Electromagnetically induced waveguiding in double-Λ systems

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Near the threshold for electromagnetically induced transparency (EIT) or coherent population trapping (CPT), two-photon-resonance-enhanced self-focusing of a Λ system can be exploited to induce spatial confinement in a second, diffracting Λ system. The diffracting Λ system is characterized by parameters below the EIT or CPT threshold, and the two Λ systems must be coupled to form a closed-loop double-Λ system. The waveguiding effect is shown to be strongly phase dependent, indicating that it derives from the phase-dependent effective third-order susceptibility rather than the phase-independent effective first-order susceptibility, as is the case in previously studied systems. We also show that when the second Λ system initially involves only a single laser beam, the loop is completed by the efficient generation of radiation at the four-wave-mixing frequency, within a propagation distance much shorter than the diffraction length. Both the applied and generated fields exhibit electromagnetically induced waveguiding.

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I. INTRODUCTION

Electromagnetically induced focusing, defocusing, spatial confinement (SC), and waveguiding of a weak probe laser by a strong pump laser have been demonstrated for a variety of atomic systems. These include two-level systems [1–6], three-level ladder [7,8], V [9–11], and Λ systems [12,13], four-level systems [14], and strongly driven Raman systems [15–18]. In the two-level system, both the pump and probe interact with a single transition whereas in the other systems, the pump and probe interact with different transitions. Generally, induced focusing and SC are obtained at probe detunings which are different from those of the pump so that the two-photon Raman detuning is nonzero. However, in the Λ system studied by Manassah and Gross [12] [see Fig. 1(a)], induced focusing and waveguiding of the probe was achieved, at two-photon resonance, by detuning the pump and probe far to the blue side of their respective transitions, and choosing the on-axis pump Rabi frequency to be near the minimum value required one to achieve electromagnetically induced transparency (EIT) [19,20] at that detuning. Previously, Kazinets *et al.* [21] showed that focusing and SC of two fields with equal Rabi frequencies [see Fig. 1(a)] can be obtained, at zero two-photon detuning, provided that the on-axis Rabi frequencies of the blue-detuned fields are near the threshold required to achieve coherent population trapping (CPT) [22,23]. Thus, in order to obtain induced focusing and SC in either the EIT or CPT configuration, the initial intensity at the center of the transverse intensity profile (TIP) of the laser beams, which we assume to be Gaussian, must be chosen so that the intensity in the wings is below the threshold required to obtain EIT or CPT. An alternative view, adopted in this paper, is that focusing is obtained when the deviation from two-photon coherence ($D=\rho_{11}\rho_{22}-|\rho_{21}|^2$), which has its minimum value at the center of the Gaussian profile of the pump(s), increases towards the wings of the profile.

We can envisage two possible scenarios where focusing cannot occur and the beams will diffract on propagation. In the first, the Rabi frequencies and detunings of the laser beams, or the decay rate of the two-photon coherence [24,25], are chosen so that either EIT or CPT is maintained over the whole beam profile [26–28]. In the second, rather trivial case, the Rabi frequencies and detunings are chosen so that neither EIT or CPT occurs, even at the center of the TIP. In this paper, we show that SC can be produced in the latter system by combining it with one in which both laser beams display SC over several diffraction lengths. In other words, a Λ system, in either a spatially confined EIT (SCEIT) or spatially confined CPT (SCCPT) configuration, can act as a waveguide for the laser beams in another Λ system, which would diffract in the absence of the guiding Λ system. This type of electromagnetically induced waveguiding only occurs when the Λ systems are coupled to form a closed-loop double-Λ system [29–34], with zero two-photon detuning, as shown in Fig. 1(b). It is important to note that this effect, in contrast to previously considered cases of electromagnetically induced waveguiding [1–11], is highly dependent on phase. In previous cases, the effect is due to the phase-independent effective linear susceptibility whereas, here, it

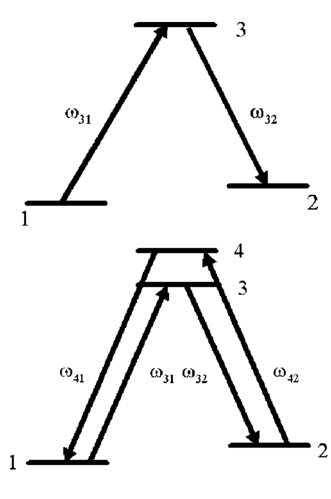

FIG. 1. Energy-level scheme for (a) single- and (b) double-Λ systems.

also derives from the phase-dependent effective third-order susceptibility.

In a previous paper [28], we discussed frequency conversion in a double-Λ system. We showed that focusing, leading to enhanced frequency conversion, can occur for propagation distances that are much shorter than the diffraction length, when the interacting fields are slighty detuned to the blue. This focusing is, however, often accompanied by ring formation which may lead to breakup [35]. We also showed that focusing, without ring formation, can occur at short propagation distances, for identical blue-detuned beams, when the initial relative phase is switched from zero where CPT occurs to π where it cannot exist [29–34]. Here, we discuss frequency conversion between the fields in the upper Λ system, for the case where the lower Λ system, which is in a SCCPT configuration, controls the two-photon coherence of the combined system. We find that the applied and generated fields are spatially confined for several diffraction lengths, and that they display oscillatory behavior which is a combination of the higher-frequency oscillations, that also occur in the absence of diffraction, with the lower-frequency breathing oscillations imposed by the guiding Λ system. It should be noted that the field generated by the four-wave-mixing (FWM) process is produced with the correct phase to close the loop. In future work, we will identify other nonlinear systems that can exhibit waveguiding when combined with SCEIT or SCCPT systems.

The double-Λ system has previously been investigated in the context of amplification without inversion [36–39], phase-sensitive laser cooling [40], the propagation of pairs of optical pulses [41], optical phase conjugation [27,42–44], phase control of photoionization [45], resonantly enhanced four-wave mixing [34,42,46–52], cavity quantum electrodynamics (QED) [53], phase control of EIT [54,55] and CPT [56], Ramsey fringes [57], light storing of a pair of pulses [58,59], quantum control of entanglement [60,61], and dynamic optical bistability [62].

II. THE MODEL

A. The Bloch equations

The three-level Λ and four-level double-Λ system are depicted in Figs. 1(a) and 1(b). The single-Λ system, which is also the lower Λ system of the double-Λ system, consists of the states $|1\rangle$, $|2\rangle$, and $|3\rangle$, whereas the upper Λ system consists of the states $|1\rangle$, $|2\rangle$, and $|4\rangle$. Each $|j\rangle \to |i\rangle$ transition (with $j=1,2$ and $i=3,4$ throughout the paper) interacts with an electromagnetic field

$$\vec{E}_{ij}(\vec{r},t) = (1/2)\hat{x}_{ij}E_{ij}(r)\exp[-i(\omega_{ij}t - k_{ij}z + \varphi_{ij})] + \text{c.c.}, \tag{1}$$

with unit polarization vector $\hat{x}_{ij}$, frequency ω_{ij}, wave vector k_{ij}, and initial phase φ_{ij}, whose detuning from the transition frequency ω'_{ij} is $\Delta_{ij}=\omega'_{ij}-\omega_{ij}$ and whose Rabi frequency is $2V_{ij}(r)=\mu_{ij}E_{ij}(r)/\hbar$.

