

Conclusive and arbitrarily perfect quantum-state transfer using parallel spin-chain channels

Daniel Burgarth and Sougato Bose

Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

(Received 28 June 2004; published 13 May 2005)

We suggest a protocol for perfect quantum communication through spin-chain channels. By combining a dual-rail encoding with measurements only at the receiving end, we can get conclusively perfect state transfer, whose probability of success can be made arbitrarily close to unity. As an example of such an *amplitude-delaying channel*, we show how two parallel Heisenberg spin chains can be used as quantum wires. Perfect state transfer with a probability of failure lower than P in a Heisenberg chain of N spin- $\frac{1}{2}$ particles can be achieved in a timescale of the order of $(0.33\hbar/J)N^{1.7}|\ln P|$. We demonstrate that our scheme is more robust to decoherence and nonoptimal timing than any scheme using single spin chains.

DOI: 10.1103/PhysRevA.71.052315

PACS number(s): 03.67.Hk, 05.50.+q, 05.60.Gg, 75.10.Pq

I. INTRODUCTION

The development of reliable methods to transfer quantum states is of fundamental importance in quantum information theory. Usually, flying qubits, such as photons, ballistic electrons, and guided atoms and ions, are considered for this purpose. However, converting back and forth between stationary qubits (of say a quantum computer or as held by the communicating parties) and mobile carriers of quantum information and interfacing between different physical implementations of qubits is very difficult and may not be really worth it for short communication distances.

An attractive alternative is to use a finite array of interacting but stationary qubits as an information bus. It would, however, not be very useful if the interactions between various pairs of stationary qubits have to be repeatedly switched on and off to perform the communication because gating errors will then accumulate. Moreover, the local control required will be as high as that of a quantum computer. Only if we can utilize systems with permanently coupled material qubits (such as molecular spin chains), or systems without local control (such as Josephson junction arrays or optical lattices) with minimal global switchings, can we have a communication bus much before a quantum computer. In such schemes, both the amplitude and the phase damping (mechanisms of decoherence) can be ensured to be no worse than that of a single moving qubit, by ensuring that there is, at most, one excitation in the array during the communication process. With the above view in mind, recently the use of spin chains [1–12] and harmonic chains [13] as quantum wires have been proposed (see Fig. 1).

The initial system-independent proposal [1] was inspired by the natural setting for spin-chain molecules (and optical lattices): regular arrays without local accessibility. Single spin encoding was assumed to avoid quantum gates. For this simplicity, its specific realization is already being proposed [14]. However, it allows only imperfect communication fidelity and necessitates the use of entanglement distillation from a large ensemble, which destroy its simplicity. Later approaches of perfecting fidelity require either the engineering of the couplings [3,4] or an encoding of a qubit to several spins [5,6]. Independently, local measurements on each qubit along the chain [7–9] have been proposed. The above deviate

either from naturalness or from simplicity. We are thus sorely in need of a scheme that remains natural and simple, yet achieves perfect quantum communication. This is achieved in this paper by using a dual-rail encoding.

The outline of the paper is as follows. In Sec. II, we suggest a scheme for quantum communication using two parallel spin chains of the most natural type (namely, those with constant couplings). We require modest encodings (or gates) and measurements only at the ends of the chains. The state transfer is *conclusive*, which means that it is possible to tell by the outcome of a quantum measurement, without destroying the state, if the transfer took place or not. If it did, then the transfer was *perfect*. The transmission time for conclusive transfer is no longer than for single spin chains. In Sec. III, we demonstrate that our scheme offers even more: if the transfer was not successful, then we can wait for some time and just repeat the measurement, without having to resend the state. By performing sufficiently many measurements, the probability for perfect transfer approaches unity. Hence the transfer is *arbitrarily perfect*. We will show in Sec. IV that the time needed to transfer a state with a given probability scales in a reasonable way with the length of the chain. Finally, in Sec. V we show that encoding to parallel chains and the conclusiveness also makes our protocol more robust to decoherence (a hitherto unaddressed issue in the field of quantum communication through spin chains).

