Quantum-state transfer in imperfect artificial spin networks

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High-fidelity quantum computation and quantum state transfer are possible in short spin chains. We exploit a system based on a dispersive qubit-boson interaction to mimic *XY* coupling. In this model, the usually assumed nearest-neighbor coupling is no longer valid: all the qubits are mutually coupled. We analyze the performances of our model for quantum state transfer showing how preengineered coupling rates allow for nearly optimal state transfer. We address a setup of superconducting qubits coupled to a microstrip cavity in which our analysis may be applied.

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I. INTRODUCTION

Many protocols for quantum-information processing (QIP) assume an arbitrary amount of control over the evolving system. However, any external influence may introduce errors and decoherence. Moreover, having a fine single-qubit control over a large register may be hard, especially for closely spaced subsystems. This has motivated some recent proposals in order to reduce the amount of the control over a quantum computer [1]. A way to achieve control-limited QIP is given by the engineering of interactions specifically designed to accomplish a prefixed task and a process can be formulated according to the program: initialization of the register, evolution by the designed interactions, and measurement (with no other interference on the dynamics).

Looking for a scenario suitable for control-limited evolutions, quantum spin systems represent promising candidates. Spin systems are intrinsically interesting because quantum phase transitions [2] and their relations with entanglement [3] can be studied. Very recently, it has been recognized that Heisenberg and *XY* interactions with nearest-neighbor couplings can be used to transfer a quantum state through a network of qubits, virtually without external control [4–6]. For *XY* couplings, linear chains of three nearest-neighbor coupled qubits achieve a perfect transfer fidelity and can be used as building blocks for a longer communication wire. This strategy is useful in those situations where the use of a photonic bus (conventionally accepted as a good information carrier) is not easy or convenient. It is worth stressing that this approach to quantum state transfer is radically different from an architecture of the transfer process in terms, for instance, of concatenated SWAP gates. The requirement of local control, which is necessary in order to realize such a quantum circuit, is abandoned in favor of a *global* interaction that collectively involves the element of a register. This change of perspective turns out to be successfully exploitable in other contexts. Indeed, a three-qubit chain of its *XY* coupling allows for universal quantum computation [7] and quantum cloning is possible in many-qubit networks connected through Heisenberg and *XY* coupling [8].

In this paper, we address the problem of quantum state transfer through an *XY* coupling model engineered via the off-resonant interaction of a group of qubits with a common bus. Our motivations are manifold. First of all, in our model the nearest-neighbor restriction is naturally relaxed. We consider a configuration of couplings which can be efficiently described in terms of complete graphs. We study the efficiency of a state transfer process in these multi-qubit interactions showing that, for uniform coupling distributions, the effectiveness of quantum state transfer can be lost. However, by properly designing the coupling rate between only the first and last physical qubits, the transfer fidelity in the chain is restored to a nearly optimal value. Under proper conditions, once we set the strengths of the inter-qubit couplings in a chain of N elements, the quantum state to be transferred can be collected at specific times at any of the $N-k$ qubits of the chain ($k=0,1,\ldots,N-2$). This can be more advantageous than strategies that bypass the chain and connects directly the sender qubit to the receiver. In this latter case, the couplings have to be redesigned each time the receiver's position is changed.

On the other hand, the appealing possibilities offered by even simple networks motivate the research for practical systems in which qubit-qubit interactions with possibly tunable couplings can be realized. As we stress in this paper, this is the case of an *XY* coupling simulated by our dispersive qubits-boson interaction. The insight we gain through this kind of simulation under controllable conditions could then be applied to real systems of solid-state physics where the amount of control is in general smaller. A setup is addressed combining quantum optics and superconducting quantum interference devices (SQUIDs) [9] that can embody our model. This setting is chosen in light of the possibility of achieving the strong-coupling regime and because of the advantages of the fixed positions of the qubits in the cavity and the long lifetime achievable in a high-quality factor (high-Q) cavity. These features have been recently exploited for solid-state–quantum-optics interfaces [10].