The first step is to write the Bloch equations for the double-Λ system [28,34] which reduce to those of the single-Λ system [63] when $V_{3j}=0$. It should be pointed out that the Bloch equations for the off-diagonal elements of the density matrix are the same for all four-level systems that interact with four fields so that a loop is formed. The equations for the diagonal elements differ only in the decay terms. The Bloch equations are given by

$$\dot{\rho}_{11} = i(V_{13}\rho'_{31} + V_{14}\rho'_{41} - V_{31}\rho'_{13} - V_{41}\rho'_{14}) - \gamma_{12}\rho_{11} + \gamma_{21}\rho_{22} + \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44}, \tag{2}$$

$$\dot{\rho}_{22} = i(V_{23}\rho'_{32} + V_{24}\rho'_{42} - V_{32}\rho'_{23} - V_{42}\rho'_{24}) + \gamma_{12}\rho_{11} - \gamma_{21}\rho_{22} + \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44}, \tag{3}$$

$$\dot{\rho}_{33} = i(V_{31}\rho'_{13} + V_{32}\rho'_{23} - V_{13}\rho'_{31} - V_{23}\rho'_{32}) - \gamma_{3}\rho_{33} + \gamma_{43}\rho_{44}, \tag{4}$$

$$\dot{\rho}_{44} = i(V_{41}\rho'_{14} + V_{42}\rho'_{24} - V_{14}\rho'_{41} - V_{24}\rho'_{42}) - \gamma_{4}\rho_{44}, \tag{5}$$

$$\dot{\rho}'_{21} = i(V_{23}\rho'_{31} + aV_{24}\rho'_{41} - V_{31}\rho'_{23} - aV_{41}\rho'_{24}) - (\Gamma_{21} + i\Delta_{21})\rho'_{21}, \tag{6}$$

$$\dot{\rho}'_{31} = i(V_{31}\rho_{11} + V_{32}\rho'_{21} - V_{31}\rho_{33} - V_{41}\rho'_{34}) - (\Gamma_{31} + i\Delta_{31})\rho'_{31}, \tag{7}$$

$$\dot{\rho}'_{32} = i(V_{32}\rho_{22} + V_{31}\rho'_{12} - V_{32}\rho_{33} - a^{*}V_{42}\rho'_{34}) - (\Gamma_{32} + i\Delta_{32})\rho'_{32}, \tag{8}$$

$$\dot{\rho}'_{41} = i(V_{41}\rho_{11} + a^{*}V_{42}\rho'_{21} - V_{31}\rho'_{43} - V_{41}\rho_{44}) - (\Gamma_{41} + i\Delta_{41})\rho'_{41}, \tag{9}$$

$$\dot{\rho}'_{42} = i(V_{42}\rho_{22} + aV_{41}\rho'_{12} - aV_{32}\rho'_{43} - V_{42}\rho_{44}) - (\Gamma_{42} + i\Delta_{42})\rho'_{42}, \tag{10}$$

$$\dot{\rho}'_{43} = i(V_{41}\rho'_{13} + a^{*}V_{42}\rho'_{23} - V_{13}\rho'_{41} - a^{*}V_{23}\rho'_{42}) - (\Gamma_{43} + i\Delta_{43})\rho'_{43}, \tag{11}$$

where $a=\exp(i\Phi)$ and $\Phi=\varphi_{31}-\varphi_{32}+\varphi_{42}-\varphi_{41}$ is the initial relative phase, γ_{kl} is the longitudinal decay rate from state $|k\rangle \to |l\rangle$, γ_i is the total decay rate from state $|i\rangle$, and $\Gamma_{kl}=0.5(\gamma_k+\gamma_l)+\Gamma^{*}_{kl}$ is the transverse decay rate of the off-diagonal density-matrix element ρ'_{kl}, where Γ^{*}_{kl} is the rate of phase-changing collisions. The rapidly oscillating terms have been eliminated by the substitutions

$$\rho'_{ij} = \rho_{ij}\exp[-i(\Delta_{ij}t + k_{ij}z - \varphi_{ij})], \tag{12}$$

and

$$\rho'_{21} = \rho_{21}\exp\{-i[(\Delta_{31}-\Delta_{32})t + (k_{31}-k_{32})z - (\varphi_{31}-\varphi_{32})]\}, \tag{13}$$

$$\rho'_{43} = \rho_{43}\exp\{-i[(\Delta_{41}-\Delta_{31})t + (k_{41}-k_{31})z - (\varphi_{41}-\varphi_{31})]\}. \tag{14}$$

It is only possible to write the Bloch equations in this form when the multiphoton resonance condition, $\omega_{31}-\omega_{32}+\omega_{42}$

$-\omega_{41}=0$, is satisfied. This condition can be rewritten in terms of the one-photon detunings as $\Delta_{31}-\Delta_{32}=\Delta_{41}-\Delta_{42}=\Delta_{21}$, where Δ_{21} is the two-photon or Raman detuning or, alternatively, $\Delta_{41}-\Delta_{31}=\Delta_{42}-\Delta_{32}=\Delta_{43}$.

In addition to solving the steady-state Bloch equations numerically, we have also obtained analytical formulas which express the off-diagonal density matrix elements in terms of the populations of the states. These formulas are a generalization of those previously developed for the three-level Λ system [63], and for the double-Λ system [49] in the case where V_{31} and V_{42} are strong, and V_{32} and V_{41} are weak (strong-weak-strong-weak configuration), and it is assumed that the strong fields remain constant. We find that the analytical expression for ρ'_{ij} can be decomposed into the sum of terms that are linear and third order in the Rabi frequency,

$$\rho'_{ij}=\rho'^{(1)}_{ij}+\rho'^{(3)}_{ij}, \tag{15}$$

or, more explicitly, as

$$\rho'_{31}=\tilde{\chi}^{(1)}_{31}V_{31}+a\tilde{\chi}^{(3)}_{31}V_{32}V_{24}V_{41}, \tag{16}$$

$$\rho'_{32}=\tilde{\chi}^{(1)}_{32}V_{32}+a^*\tilde{\chi}^{(3)}_{32}V_{31}V_{14}V_{42}, \tag{17}$$

$$\rho'_{41}=\tilde{\chi}^{(1)}_{41}V_{41}+a^*\tilde{\chi}^{(3)}_{41}V_{42}V_{23}V_{31}, \tag{18}$$

$$\rho'_{42}=\tilde{\chi}^{(1)}_{42}V_{42}+a\tilde{\chi}^{(3)}_{42}V_{41}V_{13}V_{32}, \tag{19}$$

where $\tilde{\chi}^{(1)}_{ij}$ and $\tilde{\chi}^{(3)}_{ij}$ are proportional to the effective linear and third-order susceptibilities $\chi^{(1,3)}_{ij}$ [64]. The real and imaginary parts of the effective linear susceptibility $\chi^{(1)}_{ij}$ are proportional to the refraction and absorption of the field that interacts with the $|i\rangle\rightarrow|j\rangle$ transition, and the effective third-order susceptibility $\chi^{(3)}_{ij}$ gives the contribution to the nonlinear polarization at ω_{ij} from FWM. Unfortunately, the analytical expressions for the susceptibilities are too unwieldy to reproduce here. However, as we show in Sec. III, their numerical evaluation gives important physical insight into the behavior of the system. We assume throughout that the copropagating laser beams are close in frequency. This assumption allows us to neglect Doppler broadening.