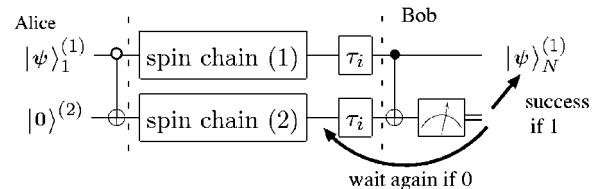


FIG. 1. Quantum circuit representation of conclusive and arbitrarily perfect state transfer. The first gate at Alice's qubits represents a NOT gate applied to the second qubit controlled by the first qubit being zero. The qubits $|\psi\rangle_1^{(1)}$ on the left-hand side represents an arbitrary input state at Alice's site, and the qubit $|\psi\rangle_N^{(1)}$ represents the same state, successfully transferred to Bob's site. The τ_i gate represents the unitary evolution of the spin chains for a time interval of τ_i .

II. SCHEME FOR CONCLUSIVE TRANSFER

We intend to propose our scheme in a system-independent way with occasional references to systems where conditions required by our scheme are achieved. We assume that our system consists of two identical uncoupled spin- $\frac{1}{2}$ -chains (1) and (2) of length N , described by the Hamiltonian

$$H = H^{(1)} \otimes I^{(2)} + I^{(1)} \otimes H^{(2)} - E_g I^{(1)} \otimes I^{(2)}. \quad (1)$$

The term *identical* states that $H^{(1)}$ and $H^{(2)}$ are the same apart from the label of the Hilbert space on which they act. The requirement of parallel chains instead of just one is not problematic because, in many experimental realizations of spin chains, it is much easier to produce a whole bunch of parallel uncoupled [15,16] chains than just a single one.

We assume that the ground state of each chain is $|\mathbf{0}\rangle^{(i)} \equiv |0_1 \dots 0_N\rangle^{(i)}$, i.e., a ferromagnetic ground state, with $H^{(i)}|\mathbf{0}\rangle^{(i)} = E_g|\mathbf{0}\rangle^{(i)}$, and that the subspace consisting of the single spin excitations $|\mathbf{n}\rangle^{(i)} \equiv \sigma_n^{+(i)}|\mathbf{0}\rangle^{(i)}$ ($n=1, \dots, N$) is invariant under $H^{(i)}$. An arbitrary qubit at the site n of system (i) can be written as

$$|\psi\rangle_n^{(i)} \equiv \alpha|\mathbf{0}\rangle^{(i)} + \beta|\mathbf{n}\rangle^{(i)}. \quad (2)$$

The dynamics restricted to this subspace can be expressed in terms of the transition amplitudes

$$f_{r,s}(t) \equiv \langle \mathbf{r} | e^{iH^{(i)}t} | \mathbf{s} \rangle^{(i)}. \quad (3)$$

The aim of our protocol is to transfer quantum information from the first (“Alice”) to the N th (“Bob”) qubit of the first chain,

$$|\psi\rangle_1^{(1)} \rightarrow |\psi\rangle_N^{(1)}. \quad (4)$$

To achieve this, we need $f_{1,N}(t) \neq 0$ (which is valid for a Heisenberg chain, for example [1]). An advantage of Heisenberg ferromagnetic chains over a noninteracting qubit array is that some XXZ anisotropy can make the states $|\mathbf{0}\rangle$ and $|\mathbf{n}\rangle$ stable against excitations at finite temperatures [17]. Even a small anisotropy in the coupling J may suffice (as J itself can be as high as 2000 K [15]). Alternatively, one can prevent thermal excitations by applying an uniform magnetic field to the chain.

The initial state of the system is $|\psi\rangle_1^{(1)} \otimes |\mathbf{0}\rangle^{(2)}$. The first step of the protocol is to encode the input qubit in a “dual rail” [18] by applying a NOT gate on the first qubit of system (2) controlled by the first qubit of system (1) being zero, resulting in a superposition of excitations in both systems,

$$|\mathbf{s}(0)\rangle = \alpha|\mathbf{0}\rangle^{(1)} \otimes |\mathbf{1}\rangle^{(2)} + \beta|\mathbf{1}\rangle^{(1)} \otimes |\mathbf{0}\rangle^{(2)}. \quad (5)$$

This is assumed to take place in a much shorter timescale than the system dynamics. Even though a two-qubit gate in solid-state systems is difficult, such a gate for charge qubits has been reported [19]. For the same qubits, Josephson arrays have been proposed as single spin chains for quantum communication [14]. For this system, both requisites of our scheme are thus available. In fact, the demand that Alice and Bob can do measurements and apply gates to their local qubits (i.e., the ends of the chains) will be naturally fulfilled in practice since we are suggesting a scheme to transfer information between quantum computers.

