Atomic quantum memory: Cavity versus single-pass schemes

A. Dantan,* A. Bramati, and M. Pinard
Laboratoire Kastler Brossel, Université Pierre et Marie Curie, Case 74, 4 place Jussieu, 75252 Paris Cedex 05, France
(Received 25 October 2004; published 4 April 2005)

We present a quantum mechanical treatment for both atomic and field fluctuations of an atomic ensemble interacting with propagating fields, either in electromagnetically induced transparency (EIT) or in a Raman situation. The atomic spin noise spectra and the outgoing field spectra are calculated in both situations. For suitable parameters both EIT and Raman schemes efficiently preserve the quantum state of the incident probe field in the transfer process with the atoms, although a single-pass scheme is shown to be intrinsically less efficient than a cavity scheme.

DOI: 10.1103/PhysRevA.71.043801 PACS number(s): 42.50.Dv, 42.50.Ct, 03.67.−a

I. INTRODUCTION

There has recently been a lot of interest in quantum communication at the light-atom interface, with the prospect of realizing quantum information networks composed of nodes of atomic ensembles connected by light [1,2]. A basic requirement of such networks is the ability to perform quantum-state exchanges between fields and atoms. There have been various proposals to write, store, and read out a field state onto a long-lived atomic spin, and several experiments have already demonstrated the possibility to manipulate quantum states between field and atoms: on the one hand, "slow-light" experiments based on electromagnetically induced transparency (EIT) [3] have shown that a pulse of light could be stored and retrieved inside an atomic cloud [4]. In the weak-probe regime conservation of the quantum character of the pulse was predicted using the concept of "dark-state polaritons" [5], but remains to be demonstrated experimentally. On the other hand, off-resonant interactions have been used to entangle two atomic ensembles and map a polarization state of light onto an atomic spin [6], and the mapping and storage of coherent states have been reported very recently by Julsgaard *et al.* [7]. There are also several proposals to realize a quantum memory using such a scheme [8]. In recent works we studied how nonclassical light states could be transferred to atoms and predicted quasi-ideal quantum-state transfer between field and atoms placed in an optical cavity, in both resonant and off-resonant EIT configurations [9,10]. In particular we showed that squeezed states and Einstein-Podolsky-Rosen (EPR) states could be mapped onto atomic ground-state spins with a high efficiency. We also developed a method to read out the atomic state in the field exiting the cavity, thus allowing quantum memory operations in a controlled and efficient manner. In these calculations the cavity plays an important part to improve the collective atom-field coupling which scales linearly with the number of atoms. Moreover, the intrinsic noise coming from spontaneous emission or ground-state decoherence is substantially damped by the cavity interaction, allowing in principle quasireversible quantum-state exchanges between the field and atoms. In the present paper we extend these cavity results to single-pass interactions and show that good-quality quantum-state transfers are also possible in either resonant and off-resonant situations. We first present a general method to calculate both the field and atomic noise spectra in a one-dimensional propagation problem. We then apply it to the case of a squeezed-vacuum input-field state and derive the outgoing-field spectrum as well as the atomic variances, first in resonant EIT and then in off-resonant EIT. For some parameters the atoms may be spin squeezed by the same amount as the incident field. We analyze the mapping efficiencies and the effect of ground-state decoherence, and compare the results obtained in the single-pass schemes with those of the cavity schemes. An important result is that, in any situation, the efficiency increases faster with the number of atoms in a cavity scheme ($\propto N$) than in a single-pass scheme ($\propto \sqrt{N}$).

II. SINGLE-PASS SCHEME

In the following sections we address the issue of one-dimensional field propagation through a dilute atomic cloud of length L, cross section $\mathcal{S}$, and containing N atoms uniformly distributed. We assume that the atomic cloud is elongated with Fresnel number of order unity, so that the emission can be considered one dimensional [11]. In order to take into account transverse effects a three-dimensional theory would be required [12], which is beyond the scope of the present paper. In the first section we introduce continuous operators by dividing the atomic medium into transverse slices, as in [13], and we give the atom-field evolution equations. In the next sections we study the continuous interaction of a coherent pump field and a squeezed-vacuum probe field with the atoms, and calculate the spectra of the field exiting the cloud in two situations: in "resonant EIT"—both fields are one- and two-photon resonant—and in an off-resonant EIT or "Raman" configuration—large one-photon detunings, but two-photon resonance is maintained. These situations have been shown to be the most favorable to the conservation of quantum states during atom-field transfer operations [9,10,15]. For each configuration we also calculate the atomic ground-state coherence variances and show that

*Electronic address: dantan@spectro.jussieu.fr

the incident-field state can be perfectly mapped onto the atoms.

