Scheme for optical implementation of orbital angular momentum beam splitter of a light beam and its application in quantum information processing

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Optical beams bearing orbital angular momentum have been recently recognized as potential candidates for realizing D-dimensional quantum systems (qudits). In this paper, we propose an optical scheme to implement an orbital angular momentum beam splitter, which changes the outgoing direction with respect to the incoming direction while leaving the qudit state unchanged. Furthermore we demonstrate that such a beam splitter can be used to sort different orbital angular states of a single photon, create arbitrary superpositions of orbital angular momentum states, and implement a high-dimensional Bennett-Brassard 1984 protocol for quantum key distribution.

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Most research in quantum computation and quantum communication, such as quantum teleportation [1], quantum dense coding [2], and quantum cryptography [3], are based on two-level quantum systems (qubits). Recently there exists a steadily growing interest in using D-dimensional quantum systems ($D\geqslant 3$) to encode quantum information since it allows realization of new types of quantum communication protocols [4–7]. For example, it has been demonstrated that key distributions based on three-level quantum systems are more secure against eavesdropping than those based on two-level systems [4]. However, the challenge is the physical realization of D-dimensional quantum systems. Recently, considerable experimental efforts have been devoted to using the orbital angular momentum (OAM) of a single photon to encode high-dimensional quantum information. In Ref. [8], Zeilinger and co-workers demonstrated that an individual photon can be prepared in superpositions of the orbital angular momentum. In Refs. [9,10], these authors also reported generation of two-photon orbital angular momentum entanglement and observed the violation of a generalized Bell inequality of three-dimensional quantum systems. These experiments open up further possibilities for generation and manipulation of higher-dimensional quantum states and implementation of higher-dimensional quantum information processing protocols by using the orbital angular momentum of photons. In fact, a scheme has been proposed for constructing high-dimensional vector states of the orbital angular momentum of a single photon [11]. In Ref. [12], several theoretical works have been done on preparation of entanglement of orbital angular momentum for the signal and idler beams in parametric down-conversion. In Ref. [13], an interferometric method was proposed for measuring the orbital angular momentum of a single photon. More recently, a scheme was proposed for distilling entangled states of photons carrying orbital angular momentum [14]. In this scheme, the authors introduced an orbital angular momentum beam splitter as the basic constructive element. The orbital angular momentum beam splitter is a device of D input ports and D output ports, whose action on the input state $|l\rangle_i$ along the input port i is defined as follows:

$$T_D|l\rangle_i = |l\rangle_{l\ominus i}, \quad i,l = 0,1,2,\ldots,D-1, \tag{1}$$

where $\ominus$ denotes subtraction modulus D and $|l\rangle$ is a single-photon state of the Laguerre-Gaussian (LG) mode LG_{0l}, which can be written as

$$|l\rangle = \int dr\, u_0^l(r)a^\dagger(r)|0\rangle \tag{2}$$

where r is the radial coordinate in the transverse X-Y plane and the normalized Laguerre-Gaussian modes in polar coordinates are given by

$$u_p^l(\rho,\varphi) = \sqrt{\frac{2p!}{\pi(|l|+p)!}}\frac{1}{w}\left(\frac{\sqrt{2}\rho}{w}\right)^{|l|}L_P^{|l|}\left(\frac{2\rho^2}{w^2}\right)e^{-\rho^2/w^2}e^{-il\varphi} \tag{3}$$

where the Z-dependent phase was omitted and $L_p^{|l|}$ denotes the associated Laguerre polynominal. The index l is referred to as the winding number and p is the number of nonaxial radial nodes. The scheme is independent of the radial index p; for convenience, we consider only the case of $p=0$. The customary Gaussian mode can be viewed as a LG mode with $l=0$. LG beams with an index l carry an orbital angular momentum of $l\hbar$ per photon. From Eq. (1), it is seen that the orbital angular momentum beam splitter is the analog of the polarization beam splitter in that it can select the optical path on the basis of orbital angular momentum, one path for each of the distinguishable states. However, a problem is left unsolved: how to realize such an orbital angular momentum beam splitter. The main aim of the present paper is to describe an interferometric method to implement the orbital angular momentum beam splitter (1). As applications, we show how to employ such a beam splitter to measure different orbital angular states of a single photon, create an arbitrary superposition of orbital angular momentum states, and implement the high-dimensional Bennett-Brassard 1984 (BB84) protocol for quantum key distribution.