Under the system Hamiltonian, the excitation in Eq. (5) will travel along the two systems. The state after the time τ_1 can be written as

$$|\phi(\tau_1)\rangle = \sum_{n=1}^N f_{n,1}(\tau_1) |\mathbf{s}(n)\rangle, \quad (6)$$

where $|\mathbf{s}(n)\rangle = \alpha|\mathbf{0}\rangle^{(1)} \otimes |\mathbf{n}\rangle^{(2)} + \beta|\mathbf{n}\rangle^{(1)} \otimes |\mathbf{0}\rangle^{(2)}$. We can decode the qubit by applying a controlled-NOT gate at Bob’s site. The state thereafter will be

$$\sum_{n=1}^{N-1} f_{n,1}(\tau_1) |\mathbf{s}(n)\rangle + f_{N,1}(\tau_1) |\psi\rangle_N^{(1)} \otimes |\mathbf{N}\rangle^{(2)}. \quad (7)$$

Bob can now perform a measurement on his qubit of system (2). If the outcome of this measurement is 1, he can conclude that the state $|\psi\rangle_1^{(1)}$ has been successfully transferred to him. This happens with the probability $|f_{N,1}(\tau_1)|^2$. If the outcome is 0, the system is in the state

$$\frac{1}{\sqrt{P(1)}} \sum_{n=1}^{N-1} f_{n,1}(\tau_1) |\mathbf{s}(n)\rangle, \quad (8)$$

where

$$P(1) = 1 - |f_{N,1}(\tau_1)|^2 \quad (9)$$

is the probability of failure for the first measurement. If the protocol stopped here, and Bob would just assume his state as the transferred one, then the channel could be described as an amplitude-damping channel, with exactly the same fidelity as the single-chain scheme discussed in [1]. But success probability is more valuable than fidelity: Bob has gained knowledge about his state and may reject it and ask Alice to retransmit. However, as we will show in Sec. III, this is not necessary.

III. ARBITRARILY PERFECT STATE TRANSFER

Because Bob’s measurement has not revealed anything about the input state, the information is still residing in the chain. By letting the state (8) evolve for another time τ_2 and applying the controlled-NOT gate again, Bob has another chance of receiving the input state. The state before performing the second measurement is easily seen to be

$$\frac{1}{\sqrt{P(1)}} \sum_{n=1}^N \{f_{n,1}(\tau_2 + \tau_1) - f_{n,N}(\tau_2)f_{N,1}(\tau_1)\} |\mathbf{s}(n)\rangle. \quad (10)$$

Hence the probability to receive the qubit at Bob’s site at the second measurement is

$$\frac{1}{P(1)} |f_{N,1}(\tau_2 + \tau_1) - f_{N,N}(\tau_2)f_{N,1}(\tau_1)|^2. \quad (11)$$

If the transfer was still unsuccessful, then this strategy can be repeated over and over. Each time Bob has a probability of failed state transfer that can be obtained from the generalization of Eq. (10) to an arbitrary number of iterations. The joint probability that Bob fails to receive the state all the time is just the product of these probabilities. We denote the joint