A. Atom-field evolution equations

In order to treat the paraxial propagation problem we write the positive frequency component of the copropagating electric fields E_j $(j=1,2)$ as $E_j^{(+)}(z,t)=\mathcal{E}_{0j}A_j(z,t)e^{i(kz-\omega_j t)}$, where ω_j is the laser frequency, $\mathcal{E}_{0j}=\sqrt{\hbar\omega_j/2\epsilon_0 \mathcal{S}L}$, and $A_j(z,t)$ is a dimensionless slowly varying envelope operator, satisfying

$$[A_j(z,t),A_j^\dagger(z',t')] = \frac{L}{c}\delta[t-t'-(z-z')/c].$$

From the single-atom operators $\sigma_{\mu\nu}^j(t)$ (in the rotating frame of their laser frequency) one can define continuous operators at position z by averaging on a slice of length Δz [13]:

$$\sigma_{\mu\nu}(z,t) = \lim_{\Delta z\to 0}\frac{L}{N\Delta z}\sum_{z\leqslant z^j\leqslant z+\Delta z}\sigma_{\mu\nu}^j(t).$$

Denoting the control field by A_1 and the probe field by A_2 the interaction Hamiltonian can then be expressed as

$$H = -\hbar\sum_{j=1,2}\int\frac{dz}{L}N[g_jA_j(z,t)\sigma_{3j}(z,t) + \text{H.c.}],$$

with $g_j=d_j\mathcal{E}_{0j}/\hbar$ the atom-field coupling constants. The field evolution equations are obtained from Maxwell's propagation equations in the slowly varying envelope approximation

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)A_j(z,t) = ig_jN\sigma_{j3}(z,t)\ \ (j=1,2). \tag{1}$$

The evolution equations for the atomic variables are given by a set of Heisenberg-Langevin equations

$$\frac{\partial}{\partial t}\sigma_{13} = -(\gamma+i\Delta_1)\sigma_{13} + ig_1A_1(\sigma_{11}-\sigma_{33}) + ig_2A_2\sigma_{21} + f_{13},$$

$$\frac{\partial}{\partial t}\sigma_{23} = -(\gamma+i\Delta_2)\sigma_{23} + ig_2A_2(\sigma_{22}-\sigma_{33}) + ig_1A_1\sigma_{12} + f_{23},$$

$$\frac{\partial}{\partial t}\sigma_{21} = -(\gamma_0-i\delta)\sigma_{21} + ig_1A_1^\dagger\sigma_{23} - ig_2A_2\sigma_{31} + f_{21},$$

$$\frac{\partial}{\partial t}\sigma_{11} = -\gamma_0\sigma_{11} + \gamma\sigma_{33} + \Lambda_1 + ig_1A_1^\dagger\sigma_{13} - ig_1A_1\sigma_{31} + f_{11},$$

$$\frac{\partial}{\partial t}\sigma_{22} = -\gamma_0\sigma_{22} + \gamma\sigma_{33} + \Lambda_2 + ig_2A_2^\dagger\sigma_{23} - ig_2A_2\sigma_{32} + f_{22},$$

$$\frac{\partial}{\partial t}\sigma_{33} = -2\gamma\sigma_{33} - (ig_1A_1^\dagger\sigma_{13} - ig_1A_1\sigma_{31}) - (ig_2A_2^\dagger\sigma_{23} - ig_2A_2\sigma_{32}) + f_{33},$$

where the Δ_i's are the detunings from resonance, $\delta=\Delta_1-\Delta_2$ the two-photon detuning, γ the optical dipole decay rate (taken equal on both transitions for simplicity), and γ_0 the decay of the ground-state coherence, modeling collisions or accounting for the transit of the atoms outside the interaction area with the light beams. The Λ_i's are chosen to maintain the total number of atoms constantly equal to N. The $f_{\mu\nu}$'s are δ-correlated Langevin operators, the correlation functions of which are of the form

$$\langle f_{\mu\nu}(z,t)f_{\rho\sigma}(z',t')\rangle = \frac{L}{N}D_{\mu\nu\rho\sigma}\delta(t-t')\delta(z-z').$$

The diffusion coefficients $D_{\mu\nu\rho\sigma}$ can be calculated via the quantum regression theorem.

B. EIT interaction

In the resonant EIT situation ($\Delta_1=\Delta_2=\delta=0$), the presence of the control field allows the probe field to propagate with little dissipation. Correlations between pump and probe fields have been investigated [13,14] and observed [16], and, recently, the quantum character of a squeezed-vacuum probe has been shown to be partially conserved in EIT [17], but little attention has been paid to the atomic variables. We will focus on the case of a coherent pump field and a zero-mean-valued probe field with some quantum fluctuations over a broad bandwidth—e.g., squeezed vacuum. In such a situation all the atoms are pumped into level 2 in steady state and the fluctuations of both fields are decoupled [9]. Moreover, the fluctuations of the probe field are only coupled to the atomic ground-state coherence and the optical coherence σ_{23}. Linearizing around this steady state one obtains, for the fluctuations of the probe field amplitude quadrature $X=A_2+A_2^\dagger$,

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)X(z,t) = -2gN\sigma_y(z,t), \tag{2}$$

$$\frac{\partial}{\partial t}\sigma_y(z,t) = -\gamma\sigma_y + \Omega j_x + \frac{g}{2}X + f_{\sigma_y}, \tag{3}$$

$$\frac{\partial}{\partial t}j_x(z,t) = -\gamma_0 j_x - \Omega\sigma_y + f_{j_x}, \tag{4}$$

where $\Omega=g_1\langle A_1\rangle$ is assumed real, and $\sigma_y=(\sigma_{23}-\sigma_{32})/2i$ and $j_x=(\sigma_{21}+\sigma_{12})/2$ are the fluctuations of the optical-dipole and ground-state coherence. Similar equations relate the field-phase quadrature, the other atomic components σ_x and j_y.

