High efficiency four-wave mixing induced by double-dark resonances in a five-level tripod system

Yueping Niu,[1] Ruxin Li,[1] and Shangqing Gong[1,2,*]

[1]*State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

[2]*Department of Physics, Shanghai University, Shanghai 200436, China*

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A five-level tripod scheme is proposed for obtaining a high efficiency four-wave-mixing (FWM) process. The existence of double-dark resonances leads to a strong modification of the absorption and dispersion properties against a pump wave at two transparency windows. We show that both of them can be used to open the four-wave mixing channel and produce efficient mixing waves. In particular, higher FWM efficiency is always produced at the transparent window corresponding to the relatively weak-coupling field. By manipulating the intensity of the two coupling fields, the conversion efficiency of FWM can be controlled.

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I. INTRODUCTION

The idea of using control lasers to manipulate the optical properties of a medium has been a subject of recent research and has met with great success since the early proposals. Harris and co-workers [1–3] have introduced the idea of electromagnetically induced transparency (EIT), which is based on the phenomenon of dark resonance. EIT modifies both absorptive and dispersion properties of an absorption medium and hence leads to suppression of linear susceptibility and enhancement of nonlinear susceptibilities [4–8]. Meanwhile, as one of the centerpieces of modern technology, the wave-mixing process has attracted the attention of many researchers and how to realize efficiency generation of short-wavelength coherent light has been of primary importance. Both EIT-based (single-photon destructive interference) [4,5,9–17] and multiphoton destructive interference-based transparency [18,19] have been proposed as efficient methods for wave-mixing processes. Deng *et al.* have proposed four-wave-mixing (FWM) schemes for enhancement of conversion efficiency based on EIT and associated ultraslow light propagation [13–15]. Their studies show that the non-existing wave-mixing channels are opened and significantly enhanced by strongly modifying the dispersion properties. A much wider transparency window, which results in a higher group velocity, will actually diminish efficient mixing wave production. Recently, Wu *et al.* have analyzed efficient hyper-Raman scattering based on EIT considering the pump field to be cw one [17]. Moreover, Wu *et al.* have also realized efficient multiwave mixing resulted from a multiphoton destructive interference [18] and dealt with a time-dependent analysis for enhancement of wave-mixing efficiency in a double-Λ system [20].

In recent years, a variety of four-level systems driven by three fields have also shown that the probe absorption is characterized by double-dark resonances [21–27]. The interaction of double-dark states has caused some original and significant effects. Lukin *et al.* have shown that in a Λ-type system where the final state has twofold levels the interaction of double-dark states can lead to a splitting of dark states and the emergence of sharp spectral features [22]. Recently, we have proposed that in the above generic four-level system the presence of double-dark states makes it possible to prepare arbitrary coherent superposition states with equal amplitudes but inverse relative phases without any resonance conditions being satisfied [28]. Moreover, Goren *et al.* have studied the sub-Doppler and subnatural narrowing of the absorption line induced by interacting dark resonances in a tripod system [29].

In the present paper, we propose a five-level tripod scheme to achieve high enhancement of FWM production. Just as discussed by Paspalakis and Knight [21], the tripod-configuration system can exhibit double transparences and at most two different group velocities against the pump wave. The double EIT, which provides strong manipulation of the absorption and dispersion properties, opens two coherent FWM channels that would otherwise not be possible because of the strong absorption. Specifically, a relatively weak-coupling field, which corresponds to a slower group velocity at the relevant transparency window, results in a much higher four-wave-mixing conversion efficiency. By manipulating the intensity of the two coupling fields, the conversion efficiency of FWM can be controlled.

II. HIGH EFFICIENCY FWM PROCESS INDUCED BY DOUBLE-DARK RESONANCES

The system under consideration is shown in Fig. 1. A pulsed pump field ω_p and two cw fields ω_c and ω_d form a tripod configuration. As pointed out in Ref. [21], this tripod configuration owns the property of double-dark resonances, which not only provide absorption and dispersion manipulation of state $|2\rangle$ but also make the manipulation behave quite differently in contrast to the single-dark-resonance system. When the pump field ω_p keeps two-photon resonance with either of the coupling fields ω_c and ω_d, a transparency window will be present and the pump field will be free of absorption. Corresponding to different transparency windows,