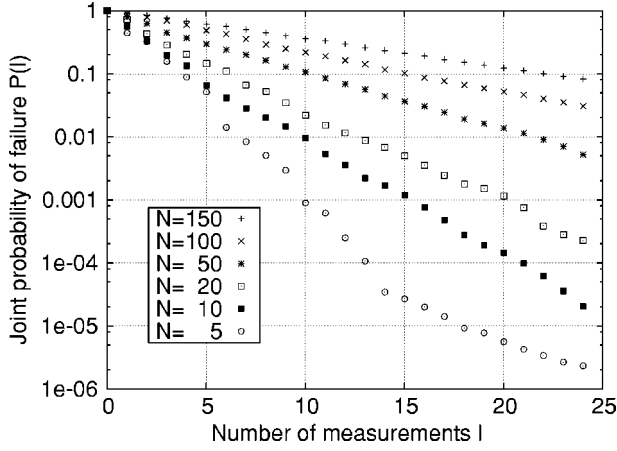


FIG. 2. Semilogarithmic plot of the joint probability of failure $P(l)$ as a function of the number of measurements l . Shown are Heisenberg spin- $\frac{1}{2}$ -chains with different lengths N . The times between measurements τ_i have been optimized numerically.

probability of failure for having done l unsuccessful measurements as $P(l)$. This probability depends on the time intervals τ_i between the $(i-1)$ th and i th measurement, and we are interested in the case where the τ_i are chosen such that the transfer is fast. It is possible to write a simple algorithm that computes $P(l)$ for any transition amplitude $f_{r,s}(t)$. Figure 2 shows some results for a Heisenberg spin- $\frac{1}{2}$ -chain with equal nearest-neighbor couplings,

$$H^{(i)} = -J \sum \vec{\sigma}_n^{(i)} \cdot \vec{\sigma}_{n+1}^{(i)}. \quad (12)$$

This model is exactly solvable, and the transition amplitude is given explicitly in [1]. However, the results are valid for a wide class of anisotropies and in the presence of a uniform magnetic field, too.

An interesting question is whether the joint probability of failure can be made arbitrarily small with a large number of measurements. Since $P(l)$ is a bounded, monotonic decreasing series, it must have a limit. In fact, the times τ_i can be chosen such that the transfer becomes arbitrarily perfect. This has been proven in [20], where a generalization of the above scheme and a much wider class of Hamiltonians is considered. In the limit of a large number of measurements, the spin channel will not damp the initial amplitude, but only *delay* it.

IV. ESTIMATION OF THE TIME SCALE OF THE TRANSFER

The achievable fidelity is an important but not the only criteria of a state transfer protocol. In this section, we give an heuristic approach to estimate the time that it needs to achieve a certain fidelity in a Heisenberg spin chain. The comparison to numeric examples is confirming this approach.

Let us first describe the dynamic of the chain in a very qualitative way. Once Alice has initialized the system, an excitation wave packet will travel along the chain. As shown in [1], it will reach Bob at a time of the order of

$$\tau_{\max} \approx \frac{\hbar N}{2J}, \quad (13)$$

with an amplitude of

$$|f_{N,1}(\tau_{\max})|^2 \approx 1.35N^{-2/3}. \quad (14)$$

It is then reflected and travels back and forth along the chain. Since the wave packet is also dispersing, it starts interfering with its tail, and after a couple of reflections the dynamic is becoming quite random. This effect becomes even stronger because of Bob's measurements, which change the dynamics by projecting away parts of the wave packet. However, $2\tau_{\max}$ (the time it takes for a wave packet to travel twice along the chain) remains a good estimate of the time scale in which significant probability amplitude peaks at Bob's site occur, and Eq. (14) remains a good estimate of the amplitude of these peaks. Therefore, the joint probability of failure is expected to scale as

$$P(l) \approx (1 - 1.35N^{-2/3})^l \quad (15)$$

in a time of the order of

$$t(l) \approx 2\tau_{\max}l = \frac{N\hbar l}{J}. \quad (16)$$

If we combine Eqs. (15) and (16) and solve for the time $t(P)$ needed to reach a certain probability of failure P , we get

$$t(P) \approx \frac{0.51\hbar N^{5/3}}{J} |\ln P|. \quad (17)$$

We compare this rough estimate with exact numerical results in Fig. 3. The best fit is given by

$$t(P) = \frac{0.33\hbar N^{5/3}}{J} |\ln P|. \quad (18)$$

We can conclude that the transmission time for arbitrarily perfect transfer is scaling not much worse with the length N of the chains than the single spin-chain schemes. Despite the logarithmic dependence on P , the time it takes to achieve high fidelity is still reasonable. For example, a system with $N=100$ and $J=20 \text{ K} \times k_B$ will take $\sim 1.3 \text{ ns}$ to achieve a fidelity of 99%. In many systems, decoherence is completely negligible within this time scale. For example, some Josephson junction systems [21] have a decoherence time of $T_\phi \approx 500 \text{ ns}$, while trapped ions have even larger decoherence times.

