}_{\overline{K}N}(t)$ is the scalar form factor [26–30,38], defining the contribution to the amplitude of $\overline{K}N$ scattering in the state with isospin $I=1$, and $t=-(\vec{k}-\vec{q})^2$ is a squared transferred momentum. In terms of the quark-field operators, the σ-term $\sigma^{(I=1)}_{KN}(t)$ is defined by [26,27,30,38]

$$\sigma^{(I=1)}_{KN}(t) = \frac{m_u + m_s}{4m_N}\langle p(-\vec{k},\sigma_p)|\overline{u}(0)u(0) + \overline{s}(0)s(0)|p(-\vec{q},\sigma_p)\rangle. \tag{3.24}$$

According to ChPT [4,5], the σ term is of order of squared 4-momenta of K^- mesons, i.e., $\sigma^{(I=1)}_{KN}(t) \sim k^2 \sim q^2$.

Accounting for the contribution of the K^--meson pole and keeping the terms of order of $O(k^2)$ and $O(q^2)$ inclusively, we get the following expression for the amplitude of elastic low-energy K^-p scattering [30]:

$$\begin{aligned} M[K^-(\vec{q})p(-\vec{q},\sigma_p) \to K^-(\vec{k})p(-\vec{k},\sigma_p)] &= \overline{u}(-\vec{k},\sigma_p)\left\{\frac{F^p_E(t) + F^p_Y(t)}{4F^2_K}(k+q)^\mu\gamma_\mu \right. \\ &\quad \left. - \frac{1}{F^2_K}[\sigma^{(I=1)}_{KN}(t) - k^\mu q^\nu W_{\mu\nu}(\vec{k},\vec{q})]\right\}u(-\vec{q},\sigma_p), \end{aligned} \tag{3.25}$$

where we have denoted

$$\langle p(-\vec{k},\sigma_p)|J^{(\mathrm{em})}_\mu(0) + Y_\mu(0)|p(-\vec{q},\sigma_p)\rangle = [F^p_E(t) + F^p_Y(t)]\overline{u}(-\vec{k},\sigma_p)\gamma_\mu u(-\vec{q},\sigma_p),$$

$$\frac{1}{2}i\int d^4x\langle p(-\vec{k},\sigma_p)|\mathrm{T}[J^{4+i5}_{5\mu}(x)J^{4-i5}_{5\nu}(0)]|p(-\vec{q},\sigma_p)\rangle = \overline{u}(-\vec{k},\sigma_p)W_{\mu\nu}(\vec{k},\vec{q})u(-\vec{q},\sigma_p). \tag{3.26}$$

Here $F^p_E(t)$ and $F^p_Y(t)$ are the form factors of the electric and hypercharge of the proton, normalized by $F^p_E(0)=F^p_Y(0)=1$. We have not taken into account the magnetic form factor, which does not contribute to the *s*- and *p*-wave amplitudes of K^-p scattering at threshold.

The last two terms in Eq. (3.25) are of order of $O(k^2)$, where $k^2 \sim q^2 \sim kq$. For the calculation of the *p*-wave scattering length of elastic K^-p scattering, the contribution of the terms of order of $O(k^2)$ can be neglected.

From Eq. (3.25) at leading order in chiral expansion [4,5], we obtain the contribution to the *p*-wave amplitude of low-energy elastic K^-p scattering,

$$(2a^{K^-p}_{3/2} + a^{K^-p}_{1/2})_{\mathrm{SKT}} = \frac{1}{16\pi}\frac{\mu}{F^2_K}\frac{1}{m^2_N} = 0.002\ \mathrm{m}^{-3}_\pi. \tag{3.27}$$

Hence, the *p*-wave scattering length $(2a^{K^-p}_{3/2}+a^{K^-p}_{1/2})_{\mathrm{SKT}}$ is smaller than the contribution of the resonance states and practically can be neglected for the calculation of the *p*-wave scattering lengths of elastic K^-p scattering and, correspondingly, for the calculation of the energy-level shift of the *np* excited state of kaonic hydrogen. This implies that the *p*-wave scattering lengths $(2a^{Y\pi}_{3/2}+a^{Y\pi}_{1/2})_{\mathrm{SKT}}$ can also be neglected in comparison with the contributions of the resonance states.