Fourier transforming these equations, one derives the outgoing-field spectrum $S_{X^{out}}(\omega)$, defined by

$$\langle X^{out}(\omega)X^{out}(\omega')\rangle = 2\pi\delta(\omega+\omega')\frac{L}{c}S_{X^{out}}(\omega). \tag{5}$$

Assuming that the incident-amplitude squeezing spectrum is constant and equal to $S_{X^{in}}$ over the frequency bandwidth considered, one gets

$$S_{X^{out}}(\omega) = 1 - [1 - S_{X^{in}}]e^{-(\alpha+\alpha^*)}, \tag{6}$$

with

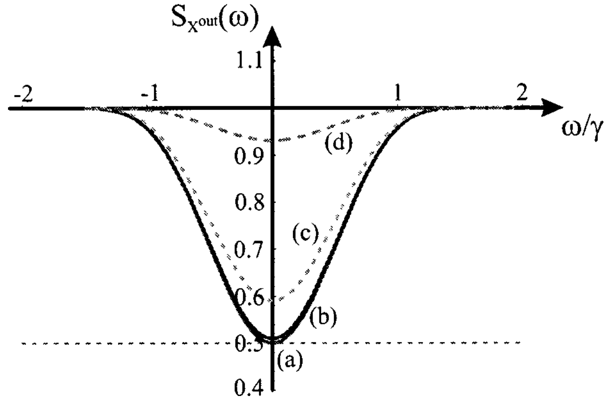

FIG. 1. Outgoing-field squeezing spectrum in EIT for different values of γ_0: (a) 0, (b) $\gamma/1000$, (c) $\gamma/100$, and (d) $\gamma/10$. Parameters: $\Gamma_E = 10\gamma$, $C=100$.

$$\alpha(\omega) = -i\frac{\omega L}{c} + C\frac{\gamma(\gamma_0 - i\omega)}{(\gamma - i\omega)(\gamma_0 - i\omega) + \Omega^2}.$$

We have denoted by $\Gamma_E = \Omega^2/\gamma$ the optical pumping at resonance and introduced a cooperativity parameter [9]

$$C = \frac{g^2 N}{\gamma}\frac{L}{c}.$$

The term in $i\omega L/c$ corresponds to the field dephasing due to the propagation in vacuum. However, in EIT conditions, the propagation is strongly modified: expanding $\alpha(\omega)$ around zero frequency yields the well-known result

$$\alpha(\omega) = A - i\omega\frac{L}{v_g} + O(\omega^2),$$

where

$$A = C\frac{\gamma_0}{\gamma_0 + \Gamma_E}, \qquad (7)$$

$$v_g = \frac{c}{1 + \frac{g^2 N}{\gamma}\frac{\Gamma_E - \gamma_0}{(\Gamma_E + \gamma_0)^2}} \qquad (8)$$

represent the absorption of the field and group velocity change in EIT, which is drastically reduced when $\gamma_0 \ll \Gamma_E \ll g^2 N/\gamma$ [5].

A typical spectrum is plotted in Fig. 1 for an initial-amplitude squeezing of 3 dB and different values of γ_0: the interesting result is that the outgoing field is squeezed only in a certain transparency window, of width

$$\Delta\omega \simeq \Gamma_E\sqrt{\frac{\ln 2}{2C}\left(1 - \frac{C\gamma_0}{\Gamma_E}\right)},$$

when $C \gg 1$ and $\Gamma_E \gg \gamma_0$. Outside this window the outgoing-field fluctuations are "absorbed" by the atoms and the field is at the shot noise. Besides, the more atoms, the larger C and the narrower the transparency bandwidth is. Last, an important parameter is γ_0, which, although it can be made very small with respect to the optical pumping and the spontaneous emission rates, can be responsible for a substantial squeezing reduction at low frequency when the number of atoms is large [see Fig. 1 and Eq. (7)].

It is also very interesting to look at what happens to the atoms. As conjectured by Fleischhauer and Lukin [5], and predicted in a cavity configuration in [9], the atomic coherence may be squeezed by almost the same amount as the incident field for a good choice of the interaction parameters. The atoms are said to be spin squeezed when the variance of one spin component in the plane orthogonal to the mean spin is less than its coherent-state value. More precisely, we define collective atomic observables by integrating the continuous operators over the cloud length:

$$J_\mu(t) = N\int \frac{dz}{L} j_\mu(z,t).$$

In our case the collective mean spin is completely polarized along z, $\langle J_z\rangle = N/2$, and, for a coherent-spin state, one has $\Delta J_x^2 = \Delta J_y^2 = N/4$. A spin-squeezed ensemble will have $\Delta J_\theta^2 < N/4$ for some component J_θ in the (x,y) plane [18].