V. DECOHERENCE AND IMPERFECTIONS

If the coupling between the spins J is very small or the chains are very long, the transmission time may no longer be negligible with respect to the decoherence time (see Sec. IV). It is interesting to note that the dual-rail encoding then offers some significant general advantages over single-chain schemes. Since we are suggesting a system-independent scheme, we will not study the effects of specific environments on our protocol, but just qualitatively point out its general advantages.

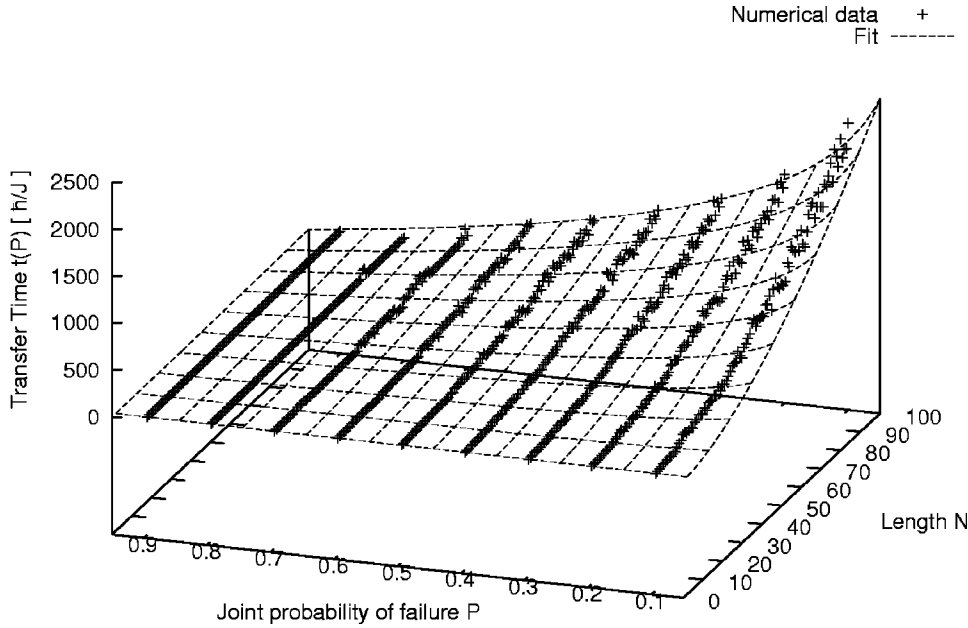


FIG. 3. Time t needed to transfer a state with a given joint probability of failure P across a chain of length N . The points denote exact numerical data, and the fit is given by Eq. (18).

At least theoretically, it is always possible to cool the system down or to apply a strong magnetic field such that the environment is not causing further excitations. Then, there are two remaining types of quantum noise that will occur: phase noise and amplitude damping. Phase noise is a serious problem and arises here *only* when an environment can distinguish between spin flips on the first chain and spin flips on the second chain. It is therefore important that the environment cannot resolve their difference. In this case, the environment will only couple with the total z component

$$S_{z,n} = \sigma_{z,n}^{(1)} + \sigma_{z,n}^{(2)} \quad (19)$$

of the spins of both chains at each position n . This has been discussed for spin-boson models in [22,23] but should also hold for spin environments as long as the chains are close enough. The qubit is encoded in a decoherence-free subspace [24], and the scheme is fully robust to phase noise. Even though this may not be true for all implementations of dual-rail encoding, it is worthwhile noting it because such an opportunity does not exist *at all* for single-chain schemes, where the coherence between two states with different total z component of the spin has to be preserved. Having shown one way of avoiding phase noise, at least in some systems, we now proceed to amplitude damping.

