Nonlocality of cluster states of qubits

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(Received 25 May 2004; revised manuscript received 9 February 2005; published 18 April 2005)

We investigate cluster states of qubits with respect to their nonlocal properties. We demonstrate that a Greenberger-Horne-Zeilinger (GHZ) argument holds for any cluster state: more precisely, it holds for any partial, thence *mixed*, state of a small number of connected qubits (five, in the case of one-dimensional lattices). In addition, we derive a Bell inequality that is maximally violated by the four-qubit cluster state and is not violated by the four-qubit GHZ state.

DOI: 10.1103/PhysRevA.71.042325 PACS number(s): 03.67.Mn, 03.65.Ud

I. INTRODUCTION

In its most widespread image, a quantum computer is depicted by an array of initially uncorrelated qubits that pass through a network of logic gates in which they become entangled [1]. In 2001, Raussendorf and Briegel noticed that one can adopt a different philosophy and described the so-called *one-way quantum computer* [2]. In this view, the entanglement is distributed once for all by preparing a peculiar entangled state of all the qubits; the logic gates are then applied as sequences of only single-qubit measurements. Remarkable entanglement properties are needed to achieve this computational power: in particular, Greenberger-Horne-Zeilinger (GHZ) states, though in some sense maximally entangled, lack this power, and indeed can be simulated by a polynomial amount of communication [3]. Suitable N-qubit states for universal, scalable quantum computation are the so-called cluster states [4]. Motivated by this discovery, many works have been devoted in the last few years to the properties of those states [5]; links have been found in particular with error-correction theory [6]. In this paper we study the cluster states under the perspective of their nonlocality properties [7].

A brief review of the definition and main properties of the cluster states, following [2,4], is a necessary introduction. For convenience, through all this paper we adopt the notation $X=\sigma_x$, $Y=\sigma_y$, and $Z=\sigma_z$ for the Pauli matrices. A *cluster* is an N-site d-dimensional square lattice with connections among the sites that define the notion of neighborhood. For any site a of the lattice, one defines the operator

$$S_a = X_a \bigotimes_{b\in \mathrm{neigh}(a)} Z_b \tag{1}$$

where neigh(a) is the set of all the neighbors of a. The operators $\{S_a, a\in \text{lattice}\}$ form a complete family of commuting operators on the lattice; a *cluster state* is any of their common eigenvectors. We note here that this construction can actually be done for any graph, not only for square lattices, leading to the notion of *graph states*; but in this paper, we stick to cluster states and will discuss possible extensions to all graph states only in the final section. For definiteness, we consider the cluster state $|\Phi_N\rangle$ associated with all eigenvalues being $+1$, that is, determined by the family of equations

$$S_a|\Phi_N\rangle = |\Phi_N\rangle \quad \text{for all } a. \tag{2}$$

We start by considering cluster states built on a one-dimensional lattice [8], which we denote $|\phi_N\rangle$. For such a lattice, $|\phi_2\rangle$ and $|\phi_3\rangle$ are locally equivalent to a maximally entangled Bell state and to a GHZ state, respectively; the nonlocality of both has been thoroughly studied. The four-qubit cluster state reads

$$|\phi_4\rangle = \frac{1}{2}|+\rangle|0\rangle|+\rangle|0\rangle + \frac{1}{2}|+\rangle|0\rangle|-\rangle|1\rangle + \frac{1}{2}|-\rangle|1\rangle|-\rangle|0\rangle + \frac{1}{2}|-\rangle|1\rangle|+\rangle|1\rangle \tag{3}$$

where the one-qubit states are defined as usual as $Z|0\rangle=|0\rangle$, $Z|1\rangle=-|1\rangle$, and $X|\pm\rangle=\pm|\pm\rangle$. Note that the state $|\phi_4\rangle$ is not locally equivalent to the four-qubit GHZ state $|\mathrm{GHZ}_4\rangle=(1/\sqrt{2})(|0000\rangle+|1111\rangle)$, but to $1/2(|0000\rangle+|0011\rangle+|1100\rangle-|1111\rangle)$.

