# Teleporting a rotation on remote photons

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Quantum remote rotation allows us to implement local quantum operation on remote systems with shared entanglement. Here we report an experimental demonstration of remote rotation on single photons using linear optical elements. The local dephase is also teleported during the process. The scheme can be generalized to any controlled rotation commuting with $\sigma_z$.

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The development of quantum information in the past two decades promises the powerful applications in information manipulation [1]. Using quantum operations, some missions, for example large number factoring, can be solved by a quantum algorithm effectively, which has not yet been shown to be possible for a classical computer. Arbitrary rotation gates on a single qubit and a controlled-NOT gate on two qubits are sufficient and necessary for universal quantum computation [2]. These two kinds of operations have been realized locally in the laboratory on physical systems such as trapped ions, neutral atoms, and so on [1]. If the qubits involved in the process are separated (e.g., distributed computation [3,4]), operations must be implemented nonlocally via shared entanglement, local operation, and classical communication, i.e., remote operation [5–10].

Suppose only two parties, Alice and Bob, are involved in the process [9]. The former has the operation gate, while the latter has the state to be operated on. The trivial realization of remote operation can be finished by the bidirectional quantum state teleportation. The target state is teleported from Bob to Alice. Then Alice applies the operation on the state and sends the resulting state back to Bob via another state teleportation. The total resources for this trivial protocol are two maximally entangled states (two ebits) shared and two bits of classical communication (two cbits) in each direction. And there is no restriction for either the target state or the operation. If the gate is restricted in the unitary operation set $U_{com}\cup U_{anti}$, where $U_{com}$ ($U_{anti}$) is the set of single-qubit operation commuting (anticommuting) with $\sigma_z$, the resource can be reduced to two ebits shared, two cbits from Bob to Alice, and one cbit reversely in the optimal nontrivial scheme. Moreover, if the set is either $U_{com}$ or $U_{anti}$, the scheme can be simplified by using only one ebit shared and one cbit in each direction [9]. Specially, if the operation at Alice's site is a collective one with an auxiliary qubit, the two-qubit gate can be performed nonlocally [5,10].

Here we present an experimental demonstration of a remote rotation on single photons. Unitary rotation commuting with $\sigma_z$ is implemented remotely on the polarization qubit that Bob holds, using the entanglement between the polarization qubit of one photon and the path qubit of another one from spontaneous parametric downconversion (SPDC).

Let us consider the remote rotation on a single qubit. Alice has an operation gate $U_{com}=e^{i\varphi\sigma_z/2}$ to implement and Bob has an arbitrary state $|\psi\rangle_3=|\psi(\theta,\phi)\rangle_3=\alpha|0\rangle_3+\beta|1\rangle_3$ to be operated on, where $\alpha=\cos\theta$, $\beta=e^{i\phi}\sin\theta$, and $\{|0\rangle,|1\rangle\}$ are the basis of $\sigma_z$. Geometrically, the operation rotates the state vector in the Bloch sphere through angle $\varphi$ about the $z$ axis, $|\psi(\theta,\phi)\rangle_3\to|\psi(\theta,\phi'=\phi+\varphi)\rangle_3$ [see Fig. 1(a)]. That is the teleportation of an angle. In particular, the gate is an identity operation for $\varphi=0$, while $\sigma_z$ for $\varphi=\pi$. Given the shared entanglement $|\Phi^+\rangle_{12}=(1/\sqrt{2})(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)$, the scheme can be realized in three steps [see Fig. 1(b)].

(i) *Encoding*. Bob performs a controlled-CNOT operation on his qubits, 2 and 3, where 3 is the controller. Then a $\sigma_z$ measurement on 2 is made by Bob and the result is sent to Alice via one cbit classical communication. Alice would perform bit-flip operation $\sigma_x$ on her qubit 1 for result $|1\rangle_2$ while doing nothing for $|0\rangle_2$. The remaining system of qubits 1 and 3 is in the state