The evolution of the system in presence of amplitude damping of a rate Γ can be easily derived using a quantum-jump approach [25]. Like for phase noise, it is necessary that the environment acts symmetrically on the chains. The dynamics is then given by an effective nonunitary Hamiltonian

$$H_{\text{eff}} = H + i\Gamma \sum_n (S_{z,n} + 2)/2 \quad (20)$$

if no jump occurs, and the effect of a jump is given by the operator

$$\sum_n S_n^-, \quad (21)$$

which will put the system in the ground state. As this can be solved analytically, we do not go into numerics. The state of the system before the first measurement conditioned on no jump is given by

$$e^{-\Gamma t} \sum_{n=1}^N f_{n,1}(t) |\mathbf{s}(n)\rangle, \quad (22)$$

and this happens with the probability of $e^{-2\Gamma t}$ (the norm of the above state). If a jump occurs, the system will be in the ground state

$$\sqrt{1 - e^{-2\Gamma t}} |\mathbf{0}\rangle^{(1)} \otimes |\mathbf{0}\rangle^{(2)}. \quad (23)$$

The density matrix at the time t is given by a mixture of (22) and (23). In case of (23), the quantum information is completely lost and Bob's error check qubit will never show success. If Bob, however, measures a success, it is clear that no jump has occurred and he has the perfectly transferred state. Therefore the protocol *remains conclusive*, but the success probability is lowered by $e^{-2\Gamma t}$. This result is still valid for multiple measurements, which leave the state (23) unaltered. The probability of a successful transfer at each particular measurement l will decrease by $e^{-2\Gamma t(l)}$, where $t(l)$ is the time of the measurement. After a certain number of measurements, the *joint* probability of failure will no longer decrease. Thus the transfer will no longer be *arbitrarily* perfect, but can still reach a very high fidelity. Some numerical examples of the minimal joint probability of failure that can be achieved,

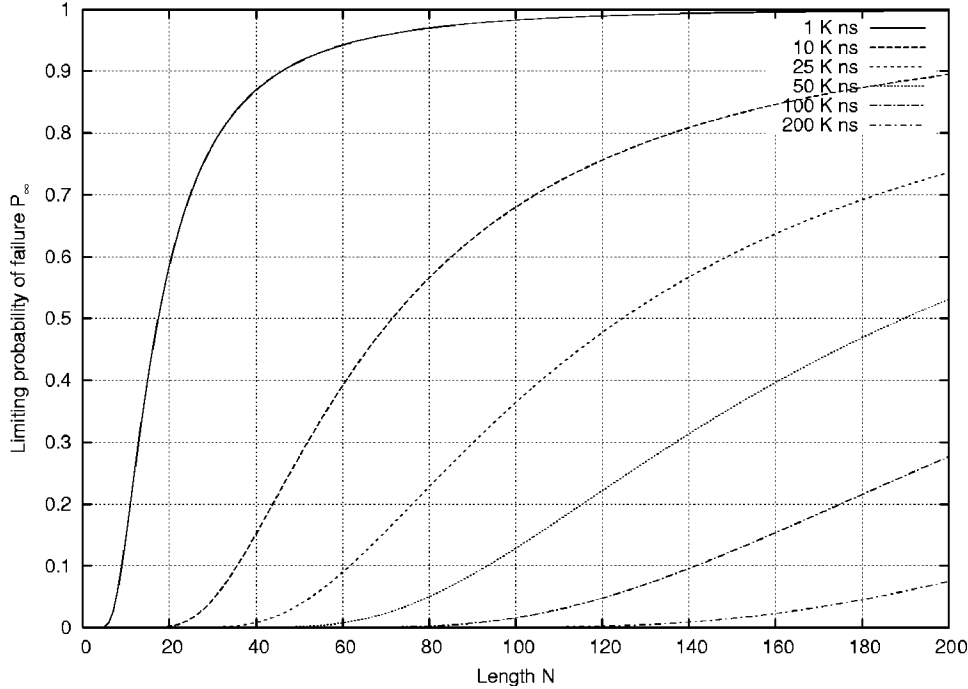


FIG. 4. The minimal joint probability of failure $P(l)$ for chains with length N in the presence of amplitude damping. The parameter J/Γ of the curves is the coupling of the chain (in kelvins) divided by the decay rate (ns^{-1}).