B. CPT, EIT, and focusing conditions

In order to understand the conditions under which focusing with low absorption occurs in a single-Λ system, we recall the analytical expressions for ρ'_{3j} in terms of the populations [63]

$$\rho'_{31}=\tilde{\chi}^{(1)}_{31}V_{31}=-\frac{V_{31}(\rho_{33}-\rho_{11})}{\Delta_{31}-i\Gamma_{31}}+\frac{V_{32}\rho'_{21}}{\Delta_{31}-i\Gamma_{31}}, \tag{20}$$

$$\rho'_{32}=\tilde{\chi}^{(1)}_{32}V_{32}=-\frac{V_{32}(\rho_{33}-\rho_{22})}{\Delta_{32}-i\Gamma_{32}}+\frac{V_{31}\rho'_{12}}{\Delta_{32}-i\Gamma_{32}}, \tag{21}$$

where

$$\rho'_{21}=-\frac{V_{31}V^*_{32}}{\tilde{\Delta}_{21}-i\tilde{\Gamma}_{21}}\left(\frac{\rho_{22}-\rho_{33}}{\Delta_{32}+i\Gamma_{32}}-\frac{\rho_{11}-\rho_{33}}{\Delta_{31}-i\Gamma_{31}}\right), \tag{22}$$

with

$$\tilde{\Delta}_{21}=\Delta_{21}-\frac{|V|^2_{32}\Delta_{31}}{\Delta^2_{31}+\Gamma^2_{31}}+\frac{|V|^2_{31}\Delta_{32}}{\Delta^2_{32}+\Gamma^2_{32}}, \tag{23}$$

$$\tilde{\Gamma}_{21}=\Gamma_{21}+\frac{|V|^2_{32}\Gamma_{31}}{\Delta^2_{31}+\Gamma^2_{31}}+\frac{|V|^2_{31}\Gamma_{32}}{\Delta^2_{32}+\Gamma^2_{32}}. \tag{24}$$

Let us consider CPT and EIT for the case where $\Delta_{31}=\Delta_{32}$ and $\Gamma_{31}=\Gamma_{32}$. For CPT where $V_{31}=V_{32}$, we see from Eqs. (20) and (21) that $\rho'_{3j}=0$ when $\rho_{11}=\rho_{22}=1/2$, $\rho_{33}=0$, and $\rho'_{21}=-1/2$, and from Eqs. (22)–(24), that $\rho'_{21}=-1/2$ when

$$2|V|^2_{31}\Gamma_{31}/(\Delta^2_{31}+\Gamma^2_{31})\gg\Gamma_{21}. \tag{25}$$

Only when this condition is obeyed will CPT occur. For the case of EIT where $V_{31}\gg V_{32}$ leads to $\rho_{22}\gg\rho_{11},|\rho_{21}|$, the condition of Eq. (25) is replaced by

$$|V|^2_{31}\Gamma_{31}/(\Delta^2_{31}+\Gamma^2_{31})\gg\Gamma_{21}. \tag{26}$$

We now choose $\Delta^2_{31}\gg\Gamma^2_{31}$ so that absorption of the laser beams is insignificant, even when the conditions for CPT and EIT are not satisfied. It can be seen from Eqs. (20) and (21) that both beams will then be focused, provided the one-photon detuning is negative. Thus, in order to obtain focusing, in either a CPT or EIT configuration, the pump Rabi frequency at the center of the TIP $V_{31}(0)$ must obey the condition

$$q|V(0)|^2_{31}\Gamma_{31}/(\Delta^2_{31}+\Gamma^2_{31})\simeq\Gamma_{21}, \tag{27}$$

where $q=1$ for the EIT situation and 2 for the CPT situation. In other words, the pump Rabi frequency must be chosen so that the threshold condition for EIT or CPT is obeyed. In addition, both the fields must be detuned to the blue of their respective transitions and two-photon resonance must be maintained. For both CPT and EIT, we will use the quantity $D=\rho_{11}\rho_{22}-|\rho_{21}|^2$ as a measure of the deviation from two-photon coherence. When this deviation has a minimum at the TIP center and increases toward its maximum value in the wings, focusing will occur.

C. Maxwell-Bloch equations

In order to study the beam propagation, we solve the Maxwell-Bloch equations, in the paraxial approximation, which may be written in the form [3–6]

$$\frac{\partial}{\partial z}V'_{ij}=\frac{i}{4L_D}\nabla^2_T V'_{ij}+\frac{i}{L_{ij}}\rho'_{ij}, \tag{28}$$

where

$$\nabla^2_T=\partial^2/\partial\xi^2+(1/\xi)\partial/\partial\xi+(1/\xi^2)\partial^2/\partial\theta^2 \tag{29}$$

is the transverse Laplacian in dimensionless cylindrical coordinates, $\xi=r/\sqrt{2}w_{31}(0)$, where $w_{31}(0)$ is the initial spot size of the field at frequency ω_{31}, $V'_{ij}=V_{ij}/\Gamma_{31}$ is the dimensionless Rabi frequency, the parameter $L_D=k[w_{31}(0)]^2$ is the diffraction length, and the parameter $L_{ij}=\hbar\Gamma_{31}/\pi kN\mu^2_{ij}=4/\alpha_{ij}(0)$, where $\alpha_{ij}(0)$ is the unsaturated line-center absorption coefficient for the $|j\rangle\rightarrow|i\rangle$ transition. In the calculations we assume that $L_{ij}=L_{\mathrm{NL}}$ (NL stands for nonlinear) for all the transitions. The ratio $L_{rel}=L_{NL}/L_D$ expresses the propagation distance at which the nonlinearity becomes important, relative to the length at which diffraction becomes important. Thus for a constant value of L_D, decreasing the value of L_{rel} ensures that the nonlinearity takes effect at a shorter propagation distance.

We solve the Maxwell-Bloch equations numerically for beams whose initial TIP's are Gaussian with the same waist sizes:

$$V'_{ij} = V'_{ij}(0)\exp(-\xi^2). \tag{30}$$

In order to compare plane-wave (PW) and Gaussian beams, we assume that the initial Rabi frequencies of the beams in the PW approximation are equal to the initial values of $V'_{ij}(0)$, the on-axis Rabi frequencies of the Gaussian TIP of the beams. In all the calculations presented here, we assume that $\Gamma'_{ij}=\Gamma_{ij}/\Gamma_{31}=1$ for all four one-photon transitions, $\gamma_{43}=0$, $\gamma'_{21}=\gamma'_{12}=\gamma_{12}/\Gamma_{31}=10^{-4}$, and $\Gamma^*_{ij}=0$.