From Eqs. (2)–(4) it is possible to compute the variances of the ground-state spin coherence. The general method to perform these calculations is detailed in the Appendix. It yields the spectrum of the collective spin-squeezed coherence J_x of the squeezed component, $S_{J_x}(\omega)$, defined as

$$\langle J_x(\omega) J_x(\omega')\rangle = 2\pi\delta(\omega + \omega') S_{J_x}(\omega).$$

The atomic spectrum is found to be the sum of three contributions:

$$S_{J_x}(\omega) = \frac{N}{4}[B_f(\omega) S_{X^{in}} + B_{coh}(\omega) + B_{spin}(\omega)]. \qquad (9)$$

The first term in Eq. (9) is the coupling with the incident squeezed-vacuum fluctuations and quantifies how much of the incident-field squeezing is transferred to the spin, whereas B_{coh} and B_{spin} give the contribution of, respectively, the atomic noise resulting from spontaneous emission and the atomic noise due to the loss of coherence in the ground state. Integrating over frequency yields the sought variance

$$\Delta J_x^2 \equiv \int \frac{d\omega}{2\pi} S_{J_x}(\omega),$$

the exact expression of which is not reproduced here. However, when the incident field is a (coherent) vacuum state—$S_{X^{in}} = 1$—the atoms are in a coherent-spin state. This implies a simple relation between the integrals of the B's:

$$\int \frac{d\omega}{2\pi}[B_f(\omega) + B_{coh}(\omega) + B_{spin}(\omega)] = 1.$$

In the case of an amplitude-squeezed input one can then measure the efficiency of the squeezing transfer by comparing the atomic squeezing to that of the incident field:

$$\eta \equiv \frac{1 - (\Delta J_x^2)/(N/4)}{1 - S_{X^{in}}}.$$

$\eta=1$ thus corresponds to perfect transfer, $\eta=0$ to no transfer at all. Using the previous relations the efficiency is equal to

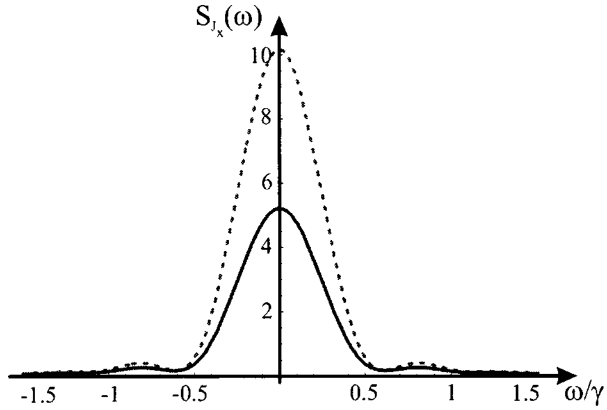

FIG. 2. Noise spectrum of the x component of the spin in EIT when the incident field is in a coherent state ($S_{X^{in}}=1$, dashed line) and squeezed by 3 dB ($S_{X^{in}}=0.5$, solid line), under the same pumping conditions ($C=100$, $\Gamma_E=10\gamma$, $\gamma_0=\gamma/1000$). The transfer efficiency is $\eta=0.91$ in the second case.

$$\eta=\int\frac{d\omega}{2\pi}B_f(\omega)=\int\frac{d\omega}{2\pi}\frac{C\Gamma_E\gamma^2}{|D|^2}\frac{|1-e^{-\alpha}|^2}{|\alpha|^2},\qquad(10)$$

with $D=(\gamma_0-i\omega)(\gamma-i\omega)+\Omega^2$. For most relevant situations, however, the field and optical dipole evolve rapidly compared to the ground-state coherence, so that it is possible to adiabatically eliminate them in Eqs. (2)–(4) and retrieve simple analytical expressions for the atomic spectrum and variance. In Fig. 2 we represent a typical atomic noise spectrum for typical experimental parameters. The atomic spectrum has a width proportional to $\Gamma_E/\sqrt{C}$ for large C. In order to maximize the transfer efficiency the pumping must be chosen in the regime $\gamma_0\ll\Gamma_E/\sqrt{C}\ll\gamma$. The efficiency can then be shown to be

$$\eta\simeq1-\frac{\sqrt{2/\pi}}{\sqrt{C}}-\frac{C\gamma_0}{\Gamma_E}\quad(C\gg1,\gamma_0\ll\gamma).\qquad(11)$$

A very good efficiency can thus be reached for a large cooperative behavior, and, as in the cavity scheme, the cooperativity is again the relevant parameter to quantify the transfer efficiency. Note also that the ground-state decay rate can also contribute to degrade the squeezing when the number of atoms grows large.