*Corresponding author: sqgong@mail.siom.ac.cn

the group velocity of the pump wave can obtain two different values. At the same time, the pulsed pump field ω_p serves as the first step of the three-photon excitation of state $|1\rangle$: the field ω_p and a resonant two-photon field ω_1 induce the four-wave mixing process $|5\rangle$-$|2\rangle$-$|1\rangle$-$|5\rangle$ and generate a coherent radiation field at frequency ω_m with $\omega_m=\omega_p+2\omega_1$. In the interaction picture, the systematic Hamiltonian can be written as $(\hbar=1)$

$$H=\begin{bmatrix} \Delta_p & -\Omega_1 e^{ik_1z} & 0 & 0 & -\Omega_m e^{ik_mz} \\ -\Omega_1 e^{-ik_1z} & \Delta_p & -\Omega_c e^{ik_cz} & -\Omega_d e^{ik_dz} & -\Omega_p e^{ik_pz} \\ 0 & -\Omega_c e^{-ik_cz} & \Delta_p-\Delta_c & 0 & 0 \\ 0 & -\Omega_d e^{-ik_dz} & 0 & \Delta_p-\Delta_d & 0 \\ -\Omega_m^* e^{-ik_mz} & -\Omega_p^* e^{-ik_pz} & 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

In the above, Ω_i $(i=1,c,d,p,m)$ denote one-half of the Rabi frequencies for the respective transitions and $\Omega_i^*=\Omega_i$ $(i=1,c,d)$ is assumed. k_i $(i=1,c,d,p,m)$ is the respective wave vector. $\Delta_p=(\varepsilon_2-\varepsilon_5)-\omega_p$, $\Delta_c=(\varepsilon_2-\varepsilon_3)-\omega_c$ and $\Delta_d=(\varepsilon_2-\varepsilon_4)-\omega_d$ represent the respective single-photon detunings. The atomic equations of motion describing this system are given as

$$\frac{\partial}{\partial t}C_1=-i(\Delta_pC_1-\Omega_1e^{-i\Delta kz}C_2-\Omega_mC_5)-\gamma_1C_1,$$

$$\frac{\partial}{\partial t}C_2=-i(\Delta_pC_2-\Omega_1e^{i\Delta kz}C_1-\Omega_cC_3-\Omega_dC_4-\Omega_pC_5)-\gamma_2C_2,$$

$$\frac{\partial}{\partial t}C_3=-i((\Delta_p-\Delta_c)C_3-\Omega_cC_2),$$

$$\frac{\partial}{\partial t}C_4=-i((\Delta_p-\Delta_d)C_4-\Omega_dC_2),$$

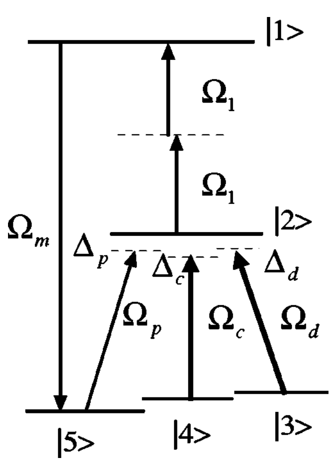

FIG. 1. Energy level diagram of the proposed scheme with relevant laser excitations.

$$\frac{\partial}{\partial t}C_5=-i(-\Omega_m^*C_1-\Omega_p^*C_2), \tag{2}$$

where γ_j $(j=1,2)$ represents the decay rates and Δk means the phase mismatching. In the following, we assume $|\Omega_{c,d}|\gg|\Omega_p|$ and the two-photon transition field Ω_1 is comparatively weak. Hence, the term $i\Omega_1e^{i\Delta kz}C_1$ in Eq. (2) can be neglected [14] and the ground-state depletion is negligible $(C_5\approx 1)$. Taking a Fourier transform of Eq. (2) and solving it, we obtain, in dimensionless form,

$$\alpha_1=\tau W_mD_m+\tau\Omega_1\tau W_pD_mD_p,$$

$$\alpha_2=\tau W_pD_p, \tag{3}$$

with $D_m=-1/\delta_1$ and $D_p=-\delta_3\delta_4/[\delta_2\delta_3\delta_4-\delta_4(\Omega_c\tau)^2-\delta_3(\Omega_d\tau)^2]$. Here, α_1, α_2, W_p, and W_m are the Fourier transforms of C_1, C_2, Ω_p, and Ω_m, respectively. $\eta=\varpi\tau$ is the dimensionless Fourier transform variable. $\delta_1=\eta-\Delta_p\tau+i\gamma_1\tau$, $\delta_2=\eta-\Delta_p\tau+i\gamma_2\tau$, $\delta_3=\eta-(\Delta_p-\Delta_c)\tau$, and $\delta_4=\eta-(\Delta_p-\Delta_d)\tau$. For simplification, we here consider the case of phase-matched FWM and hence the phase mismatching Δk is omitted in Eq. (3). The reason will be given in the following.

