

**Measurement-induced continuous-variable quantum interactions**Radim Filip,<sup>1</sup> Petr Marek,<sup>1</sup> and Ulrik L. Andersen<sup>2</sup><sup>1</sup>*Department of Optics, Palacký University, 17. listopadu 50, 772 07 Olomouc, Czech Republic*<sup>2</sup>*Institut für Optik, Information und Photonik, Max-Planck Forschungsgruppe, Universität Erlangen-Nürnberg, Staudtstrasse 7/B2, 91058 Erlangen, Germany*

(Received 15 June 2004; published 4 April 2005)

We propose feasible implementations of basic continuous-variable (CV) interactions (squeezer, parametric amplifier, and quantum nondemolition interaction) between light modes without the requirement for in-line nonlinear couplings in a strongly pumped optical medium. The method is based entirely on linear optics, homodyne detection, and off-line squeezed ancillary states and therefore represents the CV analog of the measurement-induced nonlinearity approach, previously used in single-photon qubit experiments to probabilistically implement a controlled-NOT gate.

DOI: 10.1103/PhysRevA.71.042308

PACS number(s): 03.67.-a, 42.50.-p

**I. INTRODUCTION**

Quantum optics is well suited for applications in quantum communication and information processing. To achieve high fidelity in these applications, feasible and efficient quantum interactions between light pulses are normally required. However, a successful demonstration of complex quantum protocols is often hindered by the low efficiency and poor quality of coupling between signal modes in a nonlinear medium. Recently, it has been shown that a basic two-qubit controlled-NOT gate between the individual photons, requiring nonlinear interaction, can be conditionally implemented using only linear coupling between the input and ancillary photons [1,2]. A nonlinear interaction is induced by the measurement of ancillary qubits and feedforward postselection procedure. Probabilistic gates of this kind yield the desired operation only when a specific outcome from a detector event is obtained. Thus the interaction can be implemented only conditionally, for some fraction of events.

Beyond the experiments with individual photons, there is an experimentally interesting alternative for quantum communication and information processing—namely, quantum optical experiments with continuous variables (CVs) (for a review see [3]). CV quantum information processing is usually executed by a combination of active optical devices such as squeezers, amplifiers, and quantum nondemolition (QND) interactions. They are based on strongly pumped nonlinear processes in the optical crystals or optical fibers. Therefore the efficiency of such quantum information protocols relies on the efficiency of the individual active devices. Unfortunately, however, the efficient coupling of optical fields inside a nonlinear medium poses severe practical problems rendering experimental implementation of these devices difficult. To obtain sufficient gain of the active device, the power of the pump beam must be large and the effective interaction length of the crystal or optical fiber must be long. Strong pump power can be obtained by using a very short pumping pulse and the interaction length can be enlarged by embedding the crystal inside a cavity or using a long fiber. Such enhancement techniques make it difficult to inject an unknown quantum state into the system. Coupling of the quantum state in the fiber and the cavity is a lossy process unless

extreme care has been taken to spatially and temporally overlap the fiber or cavity modes with the signal modes. Furthermore, by using cavities the impedance-matching condition can also hinder successful coupling. Another drawback of these traditional approaches is that the strongly pumped nonlinear processes are normally not pure and they introduce excess noise into the quadratures.

In this paper, we demonstrate that single-mode squeezers and phase-insensitive amplifiers as well as QND interactions can be accomplished by using only linear optical elements, homodyne measurements, and off-line squeezed light beams. In these schemes signal modes are coupled to ancillary modes in passive devices, the ancillary modes are subsequently detected, and the measurement results are used either to modulate the freely propagating output optical signals using electro-optic feedforward or equivalently to postcorrect the measured results after the protocol is terminated. Thus strongly pumped nonlinear devices are not used in line but only off line to prepare squeezed vacuum states in the ancillary modes. Contrary to the experiments with individual photons [2], such measurement-induced CV Gaussian operations are deterministic without any postselection. The proposed methods can overcome previous experimental problems in achieving efficient quantum interaction between light beams and may pave the way for further experiments in CV quantum information science.