$$P_\infty \equiv \lim_{l \rightarrow \infty} P(l) \quad (24)$$

$$\approx \prod_{l=1}^{\infty} (1 - 1.35N^{-2/3} e^{-(2\Gamma N \hbar/J)l}) \quad (25)$$

are given in Fig. 4. For $J/\Gamma = 50$ K ns nearly perfect transfer is still possible for chains up to a length of $N \approx 40$. In a single Heisenberg chain using the scheme described in [1], this system could only achieve a fidelity of 0.23 when transferring an excitation.

Even if the amplitude damping is not symmetric, its effect is weaker than in single-spin schemes. This is because it can be split into a symmetric and asymmetric part. The symmetric part can be overcome with the above strategies. For example, if the amplitude damping on the chains is Γ_1 and Γ_2 with $\Gamma_1 > \Gamma_2$, the state (22) will be

$$\sum_{n=1}^N f_{n,1}(t) \{ \alpha e^{-\Gamma_2 t} |\mathbf{0n}\rangle + \beta e^{-\Gamma_1 t} |\mathbf{n0}\rangle \} \quad (26)$$

$$= e^{-\Gamma_2 t} \sum_{n=1}^N f_{n,1}(t) \{ \alpha |\mathbf{0n}\rangle + \beta e^{-(\Gamma_1 - \Gamma_2)t} |\mathbf{n0}\rangle \} \quad (27)$$

$$\approx e^{-\Gamma_2 t} \sum_{n=1}^N f_{n,1}(t) |s(n)\rangle \quad (28)$$

provided that $t \ll (\Gamma_1 - \Gamma_2)^{-1}$. Using a chain of length $N=20$ with $J=20$ K $\times k_B$ and $\Gamma_1^{-1}=4$ ns, $\Gamma_2^{-1}=4.2$ ns we would have to fulfill $t \ll 164$ ns. We could perform ≈ 10 measurements [cf. Eq. (16)] without deviating too much from the state (28). In this time, we can use our protocol in the normal way. The resulting success probability given by the finite version of Eq. (25) would be 75%. A similar reasoning is valid for

phase noise, where the environment can be split into common and separate parts. If the chains are close, the common part will dominate and the separate parts can be neglected for short times.

Finally, let us mention another advantage of our scheme. In single-chain schemes, Bob has to extract the state precisely at an optimal time to obtain it with high fidelity. Our scheme is robust to the errors in this. Even if Bob measures to extract his state at an incorrect (nonoptimal) time, he will receive the perfect state conditional on his measurement outcome. If he is unsuccessful, then he simply tries again, without having Alice to resend. Also, because of the conclusive nature of the protocol, once Bob has received the state, the rest of the channel is automatically in the ground state and does not need to be reset for the next transfer (as opposed to many of the existing schemes [1,2,7,8]).

VI. CONCLUSIONS

In conclusion, we have presented a simple and efficient scheme for conclusive and arbitrarily perfect quantum-state transfer. To achieve this, two parallel spin chains (individually amplitude-damping channels) have been used as one *amplitude-delaying channel*. We have shown that our scheme is more robust to decoherence and imperfect timing than the single-chain schemes. Even though the encoding is simple, it has made spin-chain-based communications *both* realistic and perfect at the same time.