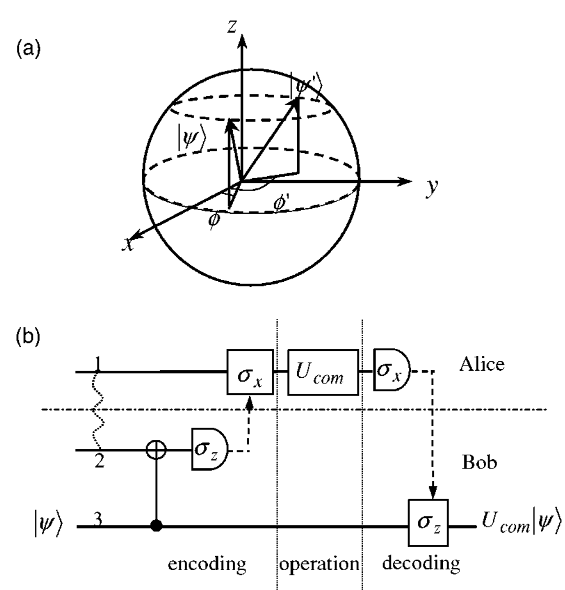


FIG. 1. (a) Geometrical interpretation of single-qubit rotation $U_{com}$, $|\psi'\rangle=U_{com}|\psi\rangle$. (b) Quantum circuit for a scheme of remote rotation on a single qubit, where qubits 1 and 2 are entangled and qubit 3 is the target to be operated on. The whole process is divided into three steps (see text for details).

 

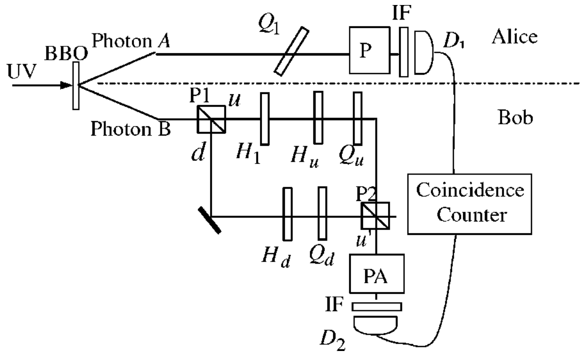


FIG. 2. Experimental setup to perform a remote rotation on a single photon. P1, P2: polarization beam splitters; $H_1$, $H_u$, $H_d$: half-wave plates; $Q_1$, $Q_u$, $Q_d$: quarter wave plates; P: polarizer; PA: polarization analyzer; IF: interference filter; $D_1$, $D_2$: single photon detectors.

$$|\psi\rangle_{13} = \alpha|0\rangle_1|0\rangle_3 + \beta|1\rangle_1|1\rangle_3. \quad (1)$$

Thus the state $|\psi\rangle_3$ to be operated on is encoded into $|\psi\rangle_{13}$ within the Hilbert subspace $H^2=\{|0\rangle_1|0\rangle_3,|1\rangle_1|1\rangle_3\}$ of the composite system, 1 and 3.

(ii) *Operating*. Alice implements the required quantum gate $U_{com}$ on her qubit 1, while Bob does nothing,

$$U_{com}\otimes I_3|\psi\rangle_{13} = \alpha e^{i\varphi/2}|0\rangle_1|0\rangle_3 + \beta e^{-i\varphi/2}|1\rangle_1|1\rangle_3. \quad (2)$$

Here the local gate plays the role of global rotation within $H^2$. The desired state $|\psi'\rangle_3=U_{com}|\psi\rangle_3$ has been just embedded in the composite system.

(iii) *Decoding*. A $\sigma_x$ measurement $\{|\pm\rangle=(1/\sqrt{2})\times(|0\rangle\pm|1\rangle)\}$ is performed on 1 by Alice and the result is sent to Bob. Qubit 3 will be found in the desired state $|\psi'\rangle_3$ for the result $|+\rangle_1$ or $|\psi''\rangle_3=U_{com}\sigma_z|\psi\rangle_3$ for $|-\rangle_1$. The latter can be converted into $|\psi'\rangle_3$ by an additional $\sigma_z$ rotation for $\sigma_z U_{com}\sigma_z=U_{com}$. That is, $|\psi'\rangle_3$ is decoded out from the composite system.

The total resources needed in the whole process are one ebit shared, one cbit from Bob to Alice for encoding, and one cbit from Alice to Bob for decoding.

To realize the protocol above, the key is how to choose the physical qubit, the local operation gate, and the entangled state. For photons, both the polarization $\{|H\rangle,|V\rangle\}$ and the path $\{|u\rangle,|d\rangle\}$ can represent the logic states $\{|0\rangle,|1\rangle\}$ for qubits. For the polarization qubit, arbitrary unitary rotation can be performed by using a half-wave plate (HWP) and a quarter-wave plate (QWP) [11]. The controlled-NOT gate between polarization qubit and path qubit of the same photon can be operated on by the polarization beam splitter (PBS), $|H\rangle|u\rangle(|H\rangle|d\rangle)\rightarrow|H\rangle|u'\rangle(|H\rangle|d'\rangle)$, $|V\rangle|u\rangle(|V\rangle|d\rangle)\rightarrow|H\rangle|d'\rangle(|H\rangle|u'\rangle)$ [12]. And the biphoton states entangled between polarization or path qubit can be generated via the SPDC process [13]. Here the three qubits are embedded in the three degrees of freedom of two photons distributed to Alice and Bob, polarization (qubit 1) of photon $A$, path (qubit 2) and polarization (qubit 3) of photon $B$.