III. NUMERICAL RESULTS

A. The single-Λ system

In this section, we show that the laser beams that interact with a single-Λ system, in either an EIT or CPT configuration, can display SC over several diffraction lengths provided the EIT or CPT condition holds at the TIP center, but not in the wings. We first discuss the EIT configuration where $V'_{31}(0)=1$, $V'_{32}(0)=0.1$, and $\Delta'_{3j}=-30$. This choice of parameters ensures that the threshold condition of Eq. (27) holds at the TIP center ($\xi=0$) but not in the wings of the profile. In Figs. 2(a) and 2(b), we show how the real and imaginary parts of $\tilde{\chi}'^{(1)}_{3j}=\tilde{\chi}^{(1)}_{3j}\Gamma_{31}$, at $z/L_D=0$, vary across the profiles of the laser beams. We see that the parts of the profiles that lie near the center experience focusing, accompanied by slight absorption, whereas the extreme wings experience some absorption but no focusing. As both transitions are far from saturation, $\tilde{\chi}'^{(1)}_{31}=\tilde{\chi}'^{(1)}_{32}$. In Fig. 2(c), we plot the deviation from two-photon coherence, D, across the profile of the pump. We see that D has a minimum ($D_{min}=0.07$) at the center of the profile and increases toward its maximum value of 0.25 in the wings ($\rho_{11}=\rho_{22}=0.5$ and $\rho_{21}=0$ when $V'_{3j}=0$ since $\gamma'_{21}=\gamma'_{12}$). In Fig. 2(d), we show the pump and probe TIP's at the center, and in Fig. 2(e), the full pump and probe TIP's, as a function of the propagation length z/L_D for $L_{rel}=3.33\times10^{-3}$. Both beams display SC with breathing for ten propagation lengths. This suggests that the effect of the pump and probe focusing is slightly greater than that of the diffraction. It should be noted that SC without breathing can be obtained by increasing the value of L_{rel} so that focusing takes effect at a longer propagation distance. From Fig. 2(d), we see clearly that the pump experiences absorption in addition to breathing.

The one-photon detuning is now increased to $\Delta'_{3j}=-150$ so that the EIT condition does not even hold at the TIP center. As a result, the deviation from two-photon coherence at the TIP center is $D_{min}=0.24$ which is close to the maximum possible deviation, and the profiles experience much less focusing [approximately 1/20 of that shown in Figs. 2(a) and 2(b)]. This focusing is rapidly overcome by diffraction, as can be seen in Fig. 2(f). We will show below that a Λ system that experiences SC, as shown in Fig. 2(d), can induce SC in a Λ system that diffracts, as shown in Fig. 2(e), provided they are linked together to form a loop.

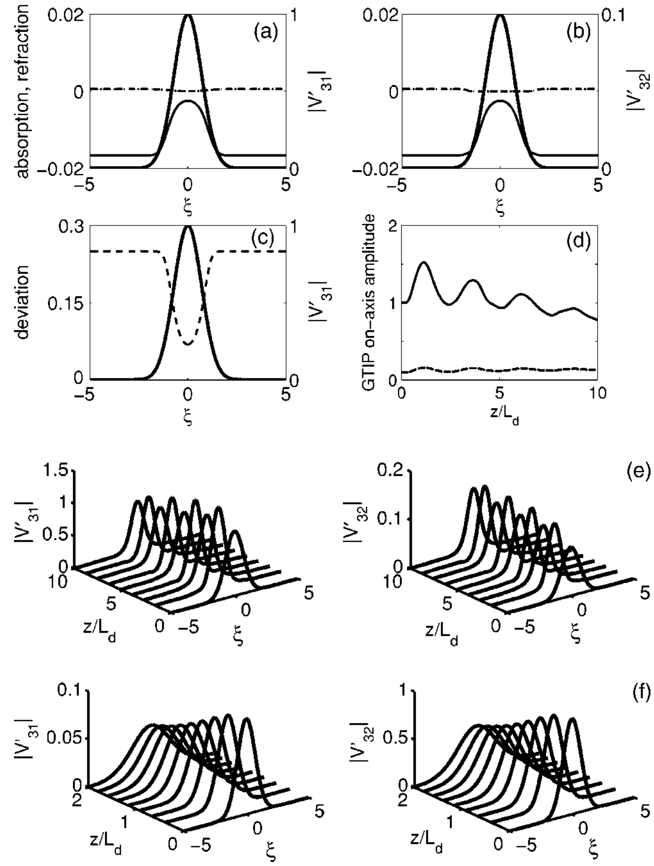

FIG. 2. Single-Λ system in EIT configuration. (a) Pump and (b) probe absorption (Im $\tilde{\chi}'^{(1)}_{3j}$, dashed line) and refraction (Re $\tilde{\chi}'^{(1)}_{3j}$, thin solid line), and V'_{3j} (thick solid line), at $z/L_D=0$, as a function of ξ, (c) D (dashed line) and V'_{31} (thick solid line), at $z/L_D=0$, as a function of ξ, (d) pump (solid line) and probe (dashed line) TIP's at $\xi=0$, as a function of z/L_D, and (e),(f) TIP's (V'_{ij} vs ξ) of propagating beams as a function of z/L_D. Note SC behavior in (e) and diffracting behavior in (f). Initial Rabi frequencies are $V'_{31}(0)=1$, $V'_{32}(0)=0.1$. Detunings are $\Delta'_{3j}=-30$ in (a)–(e), and $\Delta'_{3j}=-150$ in (f). $L_{rel}=3.33\times10^{-3}$.

We now turn to the CPT configuration and consider the parameters $V'_{3j}(0)=1$ and $\Delta'_{3j}=-30$, which are chosen so that the threshold condition of Eq. (27) holds at the TIP center but not in the wings of the profile. In Figs. 3(a) and 3(b), we show how the real and imaginary parts of $\tilde{\chi}'^{(1)}_{3j}$, at $z/L_D=0$, vary across the profiles of the laser beams. The results are similar to those shown in Figs. 2(a) and 2(b) for the EIT case except that the focusing and low absorption extend further into the wings. In Fig. 3(c), D is plotted across the beam profiles. As before, it has a minimum at the TIP center ($D_{min}=0.02$), and increases toward its maximum value in the wings, which is indicative of strong focusing. In Fig. 3(d), we show the on-axis pump and probe TIP's, and in Fig. 3(e), the full pump and probe TIP's, as a function of the propagation length z/L_D for $L_{rel}=3.33\times10^{-3}$. As expected by anal-