Some previous proposals have been made on the use of linear optics and electro-optics feed forward loops to circumvent cumbersome nonlinear interactions in the CV regime. In Ref. [4] a scheme based on linear optics and measurement was employed to perform noiseless amplification of a single quadrature and in Ref. [5] the same techniques were used to accomplish a quantum nondemolition measurement of a single quadrature. A serious drawback of this scheme is that it only performs a part of the interaction on a single quadrature of the signal. Here we propose a method how to approach the interaction which will work completely for both complementary quadratures.

In Sec. II, we introduce the measurement-induced squeezing operation which illustrates the basic concepts of our approach. In Sec. III we discuss how our method can be employed in various implementations of the QND interactions.

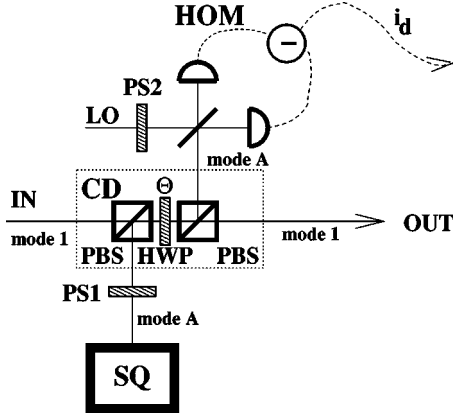


FIG. 1. Schematic setup of the measurement-induced squeezing operation. PBS, polarizing beam splitter; SQ, single-mode vacuum squeezer; PS, phase shifter; HOM, homodyne detection system; CD, coupling device; HWP, half-wave plate; and LO, local oscillator.

First we address two schemes implementing QND interaction with nonunity transfer gain where the interaction takes place either locally or at a distance, and at the end of this section we discuss an implementation of unity-gain QND. In Sec. IV we discuss the implementation of a phase-insensitive optical parametric amplifier. Finally, in Sec. V, the results are summarized with a discussion of the experimental viability.

## II. SINGLE-MODE SQUEEZER

First we discuss a basic ingredient of all the schemes proposed below—a measurement-induced single-mode phase-sensitive optical parametric amplifier or equivalently a single-mode squeezer. The optimal squeezing operation reduces the mean value and variance of a certain quadrature variable of an incident optical mode at the cost of minimal increase of the mean value and variance of the complementary quadrature. Previously, such an operation has been implemented by injecting a signal into a nonlinear medium that is intensively pumped [6].

Our alternative—the measurement-induced squeezing scheme—is depicted in Fig. 1. We assume that the signal is carried by a linear polarization mode 1 while the squeezed ancillary mode  $A$  is prepared in the orthogonal polarization mode. To controllably couple the signal mode and squeezed vacuum ancillary mode, a coupling device, CD, comprising a half-wave plate sandwiched by two polarizing beam splitters, is used. Such a device effectively controls the linear coupling

$$\begin{aligned} X'_1 &= \sqrt{T}X_1 + \sqrt{1-T}X_A, & P'_1 &= \sqrt{T}P_1 + \sqrt{1-T}P_A, \\ X'_A &= \sqrt{T}X_A - \sqrt{1-T}X_1, & P'_A &= \sqrt{T}P_A - \sqrt{1-T}P_1 \end{aligned} \quad (1)$$

between the two input polarization modes with a beam splitting ratio given by the angle  $\Theta$  of the HWP, where the transmittance  $T$  of the CD is  $T = \cos^2 \Theta$  and  $X_i, P_i$  ( $X'_i, P'_i$ ),  $i = 1, A$ , are input (output) pairs of quadrature operators associated with the light modes.