The paper is organized as follows. In Sec. II, we set the scenario in which our study is developed. We describe the general conditions under which the adiabatic evolution of a group of qubits, induced by an off-resonant photonic bus, can be derived. An effective *XY* model in a completely con-

nected graph is obtained. The features characterizing the distribution of coupling rates in the qubit network are singled out. Section III is devoted to the description of the quantum state transfer process in the specific case of the Hamiltonian model derived in Sec. II. The efficiency of the state transfer is evaluated for the case of a uniform distribution of couplings and as a function of the dimension of the network. We demonstrate that the effectiveness of the process is lost soon, in this condition. We describe in detail a strategy that allows for the restoration of the transfer efficiency by simply engineering the coupling of the first and last qubits in the chain. In Sec. IV, we analyze the details of the physical setting, based on the interaction between SQUIDs and a strip-line cavity mode, where our proposal can be implemented. Our results are summarized in Sec. V.

II. THE SYSTEM

We consider $N \geqslant 2$ qubits placed inside a cavity providing a single boson mode. We assume that, via an external potential Φ, we can modulate the transition energy $E_{q,i}$ of the qubits. This is possible in many QIP devices such as trapped ions, neutral atoms (through Stark fields), and SQUID-based systems (via a magnetic flux modulating the Josephson energy [9]).

The free Hamiltonian of the qubit system is $\Sigma_{i=1}^{N}\hat{H}_i = (1/2)\Sigma_{i=1}^{N}E_{q,i}(\Phi)\hat{\sigma}_i^z$, where $\hat{\sigma}_i^z$ is the z-Pauli operator of the ith qubit. Its eigenstates $\{|\pm\rangle\}_i$ are the basis for the ith qubit. The dependence of the qubit energies on Φ is explicitly shown. In many cases, the single addressing of the qubits in a register is a difficult task. We thus assume that Φ acts collectively on the qubits that are closely spaced in the cavity. The cavity field mode of its frequency ω_a, described by the annihilation (creation) operator $\hat{a}(\hat{a}^\dagger)$, is coupled to the qubits. The wavelength λ of the field is taken much longer than the dimension d of each qubit and their separation so that any dependence on the position in the cavity is neglected. This assumption can be relaxed if necessary. We consider the generally valid field-qubit interaction model

$$\hat{H} = \sum_{i=1}^{N}\hat{H}_{i,a} = \sum_{i}\Omega_i(\hat{a}^\dagger + \hat{a})(\hat{\sigma}_i^- + \hat{\sigma}_i^+) \quad (\hbar = 1), \tag{1}$$

where we assume the Rabi frequency Ω_i depends on a dimensionless parameter η_i which can be designed qubit by qubit and set once for all. This is in the spirit of programming the system to accomplish a given task without interferences to its evolution. We have introduced the operators $\hat{\sigma}_i^+ = (\hat{\sigma}_i^-)^\dagger = |+\rangle_i\langle -|$. Altogether, the dynamics of the system is given by $\hat{H}_{tot} = \Sigma_{i=1}^{N}\hat{H}_i + \hat{H}_a + \Sigma_{i=1}^{N}\hat{H}_{i,a}$ with $\hat{H}_a = \omega_a\hat{a}^\dagger\hat{a}$ the free energy of the field mode. Motivated by the recent achievement of high-Q cavities (planar strip-line cavities, which are characterized by quality factors larger than 10^4, are relevant to this work; experimentally, strip-line cavities with $Q \simeq 10^6$ have already been produced [12]) and the strong-coupling regime in some systems that can embody our model [10,11], we start without considering dissipation. The main sources of decoherence in our proposal will be addressed later.

We take $\Omega_i \ll \omega_a$, $E_{q,i}$ under the rotating-wave approximation (RWA). In the interaction picture and with the tunable detunings $\delta_i = \omega_a - E_{q,i}(\Phi)$, the Hamiltonian is $\hat{H}(t) = \Sigma_{i=1}^{N}\Omega_i\hat{a}^\dagger\hat{\sigma}_i^- e^{i\delta_i t} + \text{H.c.}$ If $\delta_i \gg \Omega_i$, the qubits act as a dispersive intracavity medium that changes ω_a according to $\omega_a \rightarrow \omega_a + \Sigma_i\Omega_i^2/\delta_i$. No real energy exchange is possible, in this case, which allows us to eliminate the bosonic mode from the dynamics of the qubits. Using standard techniques for adiabatic elimination [13], the interaction between the qubits is ruled by an effective time-averaged Hamiltonian which is obtained through iterative formal integrations of the corresponding Schrödinger equation. Stopping the iteration at the second step, which is justified under the above assumptions of $\Omega_i/E_{q,i} \ll 1$, the required adiabatic Hamiltonian can be formally written as $\hat{H}_e = -i\hat{H}(t)\int d\tau\hat{H}(\tau)|_{\tau=t}$, where the integral has to be considered indefinite and then evaluated at $\tau = t$ [13]. Working out this expression, we eventually find the generalized XY model