The experimental setup is shown in Fig. 2. A mode-locked Ti:sapphire pulsed laser (with the pulse width less than 200 fs, the repetition about 82 MHz, and the center wavelength at 780.0 nm) is frequency-doubled to produce the pumping source for the SPDC process. A 1.0-mm-thick BBO crystal cut for type-II phase match is used as the downconverter. By the noncollinear degenerated SPDC process, two photons, $A$ and $B$, are produced in the polarization-entangled state $|\Psi^+\rangle_{AB}=(1/\sqrt{2})(|H\rangle_A|V\rangle_B\pm|V\rangle_A|H\rangle_B)$ [13]. Bob uses PBS P1 to split photon $B$ in two paths $\{|u\rangle,|d\rangle\}$ and HWP H1 at 45° as a $\sigma_x$ gate is used to flip the polarization in path $u$. Hence the polarization entanglement between the two photons is converted into polarization-path entanglement,

$$|\Psi^+\rangle_{123} = \frac{1}{\sqrt{2}}(|H\rangle_1|u\rangle_2 + |V\rangle_1|d\rangle_2)|H\rangle_3. \quad (3)$$

The polarization of $B$ can be prepared in an arbitrary state $|\psi\rangle_3=\alpha|H\rangle_3+\beta|V\rangle_3$ with identical sets of waveplates, $\{H_u,Q_u\}$ and $\{H_d,Q_d\}$, in each path [11]. The global state is initialized in $|\Phi^+\rangle_{12}|\psi\rangle_3=(1/\sqrt{2})(|H\rangle_1|u\rangle_2+|V\rangle_1|d\rangle_2)(\alpha|H\rangle_3+\beta|V\rangle_3)$. The three steps for remote rotation are performed as follows.

(i′) *Encoding*. Path $u$ and $d$ of photon $B$ are input in a PBS P2 to perform a controlled-CNOT operation, where polarization is the controlling qubit and path is the target one. The optical path lengths of $u$ and $d$ are tuned to be equal to ensure no relative phase factor between the two terms in Eq. (3). The $\sigma_z$ measurement on qubit 2 is completed by reading out the path information of photon $B$. If $B$ is found in path $u'$, $|\psi\rangle_3$ is encoded into $|\psi\rangle_{13}=\alpha|H\rangle_1|H\rangle_3+\beta|V\rangle_1|V\rangle_3$. Or if $B$ is found in path $d'$, the two photons will be in $|\psi'\rangle_{13}=\alpha|V\rangle_1|H\rangle_3+\beta|H\rangle_1|V\rangle_3$, which can be transformed into $|\Psi\rangle_{13}$ by another HWP at 45° on photon $A$. Here we omit the latter case without loss of generality. The polarization state of photon $B$ is encoded in $\{|H\rangle_1|H\rangle_3,|V\rangle_1|V\rangle_3\}$ [14].

(ii′) *Operating*. The operation $U_{com}$ can be performed by a pair of QWP at 45° with a HWP at $(\varphi/2)-45°$ between them. Such a device has been used to verify the geometric phase of classical light and photons [15,16]. For single-qubit operation, any additional global phase is trivial, so $U_{com}$ can be replaced by $e^{i\varphi/2}U_{com}$, which can be realized by one zero-order waveplate at 0° tilted in a suitable angle (see Ref. [17] for similar application). Here we chose $\varphi=120°$ by a tilted QWP Q1.

(iii′) *Decoding*. Alice makes her $\sigma_x$ measurement $\{|D\rangle_1=(1/\sqrt{2})(|H\rangle_1+|V\rangle_1),|C\rangle_1=(1/\sqrt{2})(|H\rangle_1-|V\rangle_1)\}$ using a polarizer. Photon $A$ is detected by a single-photon detector (SPCM-AQR-14 by EG&G). Photon $B$ will be collapsed into $|\psi'\rangle_3=U_{com}|\psi\rangle_3$ for result $|+\rangle_1$, and $|\psi''\rangle_3=U_{com}\sigma_z|\psi\rangle_3$ for result $|-\rangle_1$. The latter can be converted into $|\psi'\rangle_B$ by a HWP at 0°, i.e., a $\sigma_z$ rotation. The polarization state of photon $B$ is reconstructed by quantum state tomography using polarization analyzer and detector. The measurements on $A$ and $B$ are collected for coincidence count with the window time 5 ns.