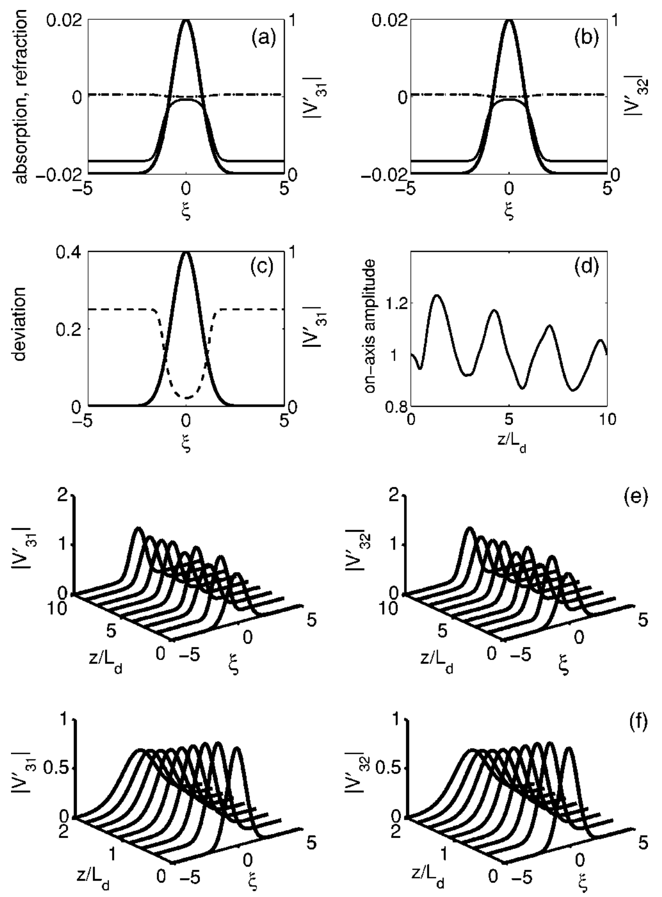

FIG. 3. Single-Λ system in CPT configuration. (a),(b) Pump absorption (Im $\tilde{\chi}'^{(1)}_{3j}$, dashed line) and refraction (Re $\tilde{\chi}'^{(1)}_{3j}$, thin solid line), and V'_{3j} (thick solid line), at $z/L_D=0$, as a function of ξ, (c) D (dashed line) and V'_{31} (thick solid line), at $z/L_D=0$, as a function of ξ, (d) pump TIP's at $\xi=0$, as a function of z/L_D, and (e),(f) TIP's (V'_{ij} vs ξ) of propagating beams as a function of z/L_D. Note SC behavior in (e) and diffracting behavior in (f). Initial Rabi frequencies are $V'_{31}(0)=V'_{32}(0)=1$. Detunings are $\Delta'_{3j}=-30$ in (a)–(e), and $\Delta'_{3j}=-150$ in (f). $L_{\rm rel}=3.33\times10^{-3}$.

ogy with the SCEIT case, the beams display SC with breathing and some absorption, over long propagation distances. The one-photon detuning is now increased to $\Delta'_{3j}=-150$ so that the CPT condition does not even hold at the TIP center. As a result, the deviation from two-photon coherence at the TIP center, $D_{\rm min}=0.19$, is considerable, and the TIP's experience focusing at the center which is approximately $1/20$ of that shown in Figs. 3(a) and 3(b). This focusing is rapidly overcome by diffraction, as can be seen in Fig. 3(f).

We now discuss a case where the CPT condition holds over the entire profile of the laser beams so that focusing does not occur at all. Consider the CPT configuration, $V'_{3j}(0)=4$ and $\Delta'_{3j}=-4$, with $L_{\rm rel}=3.33\times10^{-3}$. We see in Fig. 4(a) that the absorption and refraction are zero over all the whole pump profile, so that the pumps do not undergo reshaping due to absorption or focusing. Figure 4(b) shows that $D=0$ over the whole profile. In the absence of focusing, the beams become diffracted on propagation, as shown in Fig. 4(c).

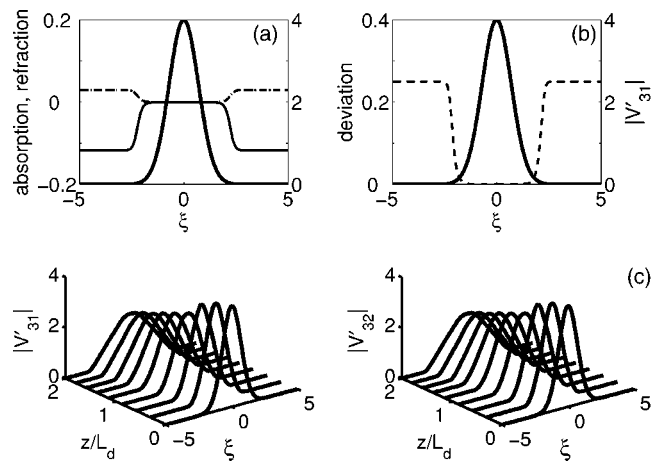

FIG. 4. Single-Λ system in CPT configuration. (a) Pump absorption (Im $\tilde{\chi}'^{(1)}_{31}=$Im $\tilde{\chi}'^{(1)}_{32}$, dashed line) and refraction (Re $\tilde{\chi}'^{(1)}_{31}$ $=$Re $\tilde{\chi}'^{(1)}_{32}$, thin solid line), and V'_{3j} (thick solid line), at $z/L_D=0$, as a function of ξ, (b) D (dashed line) and V'_{31} (thick solid line), at $z/L_D=0$, as a function of ξ. Note in (b) that $D=0$ over the whole profile and in (c) that the pumps diffract. Initial Rabi frequencies are $V'_{31}(0)=V'_{32}(0)=4$. Detunings are $\Delta'_{3j}=-4$. $L_{\rm rel}=3.33\times10^{-3}$.

The examples discussed in this section confirm the usefulness of $D(\xi)$ as an indicator of focusing in Λ EIT and CPT configurations.

B. The double-Λ system

In this section, we link two single-Λ systems [Fig. 1(a)] together so as to form a double-Λ system [Fig. 1(b)]. For the lower Λ system, we choose either a SCEIT or SCCPT configuration, and for the upper Λ system, we choose either an EIT or CPT configuration where the beams would be diffracted in the absence of the lower Λ system. We demonstrate that the lower spatially confined Λ system can act as a waveguide for the inherently diffracting upper Λ system.

We first investigate the double-Λ system formed by combining the SCEIT system of Figs. 2(a)–2(e) [$V'_{31}(0)=1$, $V'_{32}(0)=0.1$, and $\Delta'_{3j}=-30$] with the inherently diffracting system of Fig. 2(f) [$V'_{41}(0)=1$, $V'_{42}(0)=0.1$, and $\Delta'_{4j}=-150$]. As the laser beams in the lower Λ system are much closer to one-photon resonance than those in the upper Λ system, the deviation from two-photon coherence at $z/L_D=0$ is determined by the lower system and is identical to that shown in Fig. 2(c). In addition, since the E_{31} field interacts most strongly with its respective transition, the behavior of $\tilde{\chi}'^{(1)}_{31}$ $=\tilde{\chi}^{(1)}_{31}\Gamma_{31}$, at $z/L_D=0$, is unchanged, and is identical to that shown in Fig. 2(a). However, the focusing experienced by the other fields, which interact less strongly with their respective transitions, is different from that obtained in the single-Λ system. Specifically, the focusing of E_{32} is reduced by a factor of 1.5, whereas that of E_{41} is increased by a factor of 4.5, and E_{42} is even slightly defocused, at the outset. In Fig. 5(a), the TIP's of all the fields are plotted as a function of the propagation length z/L_D for $L_{\rm rel}=1.2\times10^{-3}$. We see that by linking the two Λ systems, the lower system induces spatial confinement in the upper system. While the fields

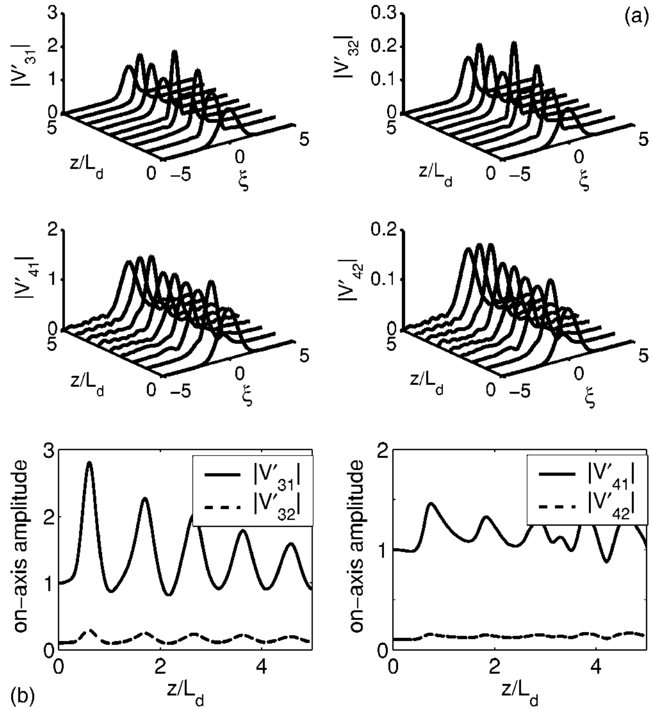

FIG. 5. Double-Λ system in which the SCEIT system of Fig. 2(e) waveguides the inherently diffracting Λ system of Fig. 2(f). (a) TIP's (V'_{ij} vs ξ) of propagating beams as a function of z/L_D, and (b) TIP's of beams at $\xi=0$, as a function of z/L_D. Initial Rabi frequencies are $V'_{i1}(0)=1$, and $V'_{i2}(0)=0.1$. Detunings are $\Delta'_{3j}=-30$, $\Delta'_{4j}=-150$, and $L_{\mathrm{rel}}=1.2\times10^{-3}$.