Our strategy can be generalized to graphs interconnecting many different users and to many other systems. As an example, we will now briefly mention how our scheme can be adapted to a *single* chain of qutrits. The correct generalization of the exchange interaction for a chain of three-level quantum system is a SU(3) chain [26]. For example, a chain of atoms with three internal levels, $|-1\rangle$, $|0\rangle$, and $|1\rangle$ in an

optical lattice, where the atoms can hop from site to site but more than one atom cannot occupy a single site, will form a $SU(3)$ chain. If we relabel our parallel spin chain states $|\mathbf{0}\rangle^{(1)}|\mathbf{0}\rangle^{(2)}$ by $|0_1, \dots, 0_n, \dots, 0_N\rangle$, $|\mathbf{n}\rangle^{(1)}|\mathbf{0}\rangle^{(2)}$ by $|0_1, \dots, 1_n, \dots, 0_N\rangle$ and $|\mathbf{0}\rangle^{(1)}|\mathbf{n}\rangle^{(2)}$ by $|0_1, \dots, -1_n, \dots, 0_N\rangle$, then our protocol can be mapped to a single chain of qutrits interacting via $SU(3)$ exchange. Though the state is no longer encoded in a decoherence free-subspace as before, in an optical lattice implementation, one can use three hyperfine

ground states of atoms as $|-1\rangle$, $|0\rangle$, and $|1\rangle$ to completely avoid amplitude damping.

ACKNOWLEDGMENT

This work was supported by the UK Engineering and Physical Sciences Research Council Grant No. GR/S62796/01 and the QIPIRC.

-
- [1] Sougato Bose, Phys. Rev. Lett. **91**, 207901 (2003).
 [2] V. Subrahmanyam, Phys. Rev. A **69**, 034304 (2004).
 [3] M. Christandl, N. Datta, A. Ekert, and A. J. Landahl, Phys. Rev. Lett. **92**, 187902 (2004).
 [4] C. Albanese, M. Christandl, N. Datta, and A. Ekert, Phys. Rev. Lett. **93**, 230502 (2004).
 [5] T. J. Osborne and N. Linden, Phys. Rev. A **69**, 052315 (2004).
 [6] H. L. Haselgrove, e-print quant-ph/0404152.
 [7] F. Verstraete, M. A. Martin-Delgado, and J. I. Cirac, Phys. Rev. Lett. **92**, 087201 (2004).
 [8] F. Verstraete, M. Popp, and J. I. Cirac, Phys. Rev. Lett. **92**, 027901 (2004).
 [9] B.-Q. Jin and V. E. Korepin, Phys. Rev. A **69**, 062314 (2004).
 [10] M. H. Yung, D. W. Leung, and S. Bose, Quantum Inf. Comput. **4**, 174 (2004).
 [11] L. Amico, A. Osterloh, F. Plastina, R. Fazio, and G. M. Palma, Phys. Rev. A **69**, 022304 (2004).
 [12] V. Giovannetti and R. Fazio, Phys. Rev. A **71**, 032314 (2005).
 [13] M. B. Plenio, J. Hartley, and J. Eisert, New J. Phys. **6**, 36 (2004).
 [14] A. Romito, R. Fazio, and C. Bruder, Phys. Rev. B **71**, 100501 (2005).
 [15] N. Motoyama, H. Eisaki, and S. Uchida, Phys. Rev. Lett. **76**, 3212 (1996).
 [16] P. Gambardella, A. Dallmeyer, K. Maiti, M. C. Malagoli, W. Eberhardt, K. Kern, and C. Carbone, Nature (London) **416**, 301 (2002).
 [17] J. B. Torrance and M. Tinkham, Phys. Rev. **187**, 587 (1969).
 [18] I. L. Chuang and Y. Yamamoto, Phys. Rev. Lett. **76**, 4281 (1996).
 [19] T. Yamamoto, Y. A. Pashkin, O. Astafiev, Y. Nakamura, and J. S. Tsai, Nature (London) **425**, 941 (2003).
 [20] D. Burgarth, S. Bose, and V. Giovannetti, e-print quant-ph/0410175.
 [21] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
 [22] G. M. Palma, K. A. Suominen, and A. K. Ekert, Proc. R. Soc. London, Ser. A **452**, 567 (1996).
 [23] W. Y. Hwang, H. Lee, D. D. Ahn, and S. W. Hwang, Phys. Rev. A **62**, 062305 (2000).
 [24] A. Beige, D. Braun, and P. L. Knight, New J. Phys. **2**, 22 (2000).
 [25] M. Plenio and P. Knight, Rev. Mod. Phys. **70**, 101 (1998).
 [26] B. Sutherland, Phys. Rev. B **12**, 3795 (1975).