The propagation of the pump field ω_p and the generated four-wave-mixing field ω_m obeys Maxwell's equations. Under the slowly varying envelope approximation, they read

$$\frac{\partial}{\partial z}W_p-i\frac{\eta}{v}W_p=i\kappa_p\alpha_2\alpha_5^*,$$

$$\frac{\partial}{\partial z}W_m-i\frac{\eta}{v}W_m=i\kappa_m\alpha_1\alpha_5^*, \tag{4}$$

where the propagation constant $\kappa_{p(m)}=2N\omega_{p(m)}|\mu_{25(15)}|^2/\hbar v$, with N being the atomic concentration and v the speed of light in vacuum. α_5^* is the Fourier transform of C_5^* and is equal to 1. Substituting Eq. (3) into Eq. (4), we get

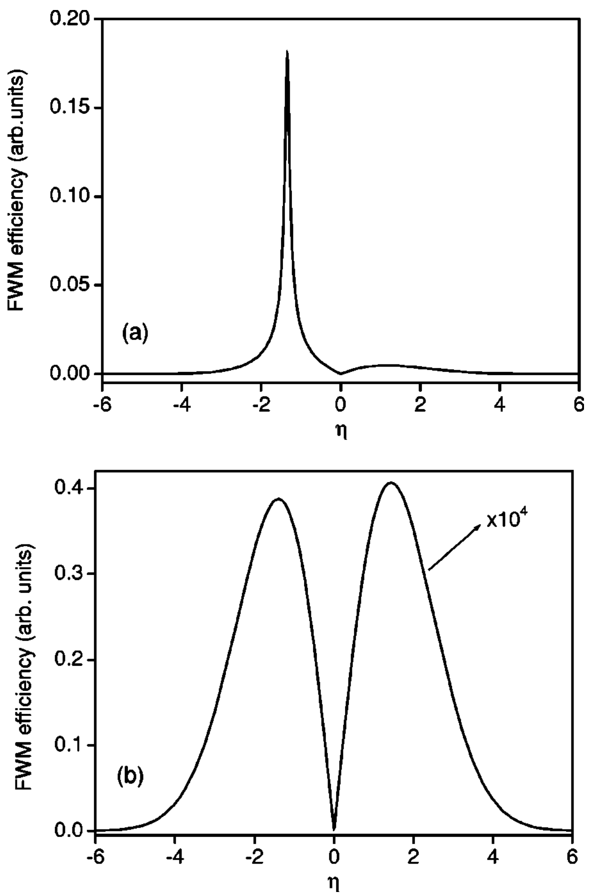

FIG. 2. The dimensionless quantity $|W_m(z,\eta)/\tau\Omega_1\tau\Omega_p(0,0)|$ as a function of the variable η. Parameters are shown in the text. (a) $\Delta_p=\Delta_d$, (b) $\Delta_p=\Delta_c$.

$$W_m(z,\eta) = (\kappa_m v\tau^2)(\Omega_1\tau)D_pD_m \times \frac{e^{i\eta z/v\tau}(e^{iD_m(\kappa_m v\tau^2)z/v\tau} - e^{iD_p(\kappa_p v\tau^2)z/v\tau})}{D_m(\kappa_m v\tau^2) - D_p(\kappa_p v\tau^2)} W_p(0,\eta). \tag{5}$$

$W_p(0,\eta)$ is the Fourier transform of $\Omega_p(z,t)$ at the entrance of the medium. For the FWM emission process, the boundary condition $W_m(0,\eta)=0$ is applied. Equation (5) gives the generated FWM field $W_m(z,\eta)$ at arbitrary frequency detunings, the inverse Fourier transform of which can be used to calculate the generated FWM power. As discussed in Ref. [13], a phase-matched FWM field can be produced if the group velocity of the pump wave ω_p is substantially reduced and about equal to the group velocity of the generated FWM wave ω_m. In order to show that the present scheme is capable of generating a phase-matched mixing wave, we now calculate the two group velocities.