C. Raman interaction

We now consider a situation in which both fields are strongly detuned with respect to the one-photon resonances ($\Delta_i\gg\gamma$), but the two-photon resonance is maintained ($\delta=0$), using a small longitudinal magnetic field, for instance. In this off-resonant EIT or "″Raman" interaction one can eliminate the optical dipole and write simplified equations for the ground-state coherence and field:

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)X(z,t)=-2\tilde{g}Nj_y(z,t),$$

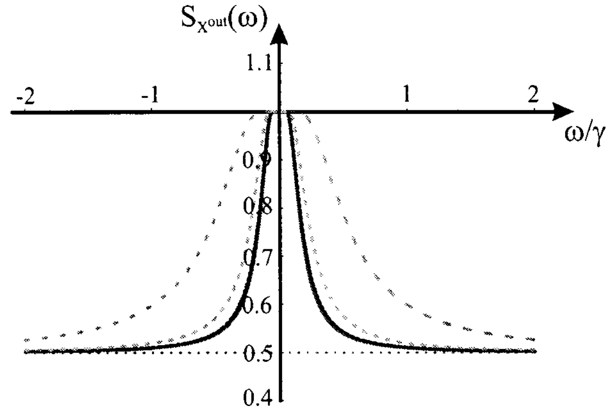

FIG. 3. Outgoing-field noise spectrum in a Raman situation for the same values of γ_0 as in Fig. 1. Parameters: $\Gamma_R=\gamma/100$, $C=100$.

$$\frac{\partial}{\partial t}j_y(z,t)=-(\gamma_0+\Gamma_R)j_y+\frac{\tilde{g}}{2}X+f_{j_y},$$

where $\Gamma_R=\gamma\Omega^2/\Delta^2$ is the Raman optical pumping rate (assumed much smaller than γ) and $\tilde{g}=g\Omega/\Delta$ is the effective atom-field coupling constant. Note that in this case the amplitude fluctuations are coupled to those of j_y. Following a method analogous to the previous section the outgoing-field noise spectrum can be written as

$$S_{X^{out}}(\omega)=1-[1-S_{X^{in}}]e^{-(\alpha'+\alpha'^*)},\qquad(12)$$

with

$$\alpha'(\omega)=-i\frac{\omega L}{c}+\frac{C\Gamma_R}{\Gamma_R+\gamma_0-i\omega}.$$

The spectrum, plotted in Fig. 3, is radically different from the EIT one; the squeezing is now absorbed around zero frequency on a width

$$\Delta\omega'\simeq\sqrt{\frac{2}{\ln 2}}\sqrt{C\Gamma_R(\Gamma_R+\gamma_0)},$$

and the spectrum broadens when the number of atoms (or the propagation length) is increased. For higher frequencies the field comes out unchanged. Note also that, the optical pumping rate being kept constant, the width of the spectrum noticeably depends on the value of the ground-state decay rate: as can be seen from Fig. 3 the central absorption peak width increases with γ_0 as soon as $\gamma_0\sim\Gamma_R$.

Concerning the atoms, one finds an atomic spectrum for the spin component coupled to X^{in} as represented in Fig. 4. Although it is rather different from the EIT spectrum, the transfer efficiency is remarkably similar:

$$\eta'=\int\frac{d\omega}{2\pi}\frac{C\Gamma_R}{(\Gamma_R+\gamma_0)^2+\omega^2}\frac{|1-e^{-\alpha'}|^2}{|\alpha'|^2}.\qquad(13)$$

In this case the atomic noise spectrum width depends on $\sqrt{C}\Gamma_R$, so that the good regime for quantum-state transfer is this time $\gamma_0\ll\sqrt{C}\Gamma_R\ll\gamma$. The efficiency, as in EIT, increases to 100% with the cooperativity as $C^{-1/2}$, but shows a differ-

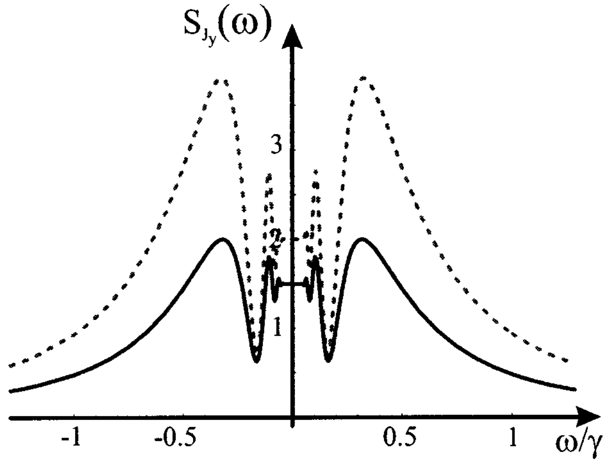

FIG. 4. Noise spectrum of the y component of the spin for a Raman interaction is in a coherent state ($S_{X^{in}}=1$, dashed line) and squeezed by 3 dB ($S_{X^{in}}=0.5$, solid line). Parameters: $C=100$, $\Gamma_R=\gamma/100$, $\gamma_0=\gamma/1000$. The transfer efficiency is $\eta'\simeq 0.91$ for a 3-dB-squeezed input, as in EIT.

ent sensitivity to the ground-state decoherence:

$$\eta' \simeq 1 - \frac{\sqrt{2/\pi}}{\sqrt{C}}\sqrt{1+\gamma_0/\Gamma_R} \tag{14}$$

for $C \gg 1$ and $\gamma_0 \ll \gamma$.