C. *p*-wave scattering length $2a^{K^-p}_{3/2}+a^{K^-p}_{1/2}$ of elastic K^-p scattering and energy-level shift of *np* excited state of kaonic hydrogen

Substituting Eq. (3.27) into Eq. (3.15), we obtain the *p*-wave scattering length of elastic K^-p scattering,

$$2a^{K^-p}_{3/2} + a^{K^-p}_{1/2} = 0.113\ \mathrm{m}^{-3}_\pi. \tag{3.28}$$

Using Eq. (3.28), we compute the shift of the energy-level of the *np* excited state of kaonic hydrogen, given by Eq. (2.13). We get

$$\epsilon_{np} = \frac{32}{3}\frac{1}{n^3}\left(1 - \frac{1}{n^2}\right)\epsilon_{2p}, \tag{3.29}$$

where the shift of the energy-level of the 2*p* excited state is equal to

$$\epsilon_{np} = -\frac{\alpha^5}{16}\left(\frac{m_K m_N}{m_K + m_N}\right)^4(2a^{K^-p}_{3/2} + a^{K^-p}_{1/2}) = -0.6\ \mathrm{meV}. \tag{3.30}$$

Hence, the shift of the energy-level ϵ_{np} of the *np* excited state of kaonic hydrogen, induced by strong low-energy interactions, is smaller than 1 meV, i.e., $|\epsilon_{np}| < 1$ meV.

We would like to emphasize that unlike the shift of the energy-level of the *ns* state of kaonic hydrogen, which is defined by repulsive forces $\epsilon_{ns}=(203\pm15)/n^3$ eV [1], the shift of the energy-level of the *np* excited state ϵ_{np}, given by Eq. (3.29), is caused by attractive forces.

IV. *p*-WAVE SCATTERING LENGTHS $2a^{Y\pi}_{3/2}+a^{Y\pi}_{1/2}$ OF INELASTIC CHANNELS $K^-p \to Y\pi$

The imaginary part of the *p*-wave amplitude of elastic K^-p scattering at threshold, defining the total width of the excited *np* state of kaonic hydrogen, is caused by the four opened inelastic channels $K^-p \to \Sigma^+\pi^-$, $K^-p \to \Sigma^-\pi^+$, $K^-p \to \Sigma^0\pi^0$, and $K^-p \to \Lambda^0\pi^0$. At threshold, the contribution of these inelastic channels we describe by the *p*-wave scattering lengths $a^{Y\pi}_{1/2}$ and $a^{Y\pi}_{3/2}$ with $Y\pi = \Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$, respectively. The *p*-wave scattering lengths $a^{Y\pi}_{1/2}$ and $a^{Y\pi}_{3/2}$ determine low-energy transitions $K^-p \to Y\pi$ with total angular moment $J=1/2$ and $J=3/2$, respectively.

The *p*-wave scattering lengths $a^{Y\pi}_J$ we represent in the form of the superposition of the background part $(a^{Y\pi}_J)_B$ and the resonant part $\Sigma_R(a^{Y\pi}_J)_R$. It is convenient to include the contribution of the octet of low-lying baryons $B(\mathbf{8}) = (N(940), \Lambda^0(1116), \Sigma(1193))$ to the resonant part and to define the contribution of the background as $(a^{Y\pi}_J)_B = (a^{Y\pi}_J)_{\mathrm{SKT}}$. Since, as has been shown above, the contribution of the resonances $\Lambda^0(1890)$ and $\Sigma^0(1840)$ is negligibye small relative to the contribution of the resonance $\Sigma^0(1385)$, below for the calculation of the *p*-wave scattering lengths of inelastic channels $K^-p \to Y\pi$ we do not take them into account.