The plan of the paper is as follows. Section II is devoted to the nonlocality properties of $|\phi_4\rangle$: we identify a GHZ argument for nonlocality [9], from which in turn a Bell inequality can be derived. The advantage of this particular construction is that the inequality is optimized for the state $|\phi_4\rangle$: it acts as a witness discriminating between $|\phi_4\rangle$, which violates it up to the algebraic limit, and $|\mathrm{GHZ}_4\rangle$, which does not violate it at all. In Sec. III we generalize the GHZ argument to the N-qubit case for one-dimensional lattices; then in Sec. IV for d-dimensional lattices. In both cases, contrary to what happens for GHZ states, a GHZ argument for nonlocality can be found for the partial (thence *mixed*) states defined on small sets of connected qubits once all the others are traced out. The result is quite surprising, since it was commonly believed that the purity of quantum states was a necessary condition for all-or-nothing violations of local realism. Finally, in Sec. V we consider the larger family of graph states.

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II. NONLOCALITY OF THE FOUR-QUBIT STATE $|\phi_4\rangle$ ON A ONE-DIMENSIONAL CLUSTER

A. GHZ argument

The four-qubit cluster state $|\phi_4\rangle$ is defined by Eq. (2):

$$XZII = +1 \quad (E_1),$$

$$ZXZI = +1 \quad (E_2),$$

$$IZXZ = +1 \quad (E_3),$$

$$IIZX = +1 \quad (E_4). \tag{4}$$

These notations are shortcuts for $XZII|\phi_4\rangle = |\phi_4\rangle$, etc. Eleven similar equations can be obtained by multiplication using the algebra of Pauli matrices:

$$XIXZ = +1, \quad (E_1) \times (E_3), \tag{5}$$

$$ZYYZ = +1, \quad (E_2) \times (E_3), \tag{6}$$

$$XIYY = +1, \quad (E_1) \times (E_3) \times (E_4), \tag{7}$$

$$ZYXY = -1, \quad (E_2) \times (E_3) \times (E_4), \tag{8}$$

and seven others which we will not use explicitly in this paper, namely, $YYZI = XZZX = ZXIX = IZYY = YYIX = YXXY = +1$ and $YXYZ = -1$. The 15 properties can be read directly from $|\phi_4\rangle\langle\phi_4|$; they are associated with the operators that, together with the identity, form the Abelian group generated by $\{S_1, \ldots, S_4\}$, called the *stabilizer group*. Note how a minus sign arises from the multiplication of three consecutive equations, since in the common site one has $ZXZ = -X$.

Properties like (4)–(8) predict perfect correlations between the outcomes of *a priori* uncorrelated measurements on separated particles. In a classical world, once communication is prevented by realizing spacelike separated detections, correlations can arise only if the outcomes of each measurement on each particle are preestablished. In other words, a local variable is a list of 12 bits $\lambda = \{(x_k, y_k, z_k), k = 1,2,3,4\}$, where $x_1 \in \{-1, +1\}$ is the preestablished value of a measurement of X on qubit 1, and so on. The GHZ argument for nonlocality aims to show that no list $\lambda \in \{-1, +1\}^{\times 12}$ can account for all the 15 properties above. To verify this, one replaces each Pauli matrix with the corresponding preestablished value: all the properties are supposed to hold, but now they are written with ordinary numbers, whose algebra is commutative. Assuming commutativity, the multiplication of Eqs. (5)–(7) gives $z_1 y_2 x_3 y_4 = +1$, in contradiction with Eq. (8), which reads $z_1 y_2 x_3 y_4 = -1$. Therefore, no local variable λ can account for all the properties of the list. Of course, a similar argument could be worked out using others among the 15 conditions above.