The group velocity of the pump and generated waves we interest is given by

$$V_g = v \Bigg/ \left(1 + \frac{\omega}{2}\frac{\partial\,\mathrm{Re}[\chi]}{\partial\omega}\right), \tag{6}$$

with the derivative of the real part of the susceptibility being evaluated at the carrier frequency of the pump or generated FWM field. When $\Delta_c \neq \Delta_d$, the pump-wave group velocity at the nth transparency window is approximated as

$$V_g^{pn} \approx \frac{v}{1 + (k_p v\tau^2)/(\Omega_n\tau)^2}, \tag{7}$$

with $n=c,d$. Therefore, the group velocity of the pump wave may be significantly reduced, similar to a Λ-type system

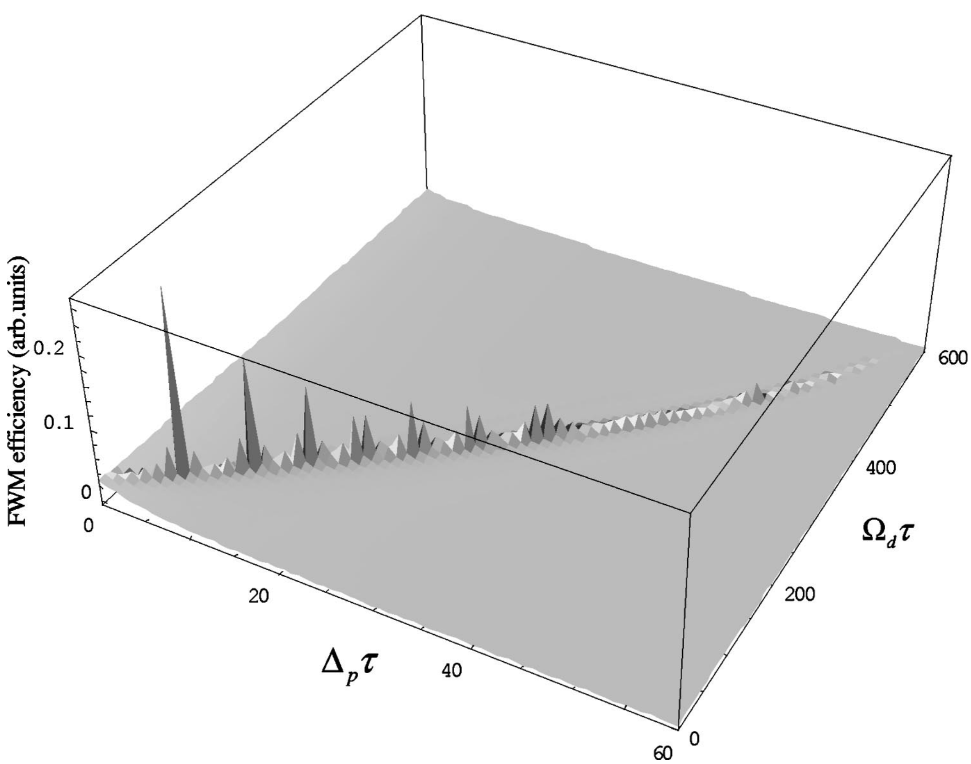

FIG. 3. The dimensionless quantity $|W_m(z,\eta)/\tau\Omega_1\tau\Omega_p(0,0)|$ as functions of $\Delta_p\tau$ and $\Omega_d\tau$. Parameters are shown in the text.

[30–32]. In this case, the pump wave can propagate with two different group velocities and can be controlled via the intensity of the coupling laser fields. Similarly, if we assume $\gamma_1^2 \ll \Delta_p^2$, the group velocity of the generated wave is approximated as

$$V_g^m \approx \frac{v}{1 + (k_m v \tau^2)/(\Delta_p \tau)^2}. \tag{8}$$

Equations (7) and (8) indicate that group velocity matching between the pump and generated waves is possible. It is noteworthy that the detuning Δ_p here should be smaller than the coupling field Ω_n since $\kappa_m < \kappa_p$ typically. The above analysis shows us that the pump wave can propagate with a minimum absorption and ultraslow group velocity as long as it keeps two-photon resonance with either of the coupling fields ω_c and ω_d. Accordingly, the generated FWM field can propagate with two ultraslow group velocities, each of which matches the corresponding ultraslow pump wave. Therefore, two FWM channels that are otherwise prohibited are now opened. Furthermore, by manipulating the intensity of the coupling fields Ω_c and Ω_d, the conversion efficiency of FWM can be controlled.