III. SINGLE-PASS vs CAVITY

It was shown in [9] that, if the atomic cloud was placed inside a single-ended optical cavity, with an output coupling mirror transmission T, a quasi-ideal mapping of the incident field is possible either in EIT or Raman. If we first consider the EIT situation, the atomic spectrum is Lorentzian shaped with width $\tilde{\gamma}_0=\gamma_0+\Gamma_E/(1+2C)$, whereas the field exiting the cavity is squeezed by approximately the same amount as the incident field for all frequencies. This is a strong difference with the single-pass scheme in which the outgoing field is squeezed only in the transparency window—i.e., for low frequencies. This is clearer when looking at the intracavity-field fluctuations X and relating them to those of the output field, $X^{out}=\sqrt{T}X-X^{in}$:

$$X(\omega) = \frac{2}{\sqrt{T}}\frac{1}{1+2C}\left[1+\frac{2C\tilde{\gamma}_0}{\tilde{\gamma}_0-i\omega}\right]X^{in}+\frac{i\omega}{\tilde{\gamma}_0-i\omega}F,$$

with F some atomic noise operator. For frequencies $\omega \ll \tilde{\gamma}_0$, $X\sim 2/\sqrt{T}X^{in}$, and the output-field fluctuations are those of the incident field: $X^{out}\sim X^{in}$. This means that, the medium being transparent in this frequency window, the intracavity field is simply the incident field, and since there is no field radiated by the atoms, the output field is the same as the input. However, at high frequencies, the intracavity-field fluctuations are in $O(1/C)$ and the output field is equal to the reflected field: $X^{out}\sim -X^{in}$. Indeed, outside the transparency window the incident-field fluctuations are absorbed by the atoms which radiate a field interacting destructively with the incident field, $X_r\sim -X^{in}$, so that $X\propto X_r+X^{in}\simeq 0$. In contrast, in the single-pass scheme, this reflected-field contribution to the output field is of course not present, so that the squeezing disappears outside the transparency window.

If we now compare with the Raman situation, this frequency dependence is opposite. The intracavity-field fluctuations can be written as

$$X=\frac{2}{\sqrt{T}}\frac{\Gamma_R-i\omega}{\tilde{\gamma}_0-i\omega}X^{in}+\frac{F'}{\tilde{\gamma}_0-i\omega},$$

where the effective atomic decay rate is now $\tilde{\gamma}_0=\gamma_0+(1+2C)\Gamma_R$ and, again, F' some atomic noise operator. At low frequencies, one has $X\simeq 0$, so that the output fluctuations are those of the reflected field, $X^{out}\sim -X^{in}$. On the contrary, for frequencies $\omega \gg \tilde{\gamma}_0$, $X\sim 2/\sqrt{T}X^{in}$ and $X^{out}\sim X^{in}$. This is again in good agreement with what was found for the single-pass scheme.

Coming now to the atoms, a noticeable difference is the transfer efficiency: in the cavity scheme the efficiency increases to 1 as $1/C$, whereas, in the single-pass scheme, the increase is slower—in $1/\sqrt{C}$. Physically, it means that the atom-field interaction in a cavity with N atoms and an output mirror transmission T is not equivalent to a single-pass interaction with N/T atoms, even though the cooperativities are then equal in both cases ($C=g^2N/T\gamma$ in a cavity [9]). This is naturally due to the fact that the incident squeezing is recycled inside the cavity on each round trip, whereas, in the single-pass scheme, the atoms "see" less and less squeezing along the propagation pass. This accounts for the fact that the cavity scheme intrinsic atomic noise decreases as $1/N$ and as $1/\sqrt{N}$ in a single-pass scheme. First, in an EIT cavity configuration and for $C\gg 1$, $\Gamma_E \gg \gamma_0$, the efficiency can be written as [9]

$$\eta=\frac{2C}{1+2C}\frac{\Gamma_E/(1+2C)}{\gamma_0+\Gamma_E/(1+2C)}\simeq 1-\frac{1}{1+2C}-\frac{(1+2C)\gamma_0}{\Gamma_E}. \tag{15}$$

Comparing Eq. (15) with Eq. (11) it is clear that the difference in efficiency comes from the sensitivity to noise coming from spontaneous emission, damped by a factor $(1+2C)$ in a cavity configuration and by $\sqrt{C}$ in single pass. Note, however, that the robustness of the mapping operation with respect to the ground-state decoherence is the same in the cavity and single pass, because the absorption is then linear in the number of atoms effectively seen by the field—i.e., proportional to C in both cases. This drawback can be overcome with the use of a buffer gas, which can significantly reduce the ground-state decay rate and increase the transfer efficiency [4].