All in all, by inspection one sees that local variables can account for 13 out of the 15 properties associated with commuting observables: e.g., using $+1$ as the preestablished value for all 12 measurement, one satisfies all properties but (8) and $(E_1) \times (E_2) \times (E_3)$, which reads $YXYZ = -1$. The same is true for the four-qubit GHZ state $|\mathrm{GHZ}_4\rangle$. This rapid argument would suggest that the nonlocality of the cluster state is after all not too different from that of the GHZ state. The rest of the paper will show that the opposite is true.

B. Bell-type inequality

The GHZ argument for nonlocality involves identifying properties that are satisfied with certainty. Thus, this approach strongly relies on the details of the state and is not suited for comparison between different states; nor can it incorporate the effect of noise in a simple way. Therefore, especially to deal with experimental results, it is convenient to introduce linear Bell inequalities.

The best-known inequality for four-qubit states is the Mermin-Ardehali-Belinski-Klyshko (MABK) [10] inequality $\mathcal{M}_4$. For the cluster state, after optimizing on the settings, one finds $\langle \mathcal{M}_4 \rangle_{\phi_4} = 2\sqrt{2}$ where the local-variable bound is set at 2. This is indeed a violation, but a rather small one: a two-qubit singlet attains this amount as well, and $|\mathrm{GHZ}_4\rangle$ reaches up to $4\sqrt{2}$. It is well known by now that the MABK inequality detects optimally GHZ-type nonlocality, but it can be beaten by other inequalities for other families of states [11].

In our case, it is natural to guess a Bell inequality out of the GHZ argument: one takes the very same four conditions (5)–(8) that have led to the GHZ argument, and writes a suitable linear combination of them. Specifically, on the one hand, the previous results imply that the Bell operator

$$\mathcal{B} = AIC'D + AICD' + A'BCD - A'BC'D' \tag{9}$$

reaches 4 when evaluated on $|\phi_4\rangle$ for the setting $A = X$, $A' = Z$, $B = Y$, $C = Y$, $C' = X$, $D = Z$, and $D' = Y$. This is the algebraic value; obviously no state can ever give a larger value (in particular, $|\phi_4\rangle$ is an eigenstate of $\mathcal{B}$ for these settings). On the other hand, the classical polynomial corresponding to $\mathcal{B}$ satisfies the inequality

$$|ac'd + acd' + a'bcd - a'bc'd'| \leqslant 2, \tag{10}$$

as one can verify either by direct check, or by grouping 1 and 2 together, thus recovering the polynomial that defines the three-party Mermin inequality [10,12]. The so-defined four-qubit Bell inequality cannot be formulated using only four-party correlation coefficients, thus it does not belong to the restricted set classified by Werner and Wolf and by Żukowski and Brukner [13]. Moreover, on particle 2 only one nontrivial setting is measured; in other words, no locality constraint is imposed on it [14].

Our inequality exhibits a remarkable feature: the GHZ state $|\mathrm{GHZ}_4\rangle$ does not violate it. The most elegant way to prove this statement consists in writing down explicitly the projector Q associated with $|\mathrm{GHZ}_4\rangle$: only terms with an even number of Pauli matrices appear. Consequently, $\mathrm{Tr}[Q(AIC'D)] = \mathrm{Tr}[Q(AICD')] = 0$ for any choice of the measurement directions, and so $\mathrm{Tr}(Q\mathcal{B}) = \mathrm{Tr}[Q(A'BCD)] - \mathrm{Tr}[Q(A'BC'D')]$ whose algebraic maximum is 2. We checked also our inequality on the state $|W_4\rangle = 1/2(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)$ and found numerically a violation $\langle \mathcal{B} \rangle_W \approx 2.618$. In conclusion, our specific derivation results in a Bell inequality which acts as a strong entanglement witness

for the cluster state $|\phi_4\rangle$: it is violated maximally by it, and is not violated at all by the four-qubit GHZ state [15].