In the real experiment, there are two kinds of imperfection which induce the phase decoherence. One is caused by the birefringency of BBO, which induces the partial time separation between the wave packets of two polarizations. It can be described by Kraus operators $\{\sqrt{[(1+p)/2]}I,\sqrt{[(1-p)/2]}\sigma_z\}$ on photon $A$ or $B$. Here $p$ is just the visibility

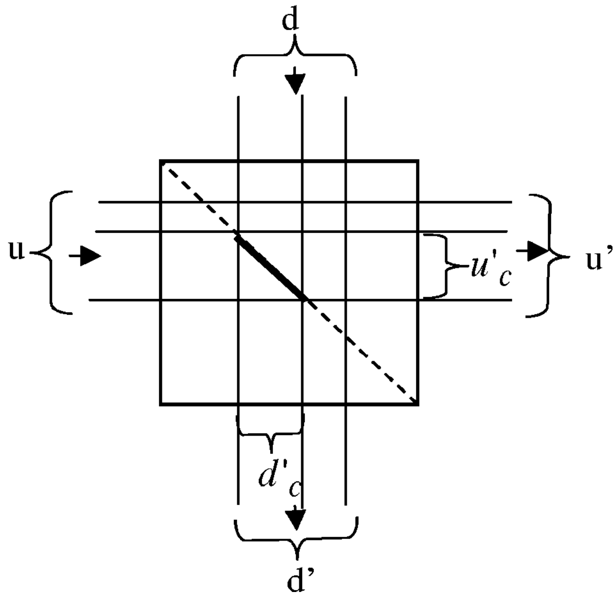


FIG. 3. Schematic drawing for the mode mismatching in a polarization beam splitter, which induces a dephase.

of the entangled state from SPDC. The other one is the mismatching of space mode in PBS P2 (see Fig. 3). The two PBSs, P1 and P2, at Bob's site consist of a Mach-Zehnder interferometer. The mode mismatch can be represented by a process dephase $\{\sqrt{[(1+\eta)/2]}I, \sqrt{[(1-\eta)/2]}\sigma_z\}$ on the paths of $B$, where $\eta$ is the visibility of the interferometer. Because of the symmetry between two qubits in $|\Phi^+\rangle$, both of the imperfections can be considered to be performed on the polarization of photon $A$. Further, it can also be regarded as a control phase operation on qubit 1 and an anxiliary system $1'$, where the latter is the target. Since the control phase gate $X_{\mathrm{CP}_{1'1}}=|0\rangle_{1'1'}\langle 0|\oplus I_1+|1\rangle_{1'1'}\langle 1|\oplus\sigma_{z1}$ commutes with $I_{1'}\otimes\sigma_{z1}$, the dephasing, or the control phase operation in the extended systems, is included in $U_{com}$. That is the nonlocal implement of a control phase gate. With a $\sigma_x$ operation on $1'$, the nonlocal CNOT has been demonstrated recently, where qubits 1 and $1'$ are path and polarization of photon $A$ [10]. Moreover, it can be generalized to the controlled rotation $X_{\mathrm{CR}}=|0\rangle_{1'1'}\langle 0|\otimes U_{com}+|1\rangle_{1'1'}\langle 1|\otimes U'_{com}$ for any $U_{com}$ and $U'_{com}$. In our experiment, $1'$ is the wave-packet distinction and the mode mismatch, both of which are traced out. So both the rotation and the dephase on photon $A$ are teleported. The dephased operation is a complete positive map $\varepsilon_d=\{\sqrt{1+p\eta/2}U_{com}, \sqrt{1-p\eta/2}U_{com}\sigma_z\}$. The final state after operation is

$$\rho_d(|\psi\rangle)=\varepsilon_d(|\psi\rangle)=\begin{pmatrix}\alpha\alpha^* & p\eta\alpha\beta^* e^{-i2\varphi}\\ p\eta\alpha^*\beta e^{i2\varphi} & \beta\beta^*\end{pmatrix}. \quad (4)$$

To completely characterize the remote operation $\varepsilon_e$ in our experiment, four states $\{|H\rangle,|V\rangle,|D\rangle,|R\rangle=(1/\sqrt{2})(|H\rangle-i|V\rangle)\}$ are input for the quantum process tomography [1,20]. The process is represented by a positive Hermitian matrix $\chi=\{\chi_{mn}\}$, which satisfies $\varepsilon(\rho)=\Sigma_{mn}\chi_{mn}E_m\rho E_n^\dagger(\{E_m\}=\{I,\sigma_x,\sigma_y,\sigma_z\})$. The matrices are shown in Fig. 4 for ideal