interacting with the lower, waveguiding Λ system experience almost no diffraction, those interacting with the upper, waveguided Λ system experience some diffraction in the wings. The extent of this diffraction is highly dependent on the value of L_{rel}. Unfortunately, we could not find a value of L_{rel} that completely eliminates the effect. In Fig. 5(b), we plot the on-axis TIP's as a function of z/L_D. The difference between the behavior of $V'_{3j}(0)$ in the double-Λ system, shown in Fig. 5(b), and in the single-Λ system, shown in Fig. 2(d), is due only to the different value of L_{rel} used in each case.

We now combine the SCCPT system of Figs. 3(a)–3(e) $[V'_{3j}(0)=1$ and $\Delta'_{3j}=-30]$ with the inherently diffracting system of Fig. 3(f) $[V'_{4j}(0)=1$ and $\Delta'_{4j}=-150]$. As in the EIT case, the deviation from two-photon coherence is determined by the lower Λ system and is identical to that shown in Fig. 3(c). The initial focusing experienced by the fields E_{3j} is slightly weaker than in the single-Λ system, whereas that experienced by the fields E_{4j} is significantly reduced to only 0.08 of the value obtained in the single-Λ system. In Figs. 6(a) and 6(b), the full TIP's and the TIP's at their center are plotted for all the fields, as a function of the propagation length z/L_D for $L_{\mathrm{rel}}=1.0\times10^{-3}$. Again, we see that the lower Λ system acts as a waveguide for the upper Λ system.

In Figs. 6(a) and 6(b), we took the initial relative phase Φ to be zero. Let us now consider the case where $\Phi=\pi$. The initial absorption, refraction, and deviation from two-photon coherence are all the same as in the case where $\Phi=0$, due to the dominance of the lower Λ system. As can be seen in Fig.

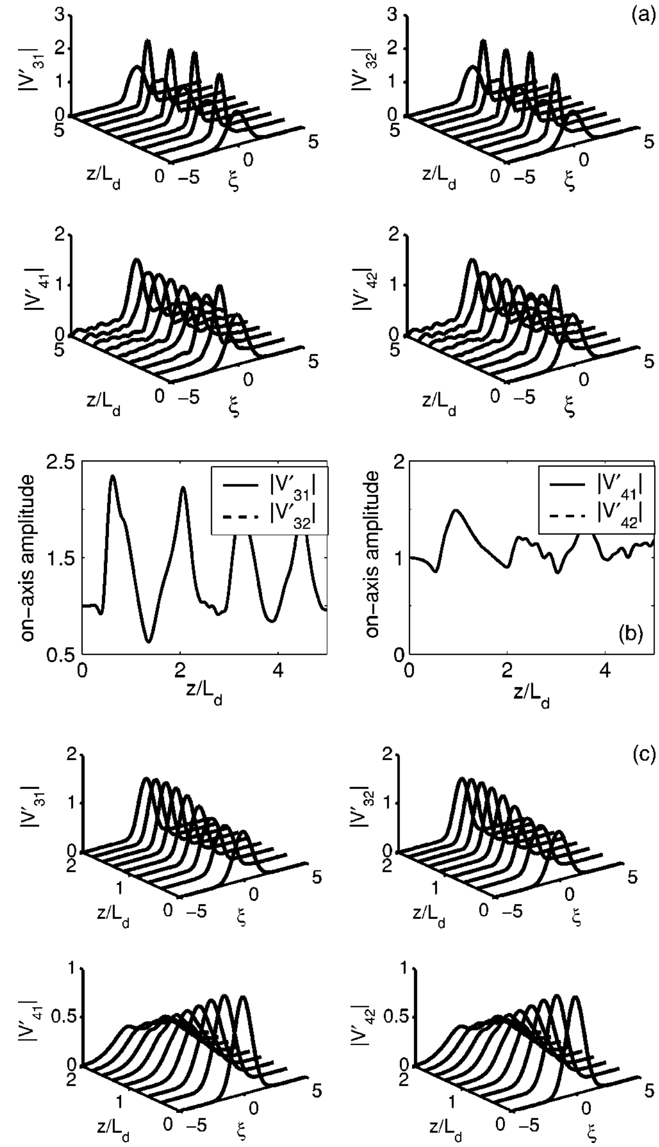

FIG. 6. Double-Λ system in which the SCCPT system of Fig. 3(e) waveguides the inherently diffracting Λ system of Fig. 3(f). (a) TIP's (V'_{ij} vs ξ) of propagating beams as a function of z/L_D, (b) TIP's of beams at $\xi=0$, as a function of z/L_D, and (c) TIP's vs ($V'_{ij}\xi$) of propagating beams as a function of z/L_D, for $\Phi=\pi$. Initial Rabi frequencies are $V'_{ij}(0)=1$. Detunings are $\Delta'_{3j}=-30$, $\Delta'_{4j}=-150$, and $L_{\mathrm{rel}}=1.0\times10^{-3}$ in (a) and (b), and $L_{\mathrm{rel}}=3.33\times10^{-3}$ in (c).

6(c), where the TIP's of all four fields are plotted, the lower Λ system is unchanged by the change in phase. However, it is no longer capable of inducing waveguiding in the upper Λ system. This suggests that the waveguiding effect derives from the phase-dependent effective third-order contribution to the density matrix as well as the phase-independent first-order contribution [see Eqs. (16)–(19)]. If the phase is changed to $\Phi=\pi$ in the case shown in Fig. 5, where the lower system is in a SCEIT configuration, not only is waveguiding of the upper Λ system not obtained but, in addition, the probe field in the lower Λ system ceases to be spatially confined.