We now present a scheme to realize the orbital angular momentum beam splitter. The experimental setup is depicted in Fig. 1, which consists of two symmetric D-port devices F_a,F_b and $D-1$ Dove prisms. An extended introduction to

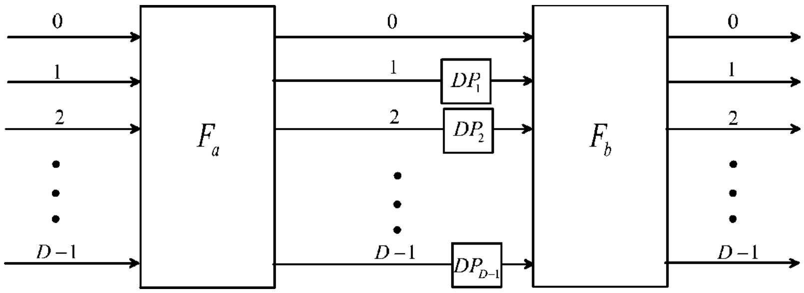

FIG. 1. The schematic shows the implementation of the orbital angular momentum beam splitter. F_a and F_b are two symmetric multiport devices and DP_i denotes the the Dove prism.

the symmetric multiport device is given in Ref. [15]. The action of the symmetric D-port device can be described by the unitary operator U_D, which transforms the creation operator $a_m^\dagger$ in the input port m as follows:

$$U^D a_m^\dagger U^{D\dagger} = \sum_{n=0}^{D-1} U_{m,n}^D a_n^\dagger \tag{4}$$

where

$$U_{m,n}^D = \frac{1}{\sqrt{D}} \exp\left(\frac{i2\pi mn}{D}\right), \tag{5}$$

which gives the probability amplitude for a single photon entering via the input port m and leaving the device by the output port n $(m,n=0,\ldots,D-1)$. Reck *et al.*[16] have shown that it is possible to construct a multiport device from mirrors, beam splitters, and phase shifters that will transform the input modes into the output modes in accord with any $D \times D$ unitary matrix. The action of the Dove prism is to flip the transverse cross section of any transmitted beam [13]. If the incident mode is a LG_{0l} mode which is of the form $\exp(-il\varphi)$, the Dove prism rotates a passing beam through an angle α, and the phase dependence of the transverse mode of the transmitted beam becomes $\exp[-il(\varphi+\alpha)]$. This demonstrates that the Dove prism can act as a phase shift corresponding to the LG_{0l} mode.

Now we present a detailed analysis of the proposed scheme shown in Fig. 1. We assume that the light beam of the LG_{0l} mode is emitted into the input port m of the symmetric D-port device F_a, and other input modes are in the vacuum states. After these modes pass through the device F_a, the state of the system becomes

$$\int dr\, u_0^l(r) \sum_{n=0}^{D-1} U_{m,n}^D a_n^\dagger(r)|0\rangle. \tag{6}$$

Then the light beam in the nth arm is rotated by the Dove prism with the angle α_n. Here we choose the angles to satisfy $\alpha_n = 2n\pi/D$ and the state of the system is evolved into

$$\int dr\, u_0^l(r) \sum_{n=0}^{D-1} U_{m,n}^D \exp\left(-i\frac{2nl\pi}{D}\right) a_n^\dagger(r)|0\rangle$$
$$= \int dr\, u_0^l(r) \sum_{n=0}^{D-1} U_{m\ominus l,n}^D a_n^\dagger(r)|0\rangle. \tag{7}$$

After passing through the second symmetric device F_b, the photon with the orbital angular momentum l appears in the output port $l \ominus m$. This transformation demonstrates that the proposed setup, which is shown in Fig. 1, definitely implements the orbital angular momentum beam splitter (1). It should be pointed out that the implementation of the scheme relies on the possibility of realizing the symmetric multiport device with simple optical elements. In Ref. [16], Reck *et al.* provide an algorithm to use a triangular array of standard beam splitters, phase shifters, and mirrors to realize any $D \times D$ unitary matrix. In this approach, each diagonal row in the triangle performs a transformation reducing the effective dimension of the Hilbert space by 1 [16]. The number of beam splitters needed for a general triangular array increases quadratically with D. The simplest symmetric multiport device is a symmetric beam splitter. In Fig. 2, we show how to realize a symmetric three-port device with optical elements. The optical implementation of the symmetric four- and five-port devices is shown in Refs. [17,18].