If the ancillary mode  $A$  is prepared in a vacuum state that is strongly squeezed in the  $X_A$  quadrature, the passive cou-

pling at the CD approaches a squeezing operation of  $X_1$  by a degree equal to the transmittance  $\sqrt{T}$ . To achieve a corresponding amplification (antisqueezing) of the  $P_1$  quadrature, we perform a measurement of the  $P'_A$  quadrature using a homodyne detection system and subsequently use the gained information to finalize the appropriate squeezing operation. This is done by amplifying photocurrent fluctuations  $i_d \propto \bar{p}_A$  (where  $\bar{p}_A$  is the measured value) with an appropriate gain  $g$  and imposing information onto the mode 1:  $P'_1 \rightarrow P'_1 + gi_d$ . This displacement operation (it is not shown in the figure) can be performed either electro-optically onto the optical modes or purely electronically onto the measurement results after the experiment has been terminated.

These two displacement transformations are thus identical under the assumption that any Gaussian operation on the signal mode following this squeezing procedure is described by linear transformations of the quadrature operators. To complete the interaction by a postcorrection of the measured data we only need to explicitly know the Gaussian operations that have been performed on the output of the operation. The data postcorrections remain deterministic and hence they do not rely on postselection, in contrast to the measurement-induced operations with single photons [2].

After an appropriate postcorrection  $P'_1 \rightarrow P'_1 + gi_d$ , the results of any measurement on the mode 1 are produced by the output quadrature operators

$$X'_1 = \sqrt{T}X_1 + \sqrt{1-T}X_A,$$

$$P'_1 = \frac{1}{\sqrt{T}}P_1 - \frac{\sqrt{(1-T)(1-\eta)}}{\sqrt{T\eta}}P_0, \quad (2)$$

where  $P_0$  is the quadrature operator of a mode exhibiting vacuum fluctuations. The overall efficiency of this homodyne detector is given by  $\eta$ . We see that the success of the squeezing operation of the input signal is limited by the squeezing of the ancillary state as well as the nonunity detector efficiency of the homodyne detection system. More precisely, the uncertainty of the squeezed quadrature is enhanced by  $(1-T)V_A$  due to finite squeezing of the ancillary state, while the uncertainty of the antisqueezed quadrature is enlarged by  $[(1-T)(1-\eta)/T\eta]V_0$  due to imperfect detectors. Here  $V_A$  and  $V_0$  are the variances of  $X_A$  and  $P_0$ , respectively. The mean values of the input signal are, however, transformed perfectly since the ancillary state is assumed to be vacuum squeezed. For a strongly squeezed input ancilla and high detection efficiency we approach the ideal squeezing operation:  $X_1 = \sqrt{T}X_1$ ,  $P_1 = (1/\sqrt{T})P_1$ . In this case the squeezing operation is optimal and the degrees of squeezing and anti-squeezing are uniquely controlled only by the beam splitter transmittance.

We can easily control the quadrature being squeezed by simple adjustment of the relative phase between the ancillary and signal modes (by using phase shifter PS1). The measured quadrature can be controlled by the phase shifter PS2 which introduces a phase shift between the local oscillator and signal mode 1. Therefore, an appropriate adjustment of the phase shifters PS1 and PS2 enables the squeezing operation to be performed on any quadrature of the signal state.

### III. QND INTERACTIONS

An optimal CV QND interaction with unit transfer gain is a coupling between two optical modes 1 and 2 described by  $X'_1 = X_1$ ,  $P'_2 = P_2$ ,  $X'_2 = X_2 + GX_1$ ,  $P'_1 = P_1 - GP_2$ , where  $X_1, P_1, X_2, P_2$  are the complementary quadrature operators [7]. After the interaction, information carried by a single *nondemolition* variable  $X_1$  ( $P_2$ ) is transferred to the output variable  $X'_2$  ( $P'_1$ ) of the other mode. The noise inevitably associated with such an interaction is transferred to the complementary output variable  $P'_1$  ( $X'_2$ ) keeping the nondemolition variable noise-free. Previously, the complete QND interaction was generated by coupling two optical modes in a nonlinear optical crystal [8]. We now outline three different methods for the complete QND interaction without the need for efficient coupling inside a nonlinear medium.