III. GHZ ARGUMENT FOR THE N-QUBIT STATE $|\phi_N\rangle$ ON A ONE-DIMENSIONAL CLUSTER

The nonlocality of the four-qubit cluster state has been studied in full detail, starting from the expression of $|\phi_4\rangle$. With the insight gained there, we can move on to look for the nonlocality of the cluster state of an arbitrary number of qubits N, still defined on a one-dimensional lattice. We do not need to give $|\phi_N\rangle$ explicitly, but can work directly on the set (2) of N eigenvalue equations that define it:

$$\begin{aligned} XZIII\cdots I &= +1 \quad (E_1),\\ ZXZII\cdots I &= +1 \quad (E_2),\\ IZXZI\cdots I &= +1 \quad (E_3),\\ IIZXZ\cdots I &= +1 \quad (E_4),\\ &\vdots\\ II\cdots IZX &= +1 \quad (E_N). \end{aligned} \tag{11}$$

The GHZ argument appears out of these equations as follows. We focus on E_2, E_3, and E_4 first. Using the algebra of Pauli matrices, one derives the three properties

$$\begin{aligned} C_1 = E_2E_3: &\quad ZYYZI\cdots I = +1,\\ C_2 = E_3E_4: &\quad IZYYZ\cdots I = +1,\\ C_3 = E_2E_3E_4: &\quad ZYXYZ\cdots I = -1. \end{aligned} \tag{12}$$

Moving to the level of local variables (that is, using commutative multiplication), $E_3C_1C_2$ leads to $z_1y_2x_3y_4z_5=+1$, which manifestly contradicts C_3. The argument is absolutely identical using $\{E_k,E_{k+1},E_{k+2}\}$, for any $k\in\{2,\ldots,N-3\}$, because all the eigenvalue equations but the first and the last one are obtained from one another by translation; and it can be verified explicitly that it holds also for $k=1$ and $k=N-2$. In conclusion, we have shown that one can build the following GHZ argument on five qubits out of any three consecutive eigenvalue equations: (1) Take the three equations $\{E_k,E_{k+1},E_{k+2}\}$, for $k\in\{1,\ldots,N-2\}$; (2) with the algebra of Pauli matrices, define $C_1=E_kE_{k+1}$, $C_2=E_{k+1}E_{k+2}$, and $C_3=E_kE_{k+1}E_{k+2}$, this last property providing the needed minus sign; (3) with commutative algebra, the condition obtained as $C_1C_2E_{k+1}$ is exactly the opposite to C_3.

Let us focus again on the GHZ paradox using $\{E_2,E_3,E_4\}$ for definiteness: this paradox involves nontrivial operators only on qubits 1–5. This means that one can forget completely about the other $N-5$ qubits, that is, the partial state ρ_{12345} obtained by tracing out all the other qubits exhibits a GHZ-type nonlocality. This state is certainly *mixed* because $|\phi_N\rangle$ is not separable according to any partition. Since this is true for any translation, we conclude that any five-qubit partial state on consecutive qubits leads to a GHZ argument for nonlocality. The converse holds too: the GHZ argument works only for *consecutive* qubits [16]. In fact, to obtain the minus sign that is necessary for the GHZ argument, one has to multiply three equations that have nontrivial operators on a common site: a rapid glance at Eq. (11) shows that this can only be the case if the three equations are consecutive. This GHZ argument for mixed states recalls the notion of "persistency" [4]: one can measure many qubits, or even throw them away, and strong locality properties are not destroyed. Finally note that in the GHZ argument involving $\{E_k,E_{k+1},E_{k+2}\}$, particles $k-1$ and $k+3$ are only asked to measure Z: as we saw for the four-qubit state, on these particles we do not impose any locality constraint, but they must be asked for cooperation in order to retrieve the GHZ argument.