$$\hat{H}_e \simeq \sum_{i<j} 2x_{ij}(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+) = \sum_{i<j} x_{ij}(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y) \tag{2}$$

with $x_{ij} = \Omega_i\Omega_j/2\delta_j$ and the sum runs over the qubits having $\delta_i = \delta_j$ [14]. In order to get this equation, highly oscillating terms (at frequency δ_i or higher) have been discarded. Their contribution, mediated over the long evolution time scale determined by the coupling rates x_{ij}, is indeed negligible compared to the contribution by the time-independent terms appearing in Eq. (2). Similar adiabatic interactions were studied elsewhere (see [15] for some recent examples). In particular, Biswas and Agarwal considered a quantum state transfer in a particle chain. For longer chains, however, their proposal turns out to be experimentally quite demanding [15]. Christandl *et al.* [5] considered an XY model showing that, in the case of all equal couplings, perfect state transfer is possible in linear chains of two and three qubits with nearest-neighbor couplings [17] or between the antipodes of a hypercube.

Equation (2) is a long-range interaction which, in general, mutually connects all the qubits in a network. In this paper, we study the effect of the redundant connections in $\hat{H}_e$ on the efficiency of quantum state transfer.

III. ENGINEERED STATE TRANSFER

We denote $|\underline{i}\rangle = (\otimes_{k\neq i}^{N}|-\rangle_k)\otimes|+\rangle_i$ for a state where only the ith qubit is in $|+\rangle_i$ and $\otimes$ denotes tensorial product. It is easy to verify that $[\hat{H}_e, \Sigma_{i=1}^{N}\hat{\sigma}_i^z] = 0$, so that the number ℓ of qubits in the excited state represents a *constant of motion* in this system. The Hilbert space is partitioned into subspaces (or sectors) labeled by ℓ, and $\hat{H}_e$ can never couple states of the total qubit system belonging to sectors having different ℓ. Thus, the Hamiltonian can be diagonalized within each subspace. In particular, the subspace with $\ell = 0$ (i.e., all the qubits in the $|-\rangle$ state) is spanned by $|\underline{0}\rangle$. The dynamic of an arbitrary state with just one excited qubit is instead confined in the subspace with $\ell = 1$, spanned by the orthonormal basis $\{|\underline{i}\rangle\}$ $(i = 1, \ldots, N)$. With the decomposition $\hat{U}_\ell(t) = e^{-i\hat{H}_{e,\ell}t}$

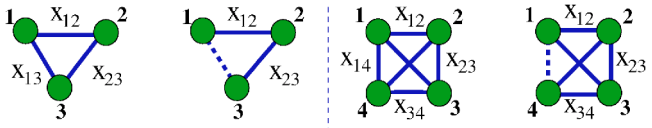

FIG. 1. Coupling configuration for $N=3$, 4. If the qubits are connected by all equal coupling rates, the systems are equivalent to triangular and square clusters that can be represented by three- and four-vertex complete graphs [panels (a) and (c)]. In this plot, each edge represents a coupling. If one connection is broken by reducing its coupling rate, the system becomes topologically equivalent to a two-qubit chain (b) or, generally, to a superposition of inequivalent chains (d).

$=\sum_{k=0}^{n_s-1} e^{-i\epsilon_\ell^k t}|\psi_\ell^k\rangle\langle\psi_\ell^k|$, we calculate the evolved state of the network. Here, $\hat{H}_{e,\ell}$ is the restriction of Eq. (2) to the n_s-dimensional subspace with a given ℓ, and ϵ_ℓ^k is the eigenvalue corresponding to the eigenstate $|\psi_\ell^k\rangle$.