#### A. Local QND interaction with nonunity transfer gain

A measurement-induced implementation of the QND interaction requiring only a single source of squeezing is depicted in Fig. 2. An ancillary linear polarization mode  $A$  is mixed with two orthogonally polarized modes 1 and 2 in the two coupling devices CD1 and CD2 with the transmission coefficients  $T_1$  and  $T_2$ , respectively. In the next step, the quadrature  $P'_A$  of the ancillary mode is measured by a homodyne detector with the efficiency  $\eta$ . Consequently, we obtain a photocurrent  $i_d$  which is proportional to the random measurement outcomes  $\bar{p}_A$ . According to these measurement outcomes, the modes 1 and 2 can be displaced  $P'_1 \rightarrow P'_1 + g_1 i_d$ ,  $P'_2 \rightarrow P'_2 + g_2 i_d$  in the data postcorrection procedure, where  $g_1$  and  $g_2$  are suitable scaling factors. Then the scheme in Fig. 2 effectively implements the following transformation:

$$\begin{aligned} X'_1 &= \sqrt{T_1}X_1 + \sqrt{1-T_1}X_A, \\ P'_1 &= \frac{1}{\sqrt{T_1}}P_1 - \frac{\sqrt{(1-T_1)(1-T_2)}}{\sqrt{T_1T_2}}P_2 - \frac{\sqrt{(1-\eta)(1-T_1)}}{\sqrt{\eta T_1T_2}}P_0, \\ X'_2 &= \sqrt{T_2}X_2 + \sqrt{(1-T_1)(1-T_2)}X_1 - \sqrt{T_1(1-T_2)}X_A, \\ P'_2 &= \frac{1}{\sqrt{T_2}}P_2 - \frac{\sqrt{(1-\eta)(1-T_2)}}{\sqrt{\eta T_2}}P_0, \end{aligned} \quad (3)$$

where  $P_0$  is the quadrature operator of a mode prepared in vacuum state. The QND interaction (3) transfers information about the quadrature  $X_1$  to the quadrature  $X'_2$  and similarly about  $P_2$  to the  $P'_1$  quadrature. The efficiency of the former information transfer is limited by the squeezing in the ancillary mode  $A$ , whereas the efficiency of the latter information transfer is degraded only by the detector efficiency  $\eta$ .

Assuming that the ancillary mode  $A$  is prepared in a strongly squeezed vacuum state and the detection efficiency  $\eta$  is almost unity, we can approach the perfect nonunity gain QND interaction:

$$X'_1 = \sqrt{T_1}X_1, \quad P'_2 = \frac{1}{\sqrt{T_2}}P_2,$$

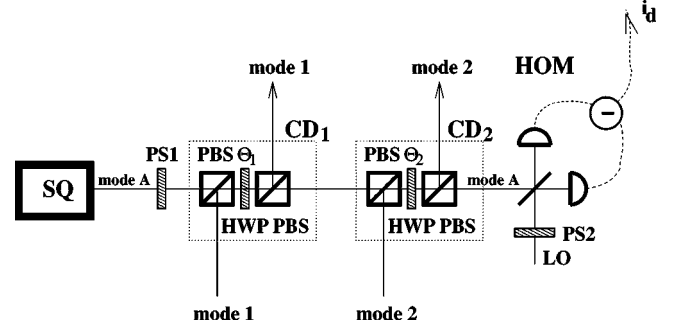


FIG. 2. Schematic setup of the measurement-induced QND interaction with nonunity transfer gains. The abbreviations are explained in the caption text of Fig. 1.

$$\begin{aligned} P'_1 &= \frac{1}{\sqrt{T_1}}P_1 - \frac{\sqrt{(1-T_1)(1-T_2)}}{\sqrt{T_1T_2}}P_2, \\ X'_2 &= \sqrt{T_2}X_2 + \sqrt{(1-T_1)(1-T_2)}X_1. \end{aligned} \quad (4)$$