A. *p*-wave scattering lengths $2a^{\Sigma^+\pi^-}_{3/2}+a^{\Sigma^+\pi^-}_{1/2}$ of inelastic channel $K^-p \to \Sigma^+\pi^-$

The resonant parts of the *p*-wave scattering lengths $a^{\Sigma^+\pi^-}_{1/2}$ and $a^{\Sigma^+\pi^-}_{3/2}$ of the reaction $K^-p \to \Sigma^+\pi^-$ are equal to

$$\sum_R (a_{1/2}^{\Sigma^+\pi^-})_R = \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha)g_{\pi NN}\right]\left[\frac{2}{\sqrt{3}}\alpha g_{\pi NN}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_1)g_{\pi NN_1}\right]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_1 g_{\pi NN_1}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda_1^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_2)g_{\pi NN_2}\right]\left[\frac{2}{\sqrt{3}}\alpha_2 g_{\pi NN_2}\right]$$
$$\times\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda_2^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha-1)g_{\pi NN}]$$
$$\times[2(1-\alpha)g_{\pi NN}]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Sigma^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha_1-1)g_{\pi NN_1}]$$
$$\times[2(1-\alpha_1)g_{\pi NN_1}]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Sigma_1^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha_2-1)g_{\pi NN_2}]$$
$$\times[2(1-\alpha_2)g_{\pi NN_2}]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Sigma_2^0}-m_K-m_N}$$
$$= (-0.015+0.006-0.001-0.005+0.001-0.002)\ \mathrm{m}_\pi^{-3} = -0.016\ \mathrm{m}_\pi^{-3}. \tag{4.1}$$

and

$$(a_{3/2}^{\Sigma^+\pi^-})_R = \frac{g_{\pi NN}^2}{36\pi m_N}\frac{1}{m_K+m_N}\frac{1}{m_{\Sigma_3^0}-m_N-m_K}\sqrt{\frac{m_\Sigma}{m_N}}\left(1+\frac{1}{4}\frac{m_K}{m_N}\frac{m_K+m_N}{m_{\Sigma_3^0}}\right) = -0.082\ \mathrm{m}_\pi^{-3}. \tag{4.2}$$

The total p-wave scattering length of the reaction $K^-p\to\Sigma^+\pi^-$ is equal to

$$2a_{3/2}^{\Sigma^+\pi^-} + a_{1/2}^{\Sigma^+\pi^-} = (2a_{3/2}^{\Sigma^+\pi^-} + a_{1/2}^{\Sigma^+\pi^-})_{\mathrm{SKT}} - 0.180\ \mathrm{m}_\pi^{-3}. \tag{4.3}$$

B. p-wave scattering lengths of $2a_{3/2}^{\Sigma^-\pi^+}+a_{1/2}^{\Sigma^-\pi^+}$ of inelastic channel $K^-p\to\Sigma^-\pi^+$

The resonant parts of the p-wave scattering lengths $a_{1/2}^{\Sigma^-\pi^+}$ and $a_{3/2}^{\Sigma^-\pi^+}$ of the reaction $K^-p\to\Sigma^-\pi^+$ are equal to

$$\sum_R (a_{1/2}^{\Sigma^-\pi^+})_R = \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha)g_{\pi NN}\right]\left[\frac{2}{\sqrt{3}}\alpha g_{\pi NN}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_1)g_{\pi NN_1}\right]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_1 g_{\pi NN_1}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda_1^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_2)g_{\pi NN_2}\right]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_2 g_{\pi NN_2}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda_2^0}-m_K-m_N} - \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha-1)g_{\pi NN}][2(1-\alpha)g_{\pi NN}]$$
$$\times\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Sigma^0}-m_K-m_N} - \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha_1-1)g_{\pi NN_1}][2(1-\alpha_1)g_{\pi NN_1}]$$
$$\times\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Sigma_1^0}-m_K-m_N} - \frac{1}{8\pi}\frac{1}{m_K+m_N}[(2\alpha_2-1)g_{\pi NN_2}][2(1-\alpha_2)g_{\pi NN_2}]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Sigma_2^0}-m_K-m_N}$$
$$= (-0.015+0.006-0.001+0.005-0.001+0.002)\ \mathrm{m}_\pi^{-3} = -0.004\ \mathrm{m}_\pi^{-3} \tag{4.4}$$

and

$$(a_{3/2}^{\Sigma^-\pi^+})_R = -(a_{3/2}^{\Sigma^+\pi^-})_R = +0.082\ \mathrm{m}_\pi^{-3}. \tag{4.5}$$

The total p-wave scattering length of the reaction $K^-p\to\Sigma^-\pi^+$ is equal to

$$2a_{3/2}^{\Sigma^-\pi^+} + a_{1/2}^{\Sigma^-\pi^+} = (2a_{3/2}^{\Sigma^-\pi^+} + a_{1/2}^{\Sigma^-\pi^+})_{\mathrm{SKT}} + 0.160\ \mathrm{m}_\pi^{-3}. \tag{4.6}$$