Let the initial state $|in\rangle$ of the network have the qubit labeled 1 in a superposition of $|-\rangle_1$ and $|+\rangle_1$ while all the other qubits are in $|-\rangle$, i.e., $|in\rangle=\beta|\underline{0}\rangle+\gamma|\underline{1}\rangle$. By choosing an input state with just one excited qubit, we automatically select the subspaces with $\ell=0$ and $\ell=1$ as those relevant to the evolution of the entire qubit network. This state evolves as

$$|in\rangle \to \beta|\underline{0}\rangle + \gamma\sum_{k=0}^{N-1} e^{-i\epsilon_1^k t}p_{k\underline{1}}|\psi_1^k\rangle \tag{3}$$

with the projections $p_{k\underline{1}}=\langle\psi_1^k|\underline{1}\rangle$. For quantum state transfer, we are interested in the transition amplitude $\langle\underline{N}|\hat{U}_1(t)|\underline{1}\rangle=\sum_{k=0}^{N-1}e^{-i\epsilon_1^k t}p_{k\underline{1}}p_{k\underline{N}}^*$, whose square modulus gives the probability for the process $|+\rangle_1\to|+\rangle_N$ to occur. In general, the Nth qubit at a time t is in a mixed state,

$$\rho_N = Tr_{12\cdots N-1}[\hat{U}(t)|in\rangle\langle in|\hat{U}^\dagger(t)]. \tag{4}$$

The fidelity of the process is defined as the quantity $\mathcal{F}(\beta,\gamma)=(\langle-|\beta^*+\langle+|\gamma^*)\rho_N(\beta|-\rangle+\gamma|+\rangle)$. To cancel dependences on the initial state, we average $\mathcal{F}(\beta,\gamma)$ over the surface of the Bloch sphere as $\overline{\mathcal{F}}=(1/4\pi)\int\mathcal{F}(\beta,\gamma)d\Sigma$ with $d\Sigma$ the surface element.

We start with a simple case that captures the spirit of this study. The topology of a network depends crucially on the configuration of the couplings x_{ij} in Eq. (2). In general, for equal δ_i's and Ω_i's, the coupled qubits form a graph of connected vertices. In Fig. 1, we show the cases of $N=3$, 4 which result in a triangular and a squared cluster, respectively [Figs. 1(a) and 1(c)]. Each vertex and solid line represent a qubit and a nonzero coupling, respectively. The natural question raised here is the performance of the state transfer in this network of connections. To analyze it, we explicitly solve the problem formulated above, considering just the relevant subspace with $\ell=1$ but for a generic number of qubits. For simplicity, we set $x_{ij}=x$. We note $n_s=N$ and find that $\hat{H}_{e,1}$ admits $N-1$ degenerate eigenvalues $\epsilon_1^k=-x(k=0,\ldots,N-2)$ and the eigenvalue $\epsilon_1^{N-1}=(N-1)x$ completes the spectrum.

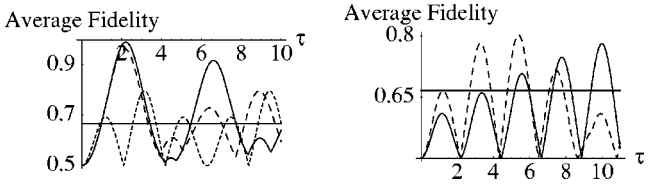

FIG. 2. (a): $\overline{\mathcal{F}}$ against the rescaled time τ for $N=3$. The straight line is the bound for classical transfer. We show $\overline{\mathcal{F}}$ for the *all-equal couplings* case (dotted line) and the *engineered* cases with $f=10$ (solid line), $f=5$ (dashed line). (b) $\overline{\mathcal{F}}$ against τ for the $1\to2$ transfer process for $f=1.1$ (dotted line) and $f=5$ (solid line).

The degenerate eigenspace is diagonalizable and an orthogonal basis can be built through the Gram-Schmidt algorithm. We get

$$\hat{U}_1(t) = e^{ixt}\sum_{k=0}^{N-2}|\psi_1^k\rangle\langle\psi_1^k| + e^{-i(N-1)xt}|\psi_1^{N-1}\rangle\langle\psi_1^{N-1}| \tag{5}$$

with $|\psi_1^{N-1}\rangle=(1/\sqrt{N})\Sigma_{i=1}^N|\underline{i}\rangle$. The transfer probability is then $|\langle\underline{N}|\hat{U}_1(\tau)|\underline{1}\rangle|^2=(2/N^2)[1-\cos(N\tau)]$, where $\tau=xt$ is the rescaled interaction time. The resulting average fidelity $\overline{\mathcal{F}}$ is plotted against τ in Fig. 2 (dotted line). The maximum of this function has to be contrasted with 2/3, the best fidelity achievable for the transfer of a qubit state through a classical channel [16]. It is clear that $\overline{\mathcal{F}}$ may be higher than the classical limit. However, trying to extend the system, we find that already for $N=4$, $\overline{\mathcal{F}}_{max}\simeq 2/3$ and the quantum channel becomes useless. For $N=3$, the unwanted coupling x_{13} does not compromise the state transfer, even if the fidelity is not optimal. However, a strategy to enlarge the range of N for which this quantum process is still worthy is desirable. An intuitive approach is to get rid of the redundant connections. For example, as shown in Figs. 1(b) and 1(d), by cutting the coupling $1\leftrightarrow N$, the clusters become equivalent to a chain of three qubits (b) and to a superposition of elementary chains [three elementary and inequivalent paths connecting 1 to 4 can be found in Fig. 1(d), namely the paths $1\to2\to4$, $1\to3\to4$, and $1\to3\to2\to4$. Any other path can be built by superimposing these three].