For a Raman interaction this sensitivity is less crucial, since its effect is to reduce the cooperativity from C to $C\Gamma_R/(\Gamma_R+\gamma_0)$, as can be seen from Eq. (14). This difference with EIT comes from the fact that the range of frequencies involved in the Raman interaction is broader (see Fig. 4) than in EIT (Fig. 2). In the latter the atomic noise reduction is greater around zero frequency where the effect of ground-state decoherence is the most important. In Fig. 5 is plotted the transfer efficiency in both schemes when the cooperativ-

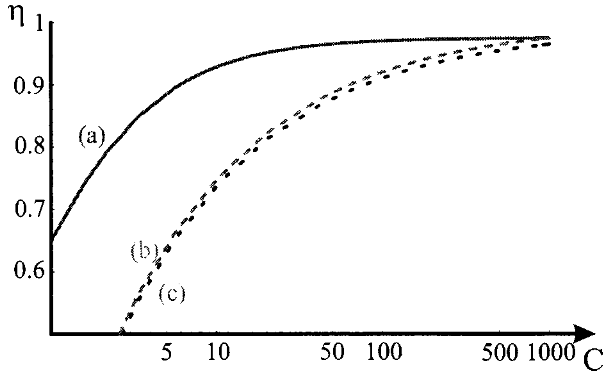

FIG. 5. Mapping efficiency versus cooperativity in a cavity scheme (a) and a single-pass interaction [(b) Raman, (c) EIT], for $\gamma_0=\gamma/1000$.

ity is varied. It is worth noticing that, even though the increase in efficiency is slower in a single pass than in a cavity, an excellent mapping—$\eta\sim 100\%$—can be achieved for all these schemes when the cooperativity is high enough.

IV. DISCUSSION AND CONCLUSION

It is therefore possible to map almost perfectly a squeezed-vacuum field state onto a ground-state spin coherence, results already predicted for a cavity scheme in [9]. The same decrease in transfer efficiency is found in EIT or Raman schemes as compared to the cavity schemes under the same conditions. However, good-quality transfer remains possible for very realistic parameters. Most properties relative to the transfer processes stress the importance of the cooperative behavior of the atoms. If qualitatively the conclusions drawn in the cavity scheme remain valid in a single-pass approach, quantitatively, however, the difference of scaling with the cooperativity shows that the cavity scheme is more efficient in many ways: writing and readout time, mapping efficiency, robustness with respect to spontaneous emission, etc. It is also interesting to note the differences and similarities between Raman and EIT interactions in terms of noise spectra, for instance. Good tests of this theory could be provided by outgoing-field noise measurements, such as those performed in [17]. We note that limitations may arise from the imperfections of the one-dimensional theory. For instance, diffraction effects or the issue of matching between the field and atomic modes are expected to play an important role [12,19,20] when the Fresnel number is not unity and when the plane-wave approximation for the field is no longer valid. In [10] it was shown that this quantum-state exchange mechanism also extends to EPR-entangled fields interacting with two ensembles in cavities. The calculations of this paper naturally extend to such states in single-pass interactions, provided suitable interaction parameters are chosen. Most ideas developed in [9,10,15] are also transposable to single-pass interactions, which should simplify the manipulation and storage of quantum states in atom-field quantum communication networks.

Note added. We recently became aware of similar work on EIT by Peng *et al.* [21], who reach the same conclusions about the outgoing-field spectrum in EIT.

APPENDIX: ATOMIC SPECTRUM CALCULATION

From Eqs. (2)–(4) it is possible to compute the variances of the ground-state coherence. We would like to stress the method to solve these space- and time-dependent coupled differential equations, a method which is actually quite general and may be applied to other situations. The idea is to perform a Fourier transform in time and a Laplace transform in space, in order to have a simple linear system. We standardly define the Laplace transform of $f(z)$ as

$$f[s]=\int_0^\infty e^{-sz}f(z)dz,$$

and the Fourier transform of $g(t)$ as

$$g(\omega)=\int_{-\infty}^\infty e^{i\omega t}g(t)d\omega.$$

The system (2)–(4) then becomes

$$(-i\omega+cs)X[s,\omega]=cX(0,\omega)-2gN\sigma_y[s,\omega],$$

$$(\gamma-i\omega)\sigma_y[s,\omega]=\Omega j_x[s,\omega]+\frac{g}{2}X[s,\omega]+f_{\sigma_y}[s,\omega],$$

$$(\gamma_0-i\omega)j_x[s,\omega]=-\Omega\sigma_y[s,\omega]+f_{j_x}[s,\omega].$$

From these equations one deduces $j_x[s,\omega]$:

$$j_x[s,\omega]=\frac{B_1}{s+s_0}X^{in}(\omega)+B_2\frac{s-b_2}{s+s_0}f_{\sigma_y}[s,\omega]+B_3\frac{s-b_3}{s+s_0}f_{j_x}[s,\omega],$$