In this limit, the QND variable  $X_1$  is not influenced by noise and information is perfectly transmitted to the variable  $X'_2$ , but at the cost of the inevitable addition of noise in  $P'_1$ . Note that if the quadrature  $P_2$  is taken to serve as the nondemolition variable, quantum optical tapping [7,9] between  $P_1$  and  $P_2$  can be achieved without any squeezing. Both the  $P_1$  and  $P_2$  variables are amplified at the outputs and the QND gain  $G = \sqrt{(1-T_1)(1-T_2)}/\sqrt{T_1T_2}$  can be arbitrarily high. On the other hand, taking  $X_1$  as the nondemolition variable, we have quantum optical tapping between  $X_1$  and  $X_2$  without the need of a measurement. Here, however, the two outputs are attenuated by the factor  $\sqrt{T_2}$ . Any quadrature of the input modes can be chosen to be the nondemolition one by adjusting the phase shifter PS1 with respect to the phase of the input modes and adjusting the phase of the local oscillator LO in the homodyne measurement accordingly (using the phase shifter PS2). We can easily prove that the outlined QND scheme described by Eqs. (4) approaches the optimal QND transformation with unit transfer gain by implementing squeezing operations with gains  $T_1$  and  $T_2$  onto the two output modes:

$$\begin{aligned} X'_1 &= X_1, \quad P'_2 = P_B, \\ P'_1 &= P_1 - \frac{\sqrt{(1-T_1)(1-T_2)}}{\sqrt{T_2}}P_2, \\ X'_2 &= X_2 + \frac{\sqrt{(1-T_1)(1-T_2)}}{\sqrt{T_2}}X_1. \end{aligned} \quad (5)$$

This transformation is the optimal QND interaction with unity transfer gain and the QND interaction gain is given by  $G = \sqrt{(1-T_1)(1-T_2)}/T_2$ . Let us stress that the squeezing operations can be implemented using the scheme discussed in Sec. II (see also Fig. 1).

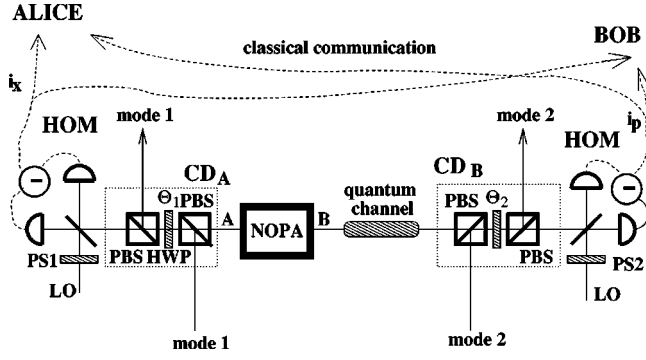


FIG. 3. Schematic setup for the measurement-induced QND interaction at a distance with nonunity transfer gain. The abbreviations are explained in the caption of Fig. 1.

### B. Distant QND interaction with nonunity transfer gain

The QND scheme can be modified using a CV entangled state shared between two distant parties (Alice and Bob). This can be useful when two distant parties want to perform a QND coupling using only local linear operations and classical communication. The proposed scheme is depicted in Fig. 3. The entanglement is carried by a two-mode squeezed vacuum state of which one half (denoted mode A) is preserved by Alice and the other half (denoted mode B) is sent to Bob via a quantum channel. The two-mode squeezed vacuum state can be prepared by mixing two orthogonally squeezed states at a balanced beam splitter. Alice mixes the modes 1 and A in the coupling device  $CD_A$  with transmittance  $T_1$ , and similarly Bob couples the modes 2 and B with the coupling device  $CD_B$  with transmittance  $T_2$ . Then Alice measures the quadrature  $X'_A$  and she obtains a photocurrent  $i_x$  proportional to the result  $\bar{x}_A$  while Bob measures the conjugate quadrature  $P'_B$  which yields the photocurrent  $i_p \propto \bar{p}_p$ . Finally, they mutually exchange these classical data by two-way classical communication and thus they can achieve the transformations