The cut may be realized by properly setting *ab initio* the Rabi frequencies Ω_j's. This is possible by an appropriate choice of each η_i defined to be related to Ω_i in Eq. (1). For $N=3$, for example, we take $\Omega_1=\Omega_3\ll\Omega_2$ to get $x_{12}=x_{23}\gg x_{13}$. This reduces the complexity of the network to a three-qubit chain. We set $x_{12,23}=x$ and $x_{13}=x/f$, with $f>1$. The decomposition of $\hat{H}_{e,1}$ can be found analytically with the eigenvalues $\epsilon_1^0=-x/f$, $\epsilon_1^{1,2}=(x/2f)(1\mp\sqrt{1+8f^2})$. They correspond to the states $|\psi_1^0\rangle=(1/\sqrt{2})(-|\underline{1}\rangle+|\underline{3}\rangle)$ and $|\psi_1^{1,2}\rangle=\mathcal{N}_{1,2}[|\underline{1}\rangle-(\epsilon_1^{2,1}/x)|\underline{2}\rangle+|\underline{3}\rangle]$ with the normalizations $\mathcal{N}_{1,2}=x[2x^2+(\epsilon_1^{2,1})^2]^{-1/2}$. We find $\langle\underline{3}|\hat{U}_1(t)|\underline{1}\rangle=-(1/2)e^{ixt/f}+\mathcal{N}_1^2e^{-i\epsilon_1^1 t}+\mathcal{N}_2^2e^{-i\epsilon_1^2 t}$, which reveals the competition between the different paths the system can follow from the sender to the receiver: the path connecting 1 to 3 via x_{13} and the one through $|\underline{2}\rangle\propto\mathcal{N}_2^{-1}|\psi_1^2\rangle-\mathcal{N}_1^{-1}|\psi_1^1\rangle$. The transfer fidelity

$\mathcal{F}(0,1) \equiv |\langle \underline{3}|\hat{U}_1(t)|\underline{1}\rangle|^2$ resulting from this interference effect is

$$\mathcal{F}(0,1) = \frac{1}{4} + 2\mathcal{N}_1^2\mathcal{N}_2^2 \cos(\Delta\epsilon_1^{21}t) + \sum_{i=1}^{2} \mathcal{N}_i^4 - \mathcal{N}_i^2 \cos(\Delta\epsilon_1^{i0}t) \tag{6}$$

with $\Delta\epsilon_1^{ij} = \epsilon_1^i - \epsilon_1^j (i,j=0,1,2)$. Equation (6) is maximized when $\Delta\epsilon_1^{21}t = 2\pi$, with $\Delta\epsilon_1^{20}t = \pi$ and $\Delta\epsilon_1^{10}t = -\pi (k \geq 1)$, which correspond to the case $f \to \infty$. In this case, the cut we have operated is perfect and the configuration reduces to a chain of three nearest-neighbor coupled qubits. For this coupling-engineered system, it is $\mathcal{F}(0,1) \simeq 0.973$ for $f=10$ and $\mathcal{F}(0,1) \simeq 0.898$ for $f=5$ at $\tau \simeq 2f\pi/\sqrt{1+8f^2}$. As time goes by, the interferences lead to collapses and revivals of the fidelity. The average fidelity $\overline{\mathcal{F}}$ can be computed and it is shown in Fig. 2(a), where it is seen that it can be much larger than the classical bound.