Since the orbital angular beam splitter is the analog of the polarization beam splitter, it may find a wide range of applications in the generation and manipulation of quantum states and implementation of quantum-information-processing protocols. In Ref. [14], such a beam splitter has been used to distill quantum entanglement of higher-dimensional quantum systems. Here we show other applications. From Eq. (1), it is easy to see that such a beam splitter can be used to distinguish individual photons in arbitrary orbital angular states. Assume that a single photon with unknown orbital angular momentum is emitted into the input port 0 of the setup shown in Fig. 1. Based on Eq. (1), we can assure that photons with orbital angular momentum l appear in the output port l. In Ref. [13], a scheme has been proposed for measuring the orbital angular momentum of a single photon. In this scheme, additional holograms are needed to sort an arbitrary orbital angular momentum. In the absence of holograms, the scheme can only be constructed to sort beams where the orbital angular momentum takes on the values of 0 or 2^n,

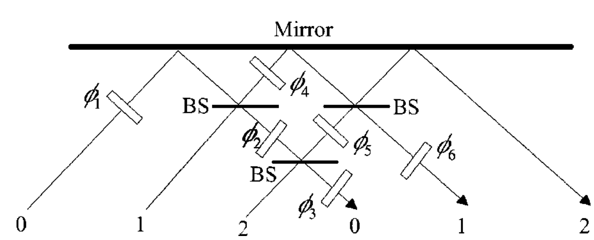

FIG. 2. Triangular array for symmetric three-port device. The lowermost beam splitter has a reflectivity $R=1/3$; for the other two $R=1/2$. The phase shifters are chosen as $\varphi_1=\varphi_6=2\varphi_2=-2\pi/3$, $\varphi_4=2\varphi_3=\pi$, and $\varphi_5=\pi/6$.

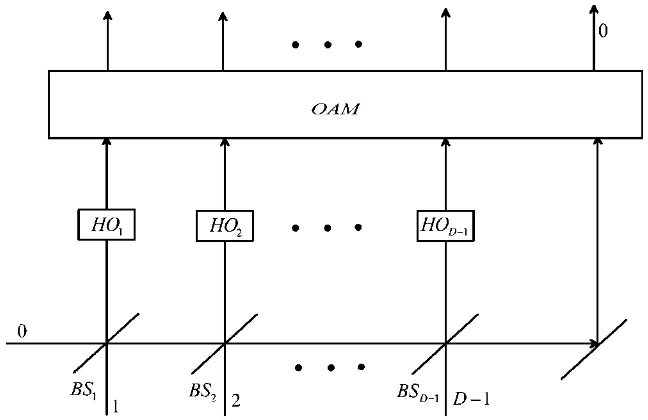

FIG. 3. The schematic shows the generation of arbitrary superpositions of the orbital angular momentum states. BS_i denotes the beam splitter and HO_i denotes the hologram. OAM denotes the orbital angular momentum beam splitter, as shown in Fig. 1.

where n is an integer. Compared with this scheme, our scheme does not need an additional hologram for sorting the arbitrary orbital angular momentum.

Next we demonstrate how to prepare a single photon in an arbitrary superposition of orbital angular momentum states. The experimental setup is shown in Fig. 3. A single photon of the Gaussian mode (LG_{00} mode) is emitted into a series of beam splitters. The other input modes of these beam splitters are in the vacuum states. If the parameter of the beam splitter BS_i is chosen to satisfy

$$\tan \theta_j e^{i\varphi_j} = C_j \Big/ \sqrt{\sum_{n=0}^{j-1} |C_j|^2} \tag{8}$$

we can prepare an arbitrary superposition of optical paths $\Sigma_{j=0}^{D-1} C_j |0\rangle_j$. Then the beam of the jth path is sent through a hologram, which transforms the incoming LG_{00} mode into the LG_{0j} mode, and we obtain

$$\sum_{j=0}^{N-1} C_j |j\rangle_j. \tag{9}$$

Finally, the beam of the j path is emitted into the ith input port of the orbital angular momentum beam splitter. Based on Eq. (1), we can assure that an arbitrary superposition of orbital angular momentum states appears in output port 0 of the setup. In principle, a scheme has been proposed for the same purpose. However, the scheme required numerically solving the inverse problem, and a definite solving scheme is not given for arbitrary N. In the present scheme, the superposition of arbitrary amplitudes and relative phases can be produced by appropriately choosing the parameters of the beam splitters.