$$\begin{aligned}
 X'_1 &= \frac{1}{\sqrt{T_1}} X_1 - \frac{\sqrt{(1-\eta)(1-T_1)}}{\sqrt{\eta T_1}} X_0, \\
 P'_1 &= \sqrt{T_1} P_1 - \frac{\sqrt{(1-T_1)(1-T_2)}}{\sqrt{T_2}} P_2 + \frac{\sqrt{(1-\eta)(1-T_1)}}{\sqrt{\eta T_2}} P_0 \\
 &\quad + \sqrt{1-T_1} (P_A + P_B), \\
 X'_2 &= \sqrt{T_2} X_2 + \frac{\sqrt{(1-T_1)(1-T_2)}}{\sqrt{T_1}} X_1 - \frac{\sqrt{(1-\eta)(1-T_2)}}{\sqrt{\eta T_1}} X_0 \\
 &\quad - \sqrt{1-T_2} (X_A - X_B), \\
 P'_2 &= \frac{1}{\sqrt{T_2}} P_2 - \frac{\sqrt{(1-\eta)(1-T_2)}}{\sqrt{\eta T_2}} P_0,
 \end{aligned} \tag{6}$$

if the local postcorrections  $X'_i \rightarrow X'_i + g_{X_i} i_x$  and  $P'_i \rightarrow P'_i + g_{P_i} i_p$ , where  $g_{X_i}, g_{P_i}$  ( $i=1,2$ ) are appropriate scaling factors, are used on the signal data after termination of the protocol. The quadratures  $X_0$  and  $P_0$  describe noncommuting

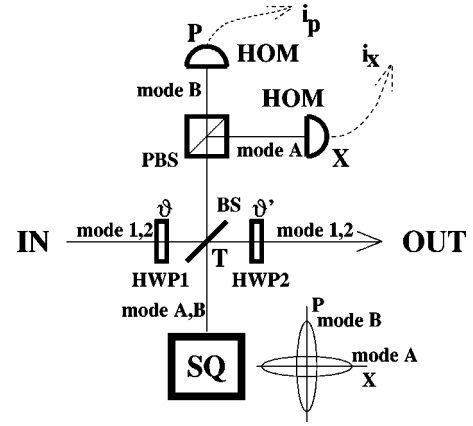


FIG. 4. Schematic setup for measurement-induced QND interaction with unity transfer gain, and the measurement-induced phase-insensitive amplifier. The abbreviations are explained in the caption of Fig. 1.

vacuum modes. As the two-mode squeezing increases (meaning that the uncertainties of the joint operators  $X_A - X_B$  and  $P_A + P_B$  decrease) and the efficiency  $\eta$  of the homodyne detectors approaches unity, the QND interaction is achieved at a distance. However, the QND interaction is performed with a nonunity transfer gain. The nondemolition variables  $X_1$  and  $P_2$  are amplified whereas the conjugate quadratures  $P_1$  and  $X_2$  are attenuated. As in the previous section the optimality of the QND interaction can be proved by implementing local single-mode squeezing operations in Alice's and Bob's modes. Then we obtain the optimal QND coupling with unity transfer gain between two distant parties with the interaction gain  $G = \sqrt{(1-T_1)(1-T_2)}/T_1 T_2$ .