III. NUMERICAL SIMULATIONS AND DISCUSSION

In order to illustrate the preceding analysis, we plot Eq. (5) by choosing $W_p(0,\eta) = \tau\Omega_p(0,0)\exp(-\eta^2/4)$ for parameters of $\kappa_m v \tau^2 = 3\times 10^9$, $\kappa_p v \tau^2 = 3\times 10^{11}$, $\Delta_c\tau = \Omega_c\tau/10$, $\Delta_d\tau = \Omega_d\tau/10$, $\gamma_1\tau = 3$, and $\gamma_2\tau = 300$ in the following. Figure 2 shows the dimensionless quantity $|W_m(z,\eta)/\tau\Omega_1\tau\Omega_p(0,0)|$ for $\Omega_c\tau = 600$ and $\Omega_d\tau = 100$. With these parameters, the group velocity at transparent windows $\Delta_p = \Delta_d$ and $\Delta_p = \Delta_c$ is approximated as $V_g^m = V_g^p = 3.3\times 10^{-8}c$ and $V_g^m = V_g^p = 1.2 \times 10^{-6}c$, respectively. That is to say, the above parameters provide ultraslow group velocity matching between the pump and generated waves at the respective transparent windows $\Delta_p = \Delta_d$ and $\Delta_p = \Delta_c$. As Fig. 2 shows, both of them can be utilized to generate efficient FWM processes. However, it is obvious that the conversion efficiency at $\Delta_p = \Delta_d$ is a several orders increase in comparison with that of the case $\Delta_p = \Delta_c$. As a whole, we plot the dependence of the FWM efficiency on the coupling field $\Omega_d\tau$ corresponding to $\eta = -1.4$ in Fig. 3. It reveals that higher conversion efficiency is always produced at the transparent window $\Delta_p = \Delta_d$ when Ω_d is weaker than Ω_c. This is because the generated field is inversely proportional to the intensity of the control field [14] and the group velocity at $\Delta_p = \Delta_d$ is much slower than that of $\Delta_p = \Delta_c$ when Ω_d is weaker than Ω_c. With the coupling field Ω_d increasing, the FWM efficiency at transparent window $\Delta_p = \Delta_d$ decreases. To the extent $\Omega_d\tau = 600$, the dimensionless quantity $|W_m(z,\eta)/\tau\Omega_1\tau\Omega_p(0,0)|$ is only 10^{-4} that of $\Omega_d\tau = 100$. In addition, we also calculate the case of $\Omega_c\tau = 100$ and $\Omega_d\tau = 100-600$. Similar results are present and one point to note is that when $\Omega_d\tau = 100$, it is about a two orders decrease in the dimensionless quantity $|W_m(z,\eta)/\tau\Omega_1\tau\Omega_p(0,0)|$ compared with that of $\Omega_d\tau = 600$. In this situation, including the case of $\Omega_d\tau = \Omega_c\tau = 600$, the double EIT windows fuse into a single one and hence the conversion efficiency decreases sharply. To sum up, we owe the existence of two FWM channels and the much higher FWM efficiency achieved at the relevant transparency window to the double-dark resonances. They not only provide absorption and dispersion manipulation of the state $|2\rangle$ but also make the manipulation behave quite differently in contrast to the single-dark-resonance system. Therefore, we can say that in the tripod-configuration FWM scheme, by properly choosing the relative coupling fields the conversion efficiency of FWM can be controlled and enhanced by several orders.

IV. CONCLUSION

In this paper, we have proposed a tripod-configuration FWM scheme for enhancement of FWM efficiency. The existence of double-dark resonances opens two FWM channels otherwise prohibited by the strong absorption of the pump wave. By adjusting the intensity of the coupling fields, a several orders increase in the conversion efficiency of FWM can be realized. Therefore, our scheme, which is feasible in practical situations, provides an effective way for controllable enhancement of wave-mixing efficiency.

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