Increasing the dimension of the network, the diagonalization of Eq. (2) becomes demanding due to the dimension of the subspace with $\ell=1$. However, a recurrence law in f and N for the spectrum of $\hat{H}_e$ can be found to allow an analytical expression for the transfer fidelity regardless of the length of the cluster. It is worth stressing here that by taking $f \neq 1$, we effectively engineer the coupling strengths of all the connections in a network. When $x_{1N} = x/f$ is the smallest coupling, we find that $\hat{H}_e$ has $N-3$ degenerate eigenvalues. The corresponding eigenvectors have no projection onto the state $|\underline{N}\rangle$. Thus, for the purpose of transferring the input state to the last physical qubit of the system, the inclusion of these degenerate eigenvectors in the decomposition of time-evolution operator is ineffective. The rest of the energy spectrum is made by the eigenvalues $\epsilon_1^{N-3} = -x/f$ and

$$\frac{\epsilon_1^{N-k}}{x} = \frac{1+(N-3)f^2}{2f} \times \left\{ 1 + \mathrm{sgn}_k \sqrt{1 + \frac{4(N-1)f^2}{[1+(N-3)f^2]^2}} \right\} \tag{7}$$

for $k=1$, 2 and $\mathrm{sgn}_1 = +$, $\mathrm{sgn}_2 = -$. $\hat{U}_1$ can thus be effectively decomposed just by considering the eigenstates $|\psi_1^{N-3}\rangle = (1/\sqrt{2})(-|\underline{1}\rangle + |\underline{N}\rangle)$ (which corresponds to ϵ_1^{N-3}) and

$$|\psi_1^{N-k}\rangle = \mathcal{N}_{N-k}\left[|\underline{1}\rangle + |\underline{N}\rangle + \frac{(\epsilon_1^{N-3} + \epsilon_1^{N-k})}{x(N-2)} \sum_{i=2}^{N-1} |\underline{i}\rangle \right], \tag{8}$$

which are relative to ϵ_1^{N-2} and ϵ_1^{N-1}. The normalization factor is written as

$$\mathcal{N}_{N-k} = x(N-2)[2(N-2)^2x^2 + (\epsilon_1^{N-3} + \epsilon_1^{N-k})^2(N-2)]^{-1/2}.$$

The fidelity keeps the structure of Eq. (6) with $\epsilon_1^k \to \epsilon_1^{N-k} (k=1,2)$, $\epsilon_1^0 \to \epsilon_1^{N-3}$. As f increases, $\mathcal{N}_{N-1} \to 0$, $\mathcal{N}_{N-2} \to 1/\sqrt{2}$, and an N-dependent value of τ such that $\mathcal{F}(0,1) \simeq 1$ can be found. This is shown for $N=4$, 5, 6 in Fig. 3(a), where $f=5$ has been taken. The plot shows the fast oscillations of $\mathcal{F}(0,1)$ around an average which is sinusoidal with τ. Practically, this may be a problem as the value of the fidelity depends on the ability of stopping the process at precise moments. The instantaneous value of $\mathcal{F}(0,1)$ thus loses significance in favor of a mean value F_m defined as the semidistance between the maximum and the minimum of the oscillations. Because of the decreasing contribution by the *beating* term $\propto \cos[\Delta\epsilon_1^{N-1,N-2}t]$, the amplitude A of the fast oscillations decreases as N and f grow. This stabilizes the fidelity around the slower behavior. A compromise between a fast process and the uncertainty in $\mathcal{F}(0,1)$ is desirable. From Fig. 3(b) (★ and ▲), we see that $f=5$ is a suitable choice, allowing a high fidelity ($F_m \gtrsim 0.9$ for $N \geq 4$) within $\tau \lesssim 60$ (up to $N=10$) and nearly optimal average fidelity $\overline{\mathcal{F}}$, as shown in Fig. 3(c). However, for $f=1.1$ and $4 < N < 10$ we find $F_m \gtrsim 0.7$ [Fig. 3(b), ■] and $\overline{\mathcal{F}} \gtrsim 0.9$. It is noticeable that, even for values of f that do not optimize the above analysis, we still get very good values of $\overline{\mathcal{F}}$. Furthermore, we stress that, once we set x_{ij}'s allowing for the "$1 \to N$" transfer, the same network can be used, under proper conditions, for "$1 \to j$" processes as well ($2 \leq j \leq N-1$). This is shown in Fig. 2(b) for $N=3$ and in Fig. 3(d) for $N=4$.