Finally we demonstrate how to realize high-dimensional generation of the BB84 protocol for quantum key distribution by using the orbital angular momentum. The BB84 protocol is briefly reviewed as follows [4]. The scheme consists in using two mutually unbiased bases. The sender Alice sends a basis state in one of these two bases chosen at random, while the receiver Bob makes a measurement in one of these two bases, again at random. The basis used by each party is subsequently disclosed on the public channel, so that Alice and Bob obtain correlated random variables if they used the same bases and there was no disturbance on the channel. The use of mutually unbiased bases implies that if Alice and Bob use different bases, Bob's measurement yields a random number that is uncorrelated with Alice's state. The raw secret key is then made out of the correlated data. This procedure ensures that any attempt by an eavesdropper to gain information on Alice's state induces errors in the transmission, which can be detected by the legitimate parties. To implement the D-dimensional BB84 protocol, we choose two mutually unbiased bases as follows:

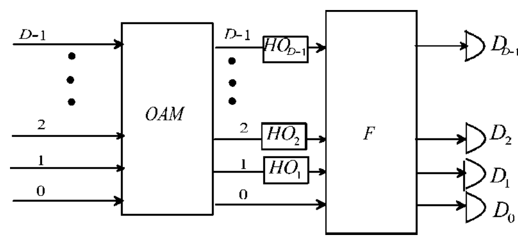

FIG. 4. The schematic shows the measurement of the states in the base B_2. OAM denotes the orbital angular momentum beam splitter, as shown in Fig. 1. F is the symmetric multiport devices; HO_i denotes the hologram.

$$B_1 = \{|k\rangle, k = 0,1,2,\ldots,D-1\} \tag{10}$$

and

$$B_2 = \left\{ |\overline{k}\rangle = \frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} e^{i2\pi nk/D} |n\rangle, \quad k = 0,1,2,\ldots,D-1 \right\} \tag{11}$$

where $|k\rangle$ is a single-photon state of the LG_{0k} mode. Thus, to implement the D-dimensional BB84 protocol, we have to be able to generate and measure the states in these two bases. Since the orbital angular momentum beam splitter can distinguish the states in the base B_1, and the experimental setup shown in Fig. 2 can prepare a single photon in an arbitrary superposition of orbital angular momentum states, the work to be done is how to distinguish the states in the base B_2. The experimental setup is shown in Fig. 4. Assume that the unknown single-photon state $|\overline{k}\rangle$ is emitted into the input port 0 of the OAM. After passing through the OAM, the state of the system becomes

$$\frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} e^{i2\pi nk/D} |n\rangle_n. \tag{12}$$

Then, the output modes n of the OAM are sent through holograms, which transform the incoming LG_{0n} mode into the LG_{00} mode $(n=1,2,\ldots,D-1)$. Thus the state of the total system becomes

$$\frac{1}{\sqrt{D}}\sum_{n=0}^{D-1} e^{i2\pi nk/D}|0\rangle_n. \tag{13}$$

Now we send the modes k into a symmetric D-port device; the state of the system becomes $|0\rangle_k$. Therefore we can assure that the photon in the state $|\overline{k}\rangle$ appears in the output port k. Thus by employing the orbital angular momentum beam splitter, we can implement the D-dimensional BB84 protocol.

In summary, we have proposed a scheme to implement an orbital angular momentum beam splitter, which is the analog of the polarization beam splitter in that it can select the optical path on the basis of orbital angular momentum, one path for each of the distinguishable states. As some applications of such a beam splitter, we show how to sort different orbital angular states of a single photon, create an arbitrary superposition of orbital angular momentum states, and implement a D-dimensional quantum key distribution.

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