C. p-wave scattering lengths of $2a_{3/2}^{\Sigma^0\pi^0}+a_{1/2}^{\Sigma^0\pi^0}$ of inelastic channel $K^-p\to\Sigma^0\pi^0$

The resonant parts of the p-wave scattering lengths $a_{1/2}^{\Sigma^0\pi^0}$ and $a_{3/2}^{\Sigma^0\pi^0}$ of the reaction $K^-p\to\Sigma^0\pi^0$ are equal to

$$\sum_R (a^{\Sigma^0\pi^0}_{1/2})_R = \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha)g_{\pi NN}\right]\left[\frac{2}{\sqrt{3}}\alpha g_{\pi NN}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_1)g_{\pi NN_1}\right]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_1 g_{\pi NN_1}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda^0_1}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}\left[\frac{1}{\sqrt{3}}(3-2\alpha_2)g_{\pi NN_2}\right]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_2 g_{\pi NN_2}\right]\frac{1}{2\sqrt{m_\Sigma m_N}}\frac{1}{m_{\Lambda^0_2}-m_K-m_N} = (-0.015+0.006-0.001)\ \mathrm{m}_\pi^{-3} = -0.010\ \mathrm{m}_\pi^{-3} \quad (4.7)$$

and

$$(a^{\Sigma^0\pi^0}_{3/2})_R = 0. \quad (4.8)$$

The total p-wave scattering length of the reaction $K^-p\to\Sigma^0\pi^0$ is equal to

$$2a^{\Sigma^0\pi^0}_{3/2} + a^{\Sigma^0\pi^0}_{1/2} = (2a^{\Sigma^0\pi^0}_{3/2} + a^{\Sigma^0\pi^0}_{1/2})_{\mathrm{SKT}} - 0.010\ \mathrm{m}_\pi^{-3}. \quad (4.9)$$

D. p-wave scattering lengths of $2a^{\Lambda^0\pi^0}_{3/2}+a^{\Lambda^0\pi^0}_{1/2}$ of inelastic channel $K^-p\to\Lambda^0\pi^0$

The resonant parts of the p-wave scattering lengths $a^{\Lambda^0\pi^0}_{1/2}$ and $a^{\Lambda^0\pi^0}_{3/2}$ of the reaction $K^-p\to\Lambda^0\pi^0$ are equal to

$$\sum_R (a^{\Lambda^0\pi^0}_{1/2})_R = \frac{1}{8\pi}\frac{1}{m_K+m_N}[-(2\alpha-1)g_{\pi NN}]\left[\frac{2}{\sqrt{3}}\alpha g_{\pi NN}\right]\frac{1}{2\sqrt{m_{\Lambda^0}m_N}}\frac{1}{m_{\Sigma^0}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}[-(2\alpha_1-1)g_{\pi NN_1}]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_1 g_{\pi NN_1}\right]\frac{1}{2\sqrt{m_{\Lambda^0}m_N}}\frac{1}{m_{\Sigma^0_1}-m_K-m_N} + \frac{1}{8\pi}\frac{1}{m_K+m_N}[-(2\alpha_2-1)g_{\pi NN_2}]$$
$$\times\left[\frac{2}{\sqrt{3}}\alpha_2 g_{\pi NN_2}\right]\frac{1}{2\sqrt{m_{\Lambda^0}m_N}}\frac{1}{m_{\Sigma^0_2}-m_K-m_N} = (0.006-0.005-0.001)\ \mathrm{m}_\pi^{-3} = 0 \quad (4.10)$$

and

$$(a^{\Lambda^0\pi^0}_{3/2})_R = \frac{\sqrt{3}g^2_{\pi NN}}{36\pi m_N}\frac{1}{m_K+m_N}\frac{1}{m_{\Sigma^0_3}-m_N-m_K}\sqrt{\frac{m_{\Lambda^0}}{m_N}}\left(1+\frac{1}{4}\frac{m_K}{m_N}\frac{m_K+m_N}{m_{\Sigma^0_3}}\right) = -0.137\ \mathrm{m}_\pi^{-3}. \quad (4.11)$$