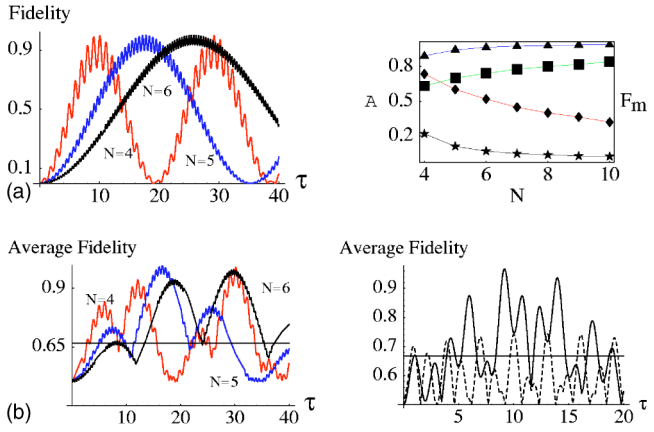

FIG. 3. (a) Fidelity $\mathcal{F}(0,1)$ against τ for $N=4$, 5, 6 and $f=5$. (b) Amplitude A of the fast oscillations of $\mathcal{F}(0,1)$ potted against N (left vertical axis) for $f=1.1$ (♦) and $f=5$ (★). The right vertical axis shows F_m against N for $f=1.1$ (■) and $f=5$ (▲). (c) Average fidelity $\overline{\mathcal{F}}$ for the values in (a). (d) $\overline{\mathcal{F}}$ of the "$1 \to 4$" (solid line) and "$1 \to 3$" (dotted line) process for $N=4$ and $f=1.1$.

IV. THE PROPOSED SETUP

As a setup combining the strong-coupling regime and fixed positions for the qubits in the cavity, we address a system of superconducting qubits in a cavity [10,11]. An array of mutually coupled Josephson junctions may be used as a high-fidelity quantum channel [18]. Our approach is different as we exploit the advantages of a dispersive bus. We take N SQUIDs [9] in a 1D superconducting strip-line resonator ($Q \gtrsim 10^4, \omega_a \simeq 10$ GHz, $\lambda \simeq 1$ cm $\gg d \simeq 1$ μm). The whole setup could be fabricated via nanolithographic techniques allowing for a precise control and calibration of the characteristics of the system. The strip-line cavity minimizes the photon losses and protects, to some extent, the qubits from the environment. The dephasing due to charge-coupled, low-frequency noise is a dangerous source of deco-

herence [19]. However, at the *degeneracy point* of the SQUIDs [9,10], each qubit is encoded in the space spanned by the equally charged states $|\pm\rangle_i=(1/\sqrt{2})(|0\rangle\pm|2e\rangle)$ ($2e$ is the charge of a Cooper pair). The environment is not able to distinguish between these states which are less sensitive to external charge fluctuations [20,21]. For $\Omega_i\sim 100$ MHz and $\delta_i\sim 1$ GHz, the Purcell effect in this off-resonant setup enhances the characteristic life-times up to $(\Omega_i/\delta_i)^{-2}\Gamma_i^{-1}\gtrsim 50\ \mu$s ($\Gamma_i$ is the spontaneous decay rate of the qubits) which is suitable for quantum state transfer up to $N=10(f=5)$.

It is possible to include the effect of losses due to the finite quality factor of the cavity in our model. In order to describe the decay of the cavity field, we have considered the usual Liouvillian used in quantum-optics-related problem. The cavity field is assumed to be in a thermal state as no coherence of the field mode is required in this dispersive scheme. In details, together with the dynamics described in Eq. (1) (within the RWA), we consider the term

$$\hat{\mathcal{L}}_{cav}\rho=\kappa(\overline{n}+1)(2\hat{a}\rho\hat{a}^{\dagger}-\{\hat{a}^{\dagger}\hat{a},\rho\})+\kappa\overline{n}(2\hat{a}^{\dagger}\rho\hat{a}-\{\hat{a}\hat{a}^{\dagger},\rho\}) \tag{9}$$

to be used in the master equation $\partial_t\rho=-i[\hat{H},\rho]+\hat{\mathcal{L}}_{cav}\rho$ (we neglect, for a moment, the spontaneous decay of the qubits). Here, ρ is the density matrix of the *qubits+cavity* system, κ is the cavity decay rate, and the curly brackets denote anticommutator. We have indicated with $\overline{n}$ the average photon number in the thermal state of the cavity field. However, it is experimentally possible to cool the resonator down to temperature such that $\overline{n}\ll 1$ [11]. For instance, for a temperature of $\simeq$300 mK, we get $\overline{n}\simeq 0.06$. Thus, from now on, we will neglect the contribution from temperature-dependent terms. This Liouvillian description of SQUID-cavity open systems has been proven to be rigorous for temperatures well above those assumed in this work. Indeed, exactly this optical master equation results starting from the Bloch-Redfield equations when the secular approximation is relaxed and a large number of elements of the Redfield tensor have to be considered [22]. However, for slow frequency noise, this approach is in general no longer valid as, usually, the correlation times are too long for a Markovian master equation to be valid.