We now combine the SCCPT system of Figs. 3(a)–3(e) $[V'_{3j}(0)=1$ and $\Delta'_{3j}=-30]$ with a Λ system that does not even

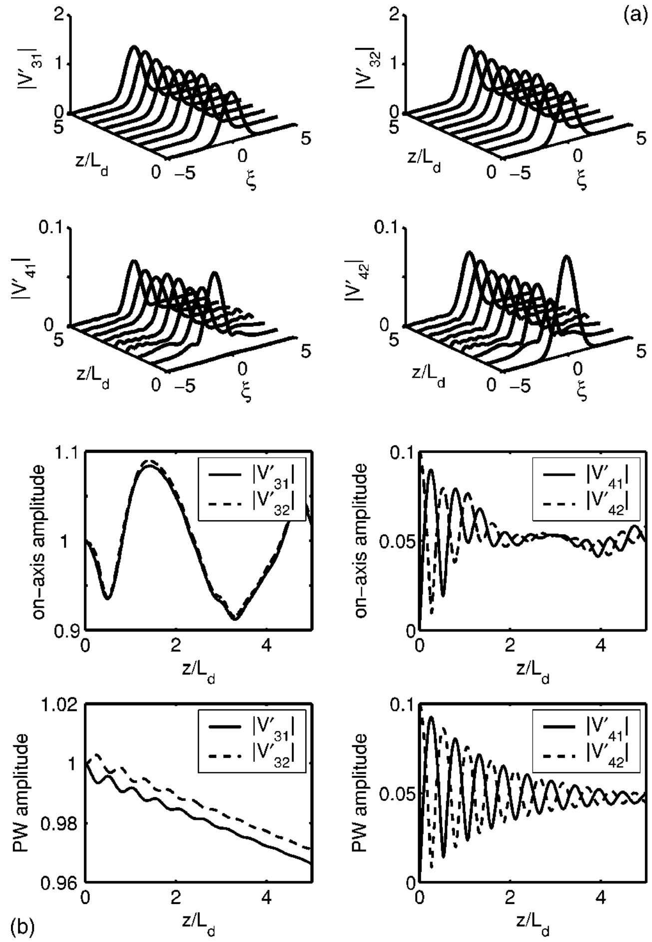

FIG. 7. Double-Λ system in which the SCCPT system of Fig. 3(e) waveguides another Λ system interacting with a weak, applied field and a field generated by FWM. (a) TIP's (V'_{ij} vs ξ) of propagating beams as a function of z/L_D. (b) Comparison of Gaussian and PW's: upper panels, TIP's of Gaussian beams at $\xi=0$, as a function of z/L_D, and lower panels, amplitude of PW's as a function of z/L_D. Initial Rabi frequencies are $V'_{3j}(0)=1$, $V'_{42}(0)=0.1$, and $V'_{41}(0)=0.001$. Detunings are $\Delta'_{3j}=-30$, $\Delta'_{4j}=-20$, and $L_{\rm rel}=4\times10^{-3}$.

satisfy the EIT condition at the TIP center, so that neither self- nor cross-focusing occurs [$V'_{41}(0)=0.001$, $V'_{42}(0)=0.1$, $\Delta'_{4j}=-20$, and $L_{\rm rel}=4\times10^{-3}$]. Our aim is to show that it is possible to spatially confine both the applied field, V'_{42}, and the field generated by FWM, V'_{41}. We discussed a similar configuration [$V'_{3j}(0)=8$, $V'_{41}(0)=0.001$, $V'_{42}(0)=0.1$, $\Delta'_{ij}=\pm4$, and $L_{\rm rel}=1.66\times10^{-4}$] in a previous paper [28]. There, D was small over the whole profile so that neither focusing nor defocusing occurred. In addition, maximum conversion of $V'_{42}(0)$ to $V'_{41}(0)$ as a result of FWM occurred after a very short propagation length (74% at $z/L_D=0.002$), long before diffraction became important. In the configuration discussed here, we see from Fig. 7(a) that both the applied and generated fields in the upper Λ system are spatially confined over a considerable propagation length. In Fig. 7(b), we compare the propagation behavior of the on-axis TIP's of the Gaussian beams with that of the PW's. As before, maximum conversion of $V'_{42}(0)$ to $V'_{41}(0)$ occurs during the first oscillation cycle (90% at $z/L_D=0.25$). In addition, we see that the on-axis amplitudes $V'_{42}(0)$ and $V'_{41}(0)$ oscillate with opposite phases at two frequencies: a faster one which is the same as that experienced by the PW's, and a slower frequency imposed by the in-phase breathing of the fields $V'_{3j}(0)$ that interact with the lower Λ system.

IV. CONCLUSION

We have confirmed that a three-level Λ system, in either an EIT or CPT configuration, can experience self- and cross-focusing, leading to SC on propagation over many diffraction lengths [13,21]. We have shown that this occurs when the lasers are detuned far to the blue, the two-photon detuning is zero, and the deviation from two-photon coherence has a minimum at the center of the transverse intensity profile of the interacting laser beams. In order to achieve this minimum, the Rabi frequencies at the center of the pump intensity profiles should be at the EIT or CPT threshold. We have demonstrated that when EIT or CPT is maintained across the whole beam profile, neither focusing nor defocusing occurs, so that the beams diffract on propagation. This also occurs when the detuning is too large or the fields too weak for EIT or CPT to occur.

We have linked a Λ system in a SCCPT configuration with one that diffracts, so that a closed loop with zero two-photon detuning is formed, and shown that the spatially confined system acts as a waveguide for the inherently diffracting system. The same phenomenon can also occur when both systems are in a CPT configuration. We also showed that when a Λ system that interacts with a weak field and a very weak (or even zero) field is linked with a Λ system in a SCCPT configuration, frequency conversion due to FWM occurs, and both the incident and generated fields experience SC over several diffraction lengths. We showed that the behavior of the applied and generated fields on propagation is a combination of the higher-frequency oscillations experienced by the PW's and the slower oscillations induced by interaction with the SCCPT configuration. Finally, the waveguiding effect was shown to be strongly phase dependent, indicating that it derives from the phase-dependent effective third-order susceptibility rather than the phase-independent effective first-order susceptibility, as is the case in previously studied systems.

The examples we have considered emphasize the importance of taking the transverse profile of the beams into account when discussing propagation, especially for distances greater than the diffraction length. All the examples discussed relate to electromagnetically induced waveguiding in the double-Λ system. However, it is clear that waveguiding over several diffraction lengths can be induced in a wide variety of inherently diffracting nonlinear systems by linking them to spatially confined systems.