### C. QND interaction with unity transfer gain

The implementation of the measurement-induced QND coupling depicted in Fig. 2 utilized only a single squeezing source to accomplish the basic features of a QND coupling in both complementary variables. However, as already pointed out, the nondemolition quadratures are not transferred with unity gain. Here we show that by adding an extra squeezed vacuum ancillary mode to the scheme we can approach the optimal QND interaction with unity transfer gain. The scheme is shown in Fig. 4. The two input signals are carried by two orthogonally polarized modes denoted 1 and 2. These modes are mixed by a half-wave plate HWP1 aligned at an angle  $\vartheta$  with respect to the optical axis. The degree of mixing can be quantified by the transmission coefficient  $T_1 = \sin \vartheta$ . Subsequently, the input signals are coupled on a polarization-insensitive beam splitter (BS with a transmission coefficient of  $T$ ) with two ancillary orthogonally polarized modes denoted A and B. The two ancillary modes are assumed to be squeezed in conjugate quadratures  $P_A$  and  $X_B$ , respectively. After the linear coupling, the ancillary modes are decoupled by a polarizing beam splitter (PBS) and the amplitude quadrature  $X'_A$  of mode A is measured (with the result  $\bar{x}_A$ ) while the phase quadrature  $P'_B$  is measured on mode B (with the result  $\bar{p}_B$ ) by using two homodyne detectors with identical efficiencies. The measurements give rise

to the photocurrents  $i_X \propto \bar{x}_A$  and  $i_P \propto \bar{p}_B$  which will be used in the postcorrection procedure. The signal outputs modes 1 and 2 are once more coupled by a half-wave plate (HWP2) which is aligned at an angle  $\vartheta'$  with respect to the optical axis corresponding to a transmission coefficient of  $T_2 = \sin \vartheta'$ . Adjusting the coupling and transmission coefficients such that

$$T_1 = 1/(1+T), \quad T_2 = T/(1+T), \quad (7)$$

we have generated the following transformation:

$$\begin{aligned} X'_1 &= X_1 - \sqrt{\alpha}X_0 - \sqrt{\beta}X_A, \\ P'_2 &= P_2 - \sqrt{\alpha}P_0 + \sqrt{\beta}P_A, \\ P'_1 &= P_1 - \left(\frac{1}{\sqrt{T}} - \sqrt{T}\right)P_2 + \sqrt{T}\beta P_A + \sqrt{\alpha}TP_0, \\ X'_2 &= X_2 + \left(\frac{1}{\sqrt{T}} - \sqrt{T}\right)X_1 + \sqrt{T}\beta X_A - \sqrt{\alpha}TX_0, \end{aligned} \quad (8)$$

if the appropriate postcorrections  $X'_i \rightarrow X'_i + g_{X_i}i_X$ ,  $P'_i \rightarrow P'_i + g_{P_i}i_P$ ,  $i=1,2$ , are applied to the signal data. Here  $\alpha=(1-T)(1-\eta)/(1+T)\eta$ ,  $\beta=(1-T)/(1+T)$ , and  $X_0, P_0$  are quadrature operators of the commuting modes exhibiting vacuum noise. As the squeezing of the ancillary modes increases and the efficiency of the homodyne detectors approaches unity, the QND interaction with unit transfer gain,

$$\begin{aligned} X'_1 &= X_1, \quad P'_2 = P_2, \\ P'_1 &= P_1 - \left(\frac{1}{\sqrt{T}} - \sqrt{T}\right)P_2, \\ X'_2 &= X_2 + \left(\frac{1}{\sqrt{T}} - \sqrt{T}\right)X_1, \end{aligned} \quad (9)$$

is generated with an arbitrary accuracy. Through control of the transmission coefficient  $T$ , the gain  $G=1/\sqrt{T}-\sqrt{T}$  of the optimal QND interaction can be freely adjusted.

#### IV. PHASE-INSENSITIVE OPTICAL PARAMETRIC AMPLIFIER

A phase-insensitive optical parametric amplifier based on off-line squeezers, linear optics, and measurements can also be implemented. The phase-insensitive amplifier optimally amplifies any quadrature of the signal with the same gain at the cost of the addition of excess noise. To date, parametric amplification has been achieved by merging signal and idler modes with a strong pump inside a nonlinear optical crystal [10]. Our implementation of the optical parametric amplifier without the necessity to couple the signal modes within a nonlinear medium is similar to the setup depicted in Fig. 4. Now, however, the wave plates HWP1 and HWP2 must be oriented such that a balanced coupling between the polarization modes is achieved, i.e.,  $\sin \vartheta = \sin \vartheta' = 1/\sqrt{2}$ . Then by controlling the transmittance  $T$  of the beam splitter and after