Finally, note that a Bell inequality can be derived from the GHZ argument as was done in Sec. II. On the five meaningful qubits, the Bell operator reads

$$\begin{aligned} \mathcal{B} = &(AB)C'(DE) + (A'B')C(DE) + (AB)C(D'E')\\ &- (A'B')C'(D'E') \end{aligned} \tag{13}$$

where we have grouped the terms in order to make explicit the analogy with Mermin's inequality [10]. The inequality for local variables reads $|\mathcal{B}|\leqslant 2$; partial states of a cluster state violate it up to the algebraic limit for $A=E=I$, $A'=E'=Z$, $B=D=Z$, $B'=D'=Y$, $C=Y$, and $C'=X$.

IV. GHZ ARGUMENT FOR CLUSTER STATES ON ANY-DIMENSIONAL CLUSTERS

As a last extension, we consider the nonlocality of a cluster state prepared on two- and higher-dimensional square lattices. It is clear why this problem is not immediately equivalent to the one we have just studied: the eigenvalue equations (2) do not have the same form as those for one-dimensional lattices (11), because the structure of the neighborhood is different. Consequently, the N-qubit cluster state on a two-dimensional lattice is different from the N-qubit cluster state on a one-dimensional lattice. Still, one expects similar properties to hold. Indeed, we provide a generalization of the GHZ argument for cluster states constructed on square lattices of any dimension.

As a case study, we consider the simplest two-dimensional square lattice, which is 3×3, because a 2×2 lattice is equivalent in terms of neighbors to a closed four-site one-dimensional loop and we have already solved that case implicitly [8]. The nine eigenvalue equations (E_{ij}), i, $j\in\{1,2,3\}$, can be written formally in a way reminiscent of the lattice:

$$\begin{pmatrix} X & Z & I\\ Z & I & I\\ I & I & I \end{pmatrix} = +1 \quad (E_{1,1}),$$

$$\begin{pmatrix} Z & X & Z\\ I & Z & I\\ I & I & I \end{pmatrix} = +1 \quad (E_{1,2}),$$

$$\vdots$$

$$\begin{pmatrix} I & Z & I \\ Z & X & Z \\ I & Z & I \end{pmatrix} = +1 \quad (E_{2,2}),$$

$$\vdots \qquad (14)$$

with obvious notations. The basic reasoning to find the GHZ argument is as before: one can find such an argument if and only if a minus sign can be produced, that is, if one takes at least three equations that lead to the product *ZXZ* in a site. In this case, the argument is constructed in a way similar to the case of one-dimensional lattices. For instance, if one takes $\{E_{1,1},E_{1,2},E_{2,2}\}$, then the *ZXZ* product can be found in site $(1,2)$ of the lattice. First, with the Pauli commutation relations, one builds $C_1=E_{1,1}E_{1,2}$, $C_2=E_{1,2}E_{2,2}$, and $C_3=E_{1,1}E_{1,2}E_{2,2}$, where the minus sign appears in C_3. Then, using commutative multiplication, one gets that $C_1C_2E_{1,2}$ is exactly the opposite property to that obtained directly from C_3. In this example, particles in sites (3, 1) and (3, 3) can be traced out because in all conditions the operator in those sites is the identity; and the four particles in sites (1, 3), (2, 1), (2, 3), and (3, 2) undergo a single measurement (Z) and are therefore there to help establish the argument.

In general, the following is easily checked.

(1) A GHZ argument can be obtained if the three sites form a neighbor-to-neighbor path, like $\{E_{1,1},E_{1,2},E_{2,2}\}$ or $\{E_{1,1},E_{1,2},E_{1,3}\}$; in this case, the argument goes as in the examples above.

(2) A GHZ argument cannot be obtained if the three sites do not form a neighbor-to-neighbor path, like $\{E_{1,1},E_{2,2},E_{3,3}\}$ or $\{E_{1,1},E_{1,2},E_{2,3}\}$.

With this characterization, it is obvious how to generalize the GHZ argument to larger two- and higher-dimensional clusters. Again, a Bell inequality can be derived from this GHZ argument, exactly as we did in the previous sections.