Under the conditions of off-resonant interaction, where the detuning of the qubits effectively participating in the evolution is much larger than both κ and $\Omega_j(\forall\ j)$, the degrees of freedom of the bath can be traced out. Using general standard techniques [23], within the first Born-Markov approximation, the reduced master equation for the qubits can be put into the form $\partial_t\rho_q=\mathrm{Tr}_{cav}\int_0^{\infty}dt'\,\hat{\mathcal{L}}_0 e^{\hat{\mathcal{L}}_{cav}t'}\hat{\mathcal{L}}_0\rho_q\otimes\rho_{cav,th}$. Here, ρ_q is the density matrix for the qubits, $\hat{\mathcal{L}}_0\rho=-i[\hat{H},\rho]$, and $\rho_{cav,th}$ stands for the thermal state of the cavity field. Behind this expression, there is the consideration that the dispersive qubit-cavity coupling is not able to affect the state of the cavity field which thus stays in its steady state throughout the interaction with the qubits. After a lengthy calculation, the reduced qubits dynamics reads

$$\partial_t\rho_q=\sum_{i,j=1}^{N}\frac{\Omega_i\Omega_j}{\delta_i^2+\kappa^2}\{-i\delta_i[\hat{\sigma}_j^{+}\hat{\sigma}_j^{-},\rho_q]+\kappa(2\hat{\sigma}_i^{-}\rho_q\sigma_j^{+}-\hat{\sigma}_i^{+}\hat{\sigma}_j^{-}\rho_q-\rho_q\hat{\sigma}_i^{+}\hat{\sigma}_j^{-})\}. \tag{10}$$

The spontaneous decay of the qubits can be easily included in this analysis as the corresponding Liouvillian superoperators do not include the field degrees of freedom and are not affected by the cavity elimination. For the range of parameters used here, taking for simplicity $\Omega_{r,s}=\Omega(\forall r,s)$ it turns out that $\Omega^2\delta_r/(\delta_r^2+\kappa^2)\simeq\Omega^2/\delta\gg\Omega^2\kappa/(\delta_r^2+\kappa^2)$. Thus, by neglecting the term proportional to κ in Eq. (10), we retrieve the Hermitian dynamics described by Eq. (2). It is thus not surprising that, by solving Eq. (10) for up to $N=5$ qubits in the network and calculating the fidelity of transport $\mathcal{F}(0,1)$, we found a similar trend in Fig. 3(a) without any significant deviation.

In this setup, the qubits energies $E_{q,i}$ are the Josephson coupling energies and the control Φ is a proper magnetic flux piercing the *whole* group of SQUIDs. Small relative differences between the detunings (up to 1 MHz) do not affect the transfer fidelity. In Refs. [10,11], qubits and cavity mode are capacitively coupled and the parameters η_i that can be set properly designing, at the building stage, the capacities between the qubits and the cavity. The initialization of the system can be performed if two values of Φ can be arranged. For $\Phi=\Phi_1$, we assume that x_{ij}'s are small enough to turn off the mutual couplings. At low temperatures (assumed to guarantee the charging regime [9]), the off-resonant coupling to the cavity makes the probability that a qubit is in $|+\rangle_i$ negligible. An electrode coupled to qubit 1, then, provides a pulse that prepares the desired state. Switching to the proper $\Phi_2\ll\Phi_1$, suitable for state transfer, the process begins. Setting back $\Phi=\Phi_1$, the interaction can be stopped.

V. REMARKS

A dispersive qubits-boson interaction is able to mimic an *XY* model with tunable couplings. However, unwanted nonnearest-neighbor connections appear, complicating the dynamics of the network. We have studied quantum state transfer in this generalized *XY* model. The presence of redundant connections affects the efficiency of the state transfer, making a quantum wire useless. We have analyzed a way in which the spoiling effect of these unwanted connections can be bypassed with only small modifications in the coupling distribution. As a practical system where our investigation can be applied, we have considered a setup combining quantum optics and SQUIDs. The relevant parameters discussed in the theoretical model have been identified and a suitable strategy for quantum state transfer in this system has been described.

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