with $X^{in}(\omega)=X(0,\omega)$, $B_1=-g\Omega/2/D$, $B_2=-\Omega/D$, $B_3=(\gamma-i\omega)/D$, $b_2=i\omega/c$, $b_3=i\omega/c-g^2N/c(\gamma-i\omega)$, $D=(\gamma_0-i\omega)(\gamma-i\omega)+\Omega^2$, and $s_0=\alpha(\omega)/L$. Using inverse Laplace transforms one then gets the fluctuations of the atomic operators at position z:

$$j_x(z,\omega)=B_1e^{-s_0z}X^{in}(\omega)+B_2\left[f_{\sigma_y}(z,\omega)-(s_0+b_2)\int_0^z dz'e^{-s_0(z-z')}f_{\sigma_y}(z',\omega)\right]+B_3\left[f_{j_x}(z,\omega)-(s_0+b_3)\int_0^z dz'e^{-s_0(z-z')}f_{j_x}(z',\omega)\right].$$

Finally, integrating over z yields the collective spin fluctuations

$$J_x(\omega)=B_1N\frac{1-e^{-\alpha}}{\alpha}X^{in}(\omega)+NB_2\int_0^L\frac{dz}{L}\left[f_{\sigma_y}(z,\omega)-\lambda_2\int_0^z dz'e^{-s_0(z-z')}f_{\sigma_y}(z',\omega)\right]+NB_3\int_0^L\frac{dz}{L}\left[f_{j_x}(z,\omega)-\lambda_3\int_0^z dz'e^{-s_0(z-z')}f_{j_x}(z',\omega)\right],$$

with $\lambda_i=s_0+b_i$ $(i=2,3)$. Using the correlation functions of the f's and of the incident field one deduces the expressions of the functions B_f, B_{coh}, and B_{spin} of the atomic-field spectrum (9).

[1] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[2] L. M. Duan, J. I. Cirac, P. Zoller, and E. S. Polzik, Phys. Rev. Lett. **85**, 5643 (2000); L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).
[3] S. E. Harris, Phys. Today **50**(x), 36 (1997).
[4] M. M. Kash *et al.*, Phys. Rev. Lett. **82**, 5229 (1999); C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001); D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001); M. Bajcsy, A. S. Zibrov, and M. D. Lukin, Nature (London) **426**, 633 (2004).
[5] M. D. Lukin and M. Fleischhauer, Phys. Rev. Lett. **84**, 5094 (2000); M. Fleischhauer and M. D. Lukin, Phys. Rev. A **65**, 022314 (2002).
[6] B. Julsgaard, A. Kozhekin, and E. S. Polzik, Nature (London) **413**, 400 (2001); C. Schori, B. Julsgaard, J. L. Sørensen, and E. S. Polzik, Phys. Rev. Lett. **89**, 057903 (2002).
[7] B. Julsgaard *et al.*,Nature (London) **432**, 482 (2004);
[8] A. Kuzmich, K. Mølmer, and E. S. Polzik, Phys. Rev. Lett. **79**, 4782 (1997); A. E. Kozhekin, K. Mølmer, and E. S. Polzik, Phys. Rev. A **62**, 033809 (2000).
[9] A. Dantan and M. Pinard, Phys. Rev. A **69**, 043810 (2004).
[10] A. Dantan, A. Bramati, and M. Pinard, Europhys. Lett. **67**, 881 (2004).
[11] M. G. Raymer and J. Mostowski, Phys. Rev. A **24**, 1980 (1981).
[12] L. M. Duan, J. I. Cirac, and P. Zoller, Phys. Rev. A **66**, 023818 (2002).
[13] M. Fleischhauer and T. Richter, Phys. Rev. A **51**, 2430 (1995); I. H. Deutsch and J. C. Garrison, *ibid.* **43**, 2498 (1991); A. Silberfarb and I. H. Deutsch, *ibid.* **68**, 013817 (2003).
[14] P. Barberis-Blostein and N. Zagury, Phys. Rev. A **70**, 053827 (2004).
[15] A. Dantan, N. Treps, A. Bramati, and M. Pinard, Phys. Rev. Lett. **94**, 050502 (2005);
[16] C. L. Garrido-Alzar, L. S. Cruz, J. G. Aguirre-Gómez, M. Frana Santos, and P. Nusszenveig, Europhys. Lett. **61**, 485 (2003).
[17] D. Akamatsu, K. Akiba, and M. Kozuma, Phys. Rev. Lett. **92**, 203602 (2004).
[18] M. Kitawaga and M. Ueda, Phys. Rev. A **47**, 5138 (1993); D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, *ibid.* **50**, 67 (1994).
[19] U. V. Poulsen and K. Mølmer, Phys. Rev. Lett. **87**, 123601 (2001).
[20] J. H. Müller *et al.*, e-print quant-ph/0403138.
[21] A. Peng, M. Johnsson, and J. J. Hope, e-print quant-ph/0409183.