V. COMPARISION WITH GRAPH STATES

A. Extension of our results

Cluster states are members of a large family of states called graph states [17]. Graph states differ from one another according to the graph on which the state is built. Since the definition of the family of commuting operators on the graph is always (1), our techniques can be applied to study the nonlocality of any graph state. However, the specific results can be strongly dependent on the graph, which here was the regular lattice or cluster. For instance, N-qubit GHZ states are graph states, but—contrary to what has just been described for cluster states—no GHZ argument for their partial states can be found, because all the partial states are separable. This derives from the connectivity of the corresponding graph, in which all the sites are connected only through a single site a; therefore, the operator S_a must be used to find any GHZ argument, and this operator is nontrivial on all sites.

B. Comparison with systematic inequalities

Very recently, a systematic way of constructing Bell's inequalities for any graph state has been found [18]. The result is very elegant: the sum of the elements of the stabilizer group provides a Bell inequality. It is instructive to apply this formalism to the four-qubit cluster state, for which we have provided inequality (10) above. We have written explicitly the equations corresponding to each element of the stabilizer group at the beginning of Sec. II A; at the end of the same section, we have stressed that only 14 (13 nontrivial plus the identity) of the 16 equations can be satisfied in a local-variable theory. By definition, QM satisfies all the properties with the cluster state. Therefore we have a Bell-type inequality $\tilde{\mathcal{B}}_{\mathrm{LV}}\leqslant 14\times(+1)+2\times(-1)=12$ [19] for which QM reaches the value $\tilde{\mathcal{B}}_{\mathrm{QM}}=16$. This inequality uses three settings per qubit.

There is a strong link between this inequality and ours (10). By summing over all the stabilizers, the polynomial $\tilde{\mathcal{B}}$ contains the four terms of Eq. (9), the four terms that build the symmetric version of it (GHZ argument based on $YXYZ=-1$), and eight more terms. These additional terms turn out to be "innocuous" as far as local variables are concerned: thus, the violation of $\widetilde{\mathcal{B}}\leqslant 12$ is nothing but the simultaneous violation of (10) and its symmetric version. However, the two inequalities are not equivalent on all quantum states, as can be seen on the GHZ state $|\mathrm{GHZ}_4\rangle$ by the same argument as in Sec. II B: eight terms in the polynomial $\widetilde{\mathcal{B}}$ are products of three Pauli operators, so $\widetilde{\mathcal{B}}_{\mathrm{GHZ}}\leqslant 8$ (and the bound can actually be attained). So, for the inequality discussed in this section, the GHZ state cannot even reach the local-variable bound.

In summary, in the case of the four-qubit cluster state $|\phi_4\rangle$, the inequality built according to the recipe of Ref. [18] exploits the same nonlocality as our inequality (10). Note also that our inequality is easier to test experimentally because it requires fewer settings (two instead of three per qubit) and fewer terms (four instead of 15). However, when we apply our method to an arbitrary number N of qubits (see the end of Sec. III) we find an inequality whose violation is always by a factor of 2, irrespective of N; whereas the inequalities discussed in Ref. [18] are such that the violation increases with N.

VI. CONCLUSION

In conclusion, we have found that a rich nonlocality structure arises from the peculiar, highly useful entanglement of cluster states of qubits. This nonlocality is very different from the one of the GHZ states: the qualitative difference is most strikingly revealed by the existence of a GHZ argument for mixed states. In the four-qubit case, we have also provided a quantitative witness in terms of a Bell inequality, which will be an important tool in planned experiments to produce photonic cluster states [20].

ACKNOWLEDGMENTS

V.S. and A.A. acknowledge hospitality from the Institut für Experimentalphysik in Vienna, where this work was

done. We thank Nicolas Gisin, Barbara Kraus, Sofyan Iblisdir, and Lluis Masanes for insightful comments. This work was supported by the European Commission (RAMBOQ), the Swiss NCCR "Quantum Photonics," the Austrian Science Foundation (FWF), the Spanish MCYT under a "Ramón y Cajal" grant, and the Generalitat de Catalunya. M.A. acknowledges additional support from the Alexander von Humboldt Foundation.

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