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[1] G. P. Agrawal, Phys. Rev. Lett. **64**, 2487 (1990).
[2] A. J. Stentz, M. Kauranen, J. J. Maki, G. P. Agrawal, and R. W. Boyd, Opt. Lett. **17**, 19 (1992).
[3] D. Bortman-Arbiv, A. D. Wilson-Gordon, and H. Friedmann, Phys. Rev. A **58**, R3403 (1998).
[4] D. Bortman-Arbiv, A. D. Wilson-Gordon, and H. Friedmann, Phys. Rev. A **61**, 033806 (2000).
[5] D. Bortman-Arbiv, A. D. Wilson-Gordon, and H. Friedmann, Phys. Rev. A **63**, 031801(R) (2001).
[6] D. Bortman-Arbiv, A. D. Wilson-Gordon, and H. Friedmann, Opt. Commun. **204**, 371 (2002).
[7] R. R. Moseley, S. Shepherd, D. J. Fulton, B. D. Sinclair, and M. H. Dunn, Phys. Rev. Lett. **74**, 670 (1995).
[8] R. R. Moseley, S. Shepherd, D. J. Fulton, B. D. Sinclair, and M. H. Dunn, Phys. Rev. A **53**, 408 (1995).
[9] A. G. Truscott, M. E. J. Friese, N. R. Heckenberg, and H. Rubinsztein-Dunlop, Phys. Rev. Lett. **82**, 1438 (1999).
[10] J. A. Andersen, M. E. J. Friese, A. G. Truscott, Z. Ficek, P. D. Drummond, N. R. Heckenberg, and H. Rubinsztein-Dunlop, Phys. Rev. A **63**, 023820 (2001).
[11] R. Kapoor and G. S. Agarwal, Phys. Rev. A **61**, 053818 (2000).
[12] J. T. Manassah and B. Gross, Opt. Commun. **124**, 418 (1996).
[13] N. A. Zharova, A. G. Litvak, and V. A. Mironov, JETP Lett. **79**, 272 (2004).
[14] T. Hong, Phys. Rev. Lett. **90**, 183901 (2003).
[15] K. S. Repasky and J. L. Carlsten, Phys. Rev. A **54**, 4528 (1996).
[16] D. R. Walker, D. D. Yavuz, M. Y. Shverdin, G. Y. Yin, A. V. Sokolov, and S. E. Harris, Opt. Lett. **27**, 2094 (2002).
[17] D. D. Yavuz, D. R. Walker, and M. Y. Shverdin, Phys. Rev. A **67**, 041803(R) (2003).
[18] M. Y. Shverdin, D. D. Yavuz, and D. R. Walker, Phys. Rev. A **69**, 031801(R) (2004).
[19] S. E. Harris, Phys. Today **50**(7), 36 (1997).
[20] J. P. Marangos, J. Mod. Opt. **45**, 471503 (1998).
[21] I. V. Kazinets, B. G. Matisov, and A. Y. Snegirev, Tech. Phys. **442**, 837 (1997).
[22] E. Arimondo, Prog. Opt. **35**, 257 (1996).
[23] For convenience, we have used the terms CPT and EIT to distinguish between the case where the fields that interact with the Λ system are of comparable strength, and that where one field is strong and the other is weak. This corresponds to the original use of these terms, as described, for example, in Ref. [20].
[24] E. B. Alexandrov, M. V. Balabas, A. S. Pasgalev, A. K. Vershovskii, and N. N. Yakobson, Laser Phys. **6**, 244 (1996).
[25] D. Budker, V. Yashchuk, and M. Zolotorev, Phys. Rev. Lett. **81**, 5788 (1998).
[26] M. Jain, A. J. Merriam, A. Kasapi, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **75**, 4385 (1995).
[27] T. T. Grove, M. S. Shahriar, P. R. Hemmer, P. Kumar, V. S. Sudarshanam, and M. Cronin-Golomb, Opt. Lett. **22**, 769 (1997).
[28] H. Shpaisman, A. D. Wilson-Gordon, and H. Friedmann, Phys. Rev. A **70**, 063814 (2004).
[29] S. P. Krinitzky and D. T. Pegg, Phys. Rev. A **33**, 403 (1986).
[30] S. J. Buckle, S. M. Barnett, P. L. Knight, M. A. Lauder, and D. T. Pegg, Opt. Acta **33**, 1129 (1986).
[31] D. Kosachiov, B. G. Matisov, and Y. V. Rozhdestvensky, Opt. Commun. **85**, 209 (1991).
[32] D. Kosachiov, B. G. Matisov, and Y. V. Rozhdestvensky, J. Phys. B **25**, 2473 (1992).
[33] W. Maichen, R. Gaggl, E. Korsunsky, and L. Windholz, Europhys. Lett. **31**, 189 (1995).
[34] E. A. Korsunsky and D. V. Kosachiov, Phys. Rev. A **60**, 4996 (1999).
[35] J. M. Soto-Crespo, E. M. Wright, and N. N. Akhmediev, Phys. Rev. A **45**, 3168 (1992).
[36] O. Kocharovskaya and P. Mandel, Phys. Rev. A **42**, 523 (1990).
[37] O. A. Kocharovskaya, G. Mauri, and E. Arimondo, Opt. Commun. **84**, 393 (1991).
[38] C. H. Keitel, O. A. Kocharovskaya, L. M. Narducci, M. O. Scully, S.-Y. Zhu, and H. M. Doss, Phys. Rev. A **48**, 3196 (1993).
[39] P. Doug, S. H. Tang, W. N. Man, and J. Y. Gao, J. Phys. B **34**, 2851 (2001).
[40] D. Kosachiov, B. G. Matisov, and Y. V. Rozhdestvensky, Europhys. Lett. **22**, 11 (1993).
[41] E. Cerboneschi and E. Arimondo, Phys. Rev. A **52**, R1823 (1995).
[42] M. D. Lukin, P. R. Hemmer, M. Loffler, and M. O. Scully, Phys. Rev. Lett. **81**, 2675 (1998).
[43] P. R. Hemmer, D. P. Katz, J. Donoghue, M. Cronin-Golomb, M. S. Shahriar, and P. Kumar, Opt. Lett. **20**, 982 (1995).
[44] T. T. Grove, E. Rousseau, X.-W. Xia, D. S. Hsiung, and M. S. Shahriar, Opt. Lett. **22**, 1677 (1997).
[45] Y. F. Li, J. F. Sun, X. Y. Zhang, and Y. C. Wang, Opt. Commun. **202**, 97 (2002).
[46] B. L. Lu, W. H. Burkett, and M. Xiao, Opt. Lett. **23**, 804 (1998).
[47] S. Barreiro and J. W. R. Tabosa, Opt. Commun. **233**, 383 (2004).
[48] S. A. Babin, U. Hinze, E. Tiemann, and B. Wellegehausen, Opt. Lett. **21**, 1186 (1996).
[49] S. A. Babin, E. V. Podivilov, D. A. Shapiro, U. Hinze, E. Tiemann, and B. Wellegehausen, Phys. Rev. A **59**, 1355 (1999).
[50] U. Hinze, B. N. Chichkov, E. Tiemann, and B. Wellegehausen, J. Opt. Soc. Am. B **17**, 2001 (2000).
[51] S. A. Babin, S. I. Kablukov, U. Hinze, E. Tiemann, and B. Wellegehausen, Opt. Lett. **26**, 81 (2001).
[52] M. G. Payne and L. Deng, Phys. Rev. A **65**, 063806 (2002).
[53] J. H. Eberly, Philos. Trans. R. Soc. London, Ser. A **355**, 2387 (1997).
[54] E. A. Korsunsky, N. Leinfellner, A. Huss, S. Baluschev, and L. Windholz, Phys. Rev. A **59**, 2302 (1999).
[55] A. F. Huss, E. A. Korsunsky, and L. Windholz, J. Mod. Opt. **49**, 141 (2002).
[56] W. Maichen, F. Renzoni, I. Mazets, E. Korsunsky, and L. Windholz, Phys. Rev. A **53**, 3444 (1996).
[57] A. S. Zibrov, I. Novikova, and A. B. Matsko, Opt. Lett. **26**, 1311 (2001).
[58] A. Raczynski and J. Zaremba, Opt. Commun. **209**, 149 (2002).
[59] A. Raczynski, J. Zaremba, and S. Zielinska-Kaniasty, Phys. Rev. A **69**, 043801 (2004).
[60] V. S. Malinovsky and I. R. Sola, Phys. Rev. A **70**, 042304 (2004).

[61] V. S. Malinovsky and I. R. Sola, Phys. Rev. A **70**, 042305 (2004).
[62] I. Novikova, A. S. Zibrov, D. F. Phillips, A. Andre, and R. L. Walsworth, Phys. Rev. A **69**, 061802(R) (2004).
[63] S. Boublil, A. D. Wilson-Gordon, and H. Friedmann, J. Mod. Opt. **38**, 1739 (1991).
[64] R. W. Boyd, *Nonlinear Optics*, 2nd ed. (Academic, San Diego, 2003).