The total p-wave scattering length of the reaction $K^-p\to\Lambda^0\pi^0$ is equal to

$$2a^{\Lambda^0\pi^0}_{3/2} + a^{\Lambda^0\pi^0}_{1/2} = (2a^{\Lambda^0\pi^0}_{3/2} + a^{\Lambda^0\pi^0}_{1/2})_{\mathrm{SKT}} - 0.274\ \mathrm{m}_\pi^{-3}. \quad (4.12)$$

E. p-wave scattering lengths of inelastic reactions $K^-p\to Y\pi$ and energy-level width of *np* excited state of kaonic hydrogen

According to the estimate Eq. (3.27), the contribution of the p-wave scattering lengths $(2a^{Y\pi}_{3/2}+a^{Y\pi}_{1/2})_{\mathrm{SKT}}$ can be neglected in comparison with the contribution of the baryon resonances. Therefore, below we neglect $(2a^{Y\pi}_{3/2}+a^{Y\pi}_{1/2})_{\mathrm{SKT}}$ for the estimate of the energy-level width of the *np* excited state of kaonic hydrogen.

Using Eqs. (4.3), (4.6), and (4.12) and substituting them into Eq. (2.13), we compute the energy-level width of the *np* excited state of kaonic hydrogen,

$$\Gamma_{np} = \frac{32}{3}\frac{1}{n^3}\left(1-\frac{1}{n^2}\right)\Gamma_{2p}. \quad (4.13)$$

The partial width Γ_{2p} of the energy-level of the $2p$ excited state of kaonic hydrogen is equal to

$$\Gamma_{2p} = \frac{\alpha^5}{24}\left(\frac{m_K m_N}{m_K+m_N}\right)^4\sum_{Y\pi}(2a^{Y\pi}_{3/2}+a^{Y\pi}_{1/2})^2 k^3_{Y\pi} = 2\ \mathrm{meV} \quad (4.14)$$

or $\Gamma_{2p}=3\times10^{12}\ \mathrm{s}^{-1}$.

The lifetime of the $2p$ state of kaonic hydrogen, defined by the decays of kaonic hydrogen into hadronic states $(K^-p)_{2p}\to Y\pi$, where $Y\pi=\Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$, is equal to $\tau_{2p}=3.4\times10^{-13}$ s. It is much smaller than the lifetime of the K^- meson, $\tau_{K^-}=1.24\times10^{-8}$ s [20], which is the upper limit on the lifetime of kaonic hydrogen. Thus, the rates of the hadronic decays of kaonic hydrogen in the *np* excited states are comparable with the rates of the deexcitation of kaonic hydrogen $np\to1s$, caused by the emission of the x rays [7–13].

The result obtained for the partial width of the excited $2p$ state of kaonic hydrogen, given by Eq. (4.14), is important for the theoretical analysis of the x-ray yields in kaonic hydrogen [7–13].

V. CONCLUSION

The quantum field theoretic model of the description of low-energy $\bar{K}N$ interaction in the s-wave state near threshold, which we have suggested in [1,6], is extended on the analysis of low-energy $\bar{K}N$ interactions in the p-wave state near threshold. We would like to emphasize that our approach to the description of low-energy $\bar{K}N$ interaction in the s-wave state near threshold agrees well with the nonrelativistic effective field theory based on ChPT by Gasser and Leutwyler, which has been recently applied by Meißner *et al.* [3] to the calculation of the energy-level displacement of the ns state of kaonic hydrogen and systematic corrections to the energy-level displacement of the ns state, caused by QCD isospin breaking and electromagnetic interactions. The result for the energy-level displacement of the ns state of kaonic hydrogen has been obtained in [3] in terms of the s-wave scattering lengths a_0^0 and a_0^1 of $\bar{K}N$ scattering with isospin $I=0$ and $I=1$, respectively. The s-wave scattering lengths a_0^0 and a_0^1 have been treated as free parameters of the approach. Using our results for the s-wave scattering lengths a_0^0 and a_0^1 [1,6] and keeping leading terms in QCD isospin breaking and electromagnetic interactions, i.e., accounting for only the contribution of Coulombic photons, we have shown that the numerical value of the energy-level displacement of the ns state of kaonic hydrogen, computed by Meißner *et al.* [3], agrees well with both our theoretical prediction [1] and recent experimental data by the DEAR Collaboration [2] within 1.5 standard deviations. Hence, our approach to the description of low-energy dynamics of strong low-energy $\bar{K}N$ interactions at threshold agrees well with a general description of strong low-energy interactions of hadrons within nonrelativistic effective field theory based on ChPT [3–5].