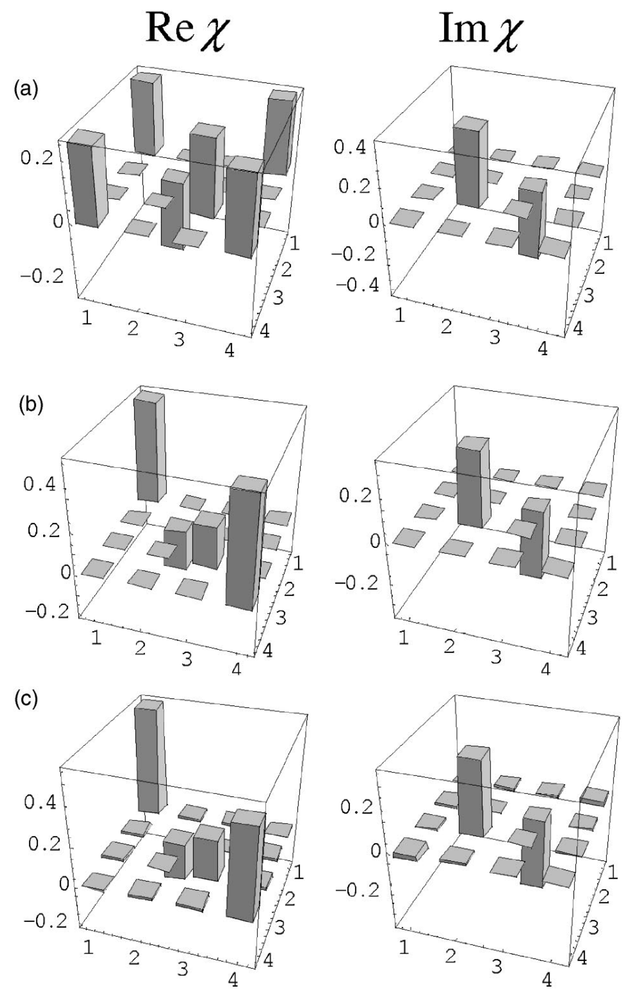


FIG. 4. $\chi$ matrices determined by (a) ideal rotation $\chi_i$, (b) dephased rotation $\chi_d$, and (c) experimental rotation $\chi_e$ from Fig. 2. The real parts of $\chi$ are on the left while the imaginary ones are on the right.

rotation $\chi_i$, dephased rotation $\chi_d$, and effective operation $\chi_e$ in our experiment, where two parameters for $\varepsilon_d$ are measured, $p\approx 0.85$ and $\eta\approx 0.92$. And the comparison of experimental operation $\varepsilon_e$ with the dephased one $\varepsilon_d$ is characterized by the average fidelity of the pure state inputted throughout the Bloch sphere $\overline{F}[\varepsilon',\varepsilon]=\int d\psi F[\varepsilon'(|\psi\rangle), \varepsilon_d(|\psi\rangle)]$, where $F[\rho,\rho']=\mathrm{Tr}[\sqrt{\sqrt{\rho'}\rho\sqrt{\rho'}}]$ [1,18,19]. From $\chi$ we get $\overline{F}[\varepsilon_i,\varepsilon_e]=0.96$ and $\overline{F}[\varepsilon_d,\varepsilon_e]=0.99$. Geometrically, the rotation can also be characterized by the actual rotation angle $\varphi=\phi'-\phi$. Here we use the angle deviation $\delta=\delta(\rho_i,\rho_e)$, which is defined by the cross angle between the Bloch vector by ideal rotation and the one we finally get, to characterize the experimental operation. The maximal angle is $\delta_{\max}=7°$, which means $\varphi=120°\pm 7°$.

In conclusion, a remote rotation on qubits throught 120° about the $z$ axis is performed using shared entanglement and local operation without rotating the target photons. And the dephase on photon $A$ can also be teleported. The whole process is measured by quantum process tomography and agrees with the theoretical prediction. The scheme can be generalized to remote implement a class of controlled rotation. Although only rotation commuting with $\sigma_z$ is used in our experiment, it is the same with operations anticommuting with $\sigma_z$, where the only difference is that an additional $\sigma_x$ rotation should be performed on qubit 3 in decoding for result $|+\rangle_1$ and $\sigma_z\sigma_x$ for $|-\rangle_1$ (see Ref. [9] for details). On the other hand, if it is unknown whether the operation commutes or anticommutes with $\sigma_z$, more resources are needed because the sufficient and necessary resources for univeral remote control is two ebits shared, two cbits from Alice to Bob, and one cbit reversely [9].

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