The detection of the x rays of the x-ray cascade processes, leading to the deexcitation of kaonic hydrogen from the excited states to the ground state, is the main experimental tool for the measurement of the energy-level displacement of the ground state of kaonic hydrogen, caused by strong low-energy interactions [2,39]. The main transitions in kaonic hydrogen, which are measured experimentally for the extraction of the energy-level displacement of the ground state, are $3p\to 1s$ and $2p\to 1s$, i.e., the reactions $(K^-p)_{3p}\to(K^-p)_{1s}+\gamma$ and $(K^-p)_{2p}\to(K^-p)_{1s}+\gamma$.

As has been pointed out by Markushin and Jensen, the yields of x rays of these transitions are quite sensitive to the value of Γ_{2p} [13]. Using Γ_{2p} as an input parameter taking values from the region $0.1\ \mathrm{meV}\leqslant\Gamma_{2p}\leqslant 0.9\ \mathrm{meV}$, Markushin and Jensen [13] have found that their theoretical predictions for the x-ray yields in kaonic hydrogen agree well with the experimental data on the x-ray yields detected by the KEK Collaboration [40], which have been used for the extraction of the energy-level displacement of the ground state of kaonic hydrogen, for $\Gamma_{2p}=0.3\ \mathrm{meV}=4.6\times 10^{11}\ \mathrm{s}^{-1}$ and $\epsilon_{1s}=320$ eV and $\Gamma_{1s}=400$ eV.

Recent experimental data on the energy-level displacement of the ground of kaonic hydrogen obtained by the DEAR Collaboration [2] were smaller by a factor of 2 than the experimental data by the KEK Collaboration [40]. Our theoretical analysis of the energy-level displacement of the $2p$ excited state of kaonic hydrogen has shown that the rate of the hadronic decays of kaonic hydrogen from the $2p$ excited state is equal to $\Gamma_{2p}=2\ \mathrm{meV}=3\times 10^{12}\ \mathrm{s}^{-1}$, which is an order of magnitude larger than the phenomenological value $\Gamma_{2p}=0.3\ \mathrm{meV}=4.6\times 10^{11}\ \mathrm{s}^{-1}$, used by Markushin and Jensen as an input parameter [13].

Thus, the computed value $\Gamma_{2p}=2$ meV of the energy-level of the $2p$ excited state of kaonic hydrogen can be applied to the theoretical analysis of the x-ray yields in kaonic hydrogen of recent experimental data by the DEAR Collaboration [2] using the the following input parameters: (1) the experimental setup [39] and (2) the theoretical predictions for the hadronic energy-level displacements of the $2p$ state, $\epsilon_{2p}=-0.6$ meV, $\Gamma_{2p}=2.0$ meV, and the ground state, $\epsilon_{1s}=203$ eV and $\Gamma_{1s}=226$ eV, of kaonic hydrogen [1].

VI. COMMENT ON THE RESULT

After this paper was accepted for publication, Faifman and Men'shikov presented the calculated yields for the K series of x rays for kaonic hydrogen in dependence of the hydrogen density [41]. They have shown that the use of the theoretical value $\Gamma_{2p}=2$ meV of the width of the $2p$ state of kaonic hydrogen, computed in our work, leads to good agreement with the experimental data, measured for the K_α line by the KEK Collaboration [40]. They have also shown that the results of cascade calculations with other values of the width of the $2p$ excited state of kaonic hydrogen, used as an input parameter, disagree with the available experimental data. The results obtained by Faifman and Men'shikov contradict those by Jensen and Markushin [13]. Therefore, as has been accentuated by Faifman and Men'shikov [41], further analysis of the experimental data by the DEAR Collaboration should allow us to perform a more detailed comparison of the theoretical value $\Gamma_{2p}=2$ meV with other phenomenological values of the width of the $2p$ state of kaonic hydrogen Γ_{2p}, used as input parameters.

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