appropriate data postcorrections  $X'_i \rightarrow X'_i + g_{X_i}i_X$ ,  $P'_i \rightarrow P'_i + g_{P_i}i_P$ ,  $i=1,2$ , we have implemented the following transformation:

$$\begin{aligned} X'_1 &= \sqrt{G}X_1 - \sqrt{G-1}X_2 + \frac{\sqrt{\lambda}}{\sqrt{2}}X_0 + \frac{\sqrt{1-T}}{\sqrt{2}}X_A, \\ P'_1 &= \sqrt{G}P_1 + \sqrt{G-1}P_2 + \frac{\sqrt{\lambda}}{\sqrt{2}}P_0 + \frac{\sqrt{1-T}}{\sqrt{2}}P_B, \\ X'_2 &= \sqrt{G}X_2 - \sqrt{G-1}X_1 - \frac{\sqrt{\lambda}}{\sqrt{2}}X_0 + \frac{\sqrt{1-T}}{\sqrt{2}}X_A, \\ P'_2 &= \sqrt{G}P_2 + \sqrt{G-1}P_1 - \frac{\sqrt{\lambda}}{\sqrt{2}}P_0 + \frac{\sqrt{1-T}}{\sqrt{2}}P_B, \end{aligned} \quad (10)$$

where  $G=\frac{1}{4}(1/\sqrt{T}+\sqrt{T})^2$  is the gain of the amplifier,  $\lambda=(1-T)(1-\eta)/T\eta$ , and  $X_0, P_0$  are quadrature operators of the commuting modes prepared in vacuum state. The performance of this device is limited, as in the previous sections, by the detection efficiency  $\eta$  of the homodyne detectors as well as by the finite squeezing of the ancillary modes. Assuming almost perfect homodyne detectors and high degrees of squeezing in the ancillary modes we approach the phase-insensitive amplifier transformation

$$\begin{aligned} X'_{1,2} &= \sqrt{G}X_{1,2} - \sqrt{G-1}X_{2,1}, \\ P'_{1,2} &= \sqrt{G}P_{1,2} + \sqrt{G-1}P_{2,1} \end{aligned} \quad (11)$$

with an arbitrary accuracy.

#### V. DISCUSSION

In this paper we have proposed schemes for measurement-induced parametric amplifiers and quantum nondemolition measurements without the necessity of a cumbersome interaction between signal and pump modes in an active medium. They avoid the difficulties mentioned in the Introduction. Linear coupling of two modes in a linear device (like a beam splitter) is relatively easy and can be performed very efficiently by carefully overlapping the modes. Therefore the proposed schemes based on linear optics and measurements represent a remarkable practical simplification of the traditional two-mode nonlinear interaction schemes. In addition to the practical simplifications, the measurement-induced interaction schemes have some distinct features. First, they are deterministic schemes that employ every input state. This is in contrast to the measurement-induced nonlinearities on single-photon level where events are probabilistically postselected which renders these protocols rather inefficient [2]. In our approaches the states are not postselected but only deterministically postcorrected (displaced) according to data from the measurement on some ancillary modes. However, similar to experiments with qubits, we can perform this displacement simply on the measured data after the execution of the optical experiment. Finally, the various trans-

formations presented above are independent of the anti-squeezed quadrature of the ancillary states. This means that the eventual excess noise in the antisqueezed quadrature of the ancilla is not transferred to the output signal state. Hence, a minimum uncertainty operation can be approached even with a nonminimum uncertainty squeezed ancillary state.

The performance of our schemes is ultimately hampered by imperfections. For a successful demonstration, strongly squeezed ancillary modes and good homodyne detectors are required. Strongly squeezed states can be prepared using present technology for a wide range of the frequencies based on either optical parametric oscillation [11] or the optical Kerr effect in fibers [12] in both the continuous-wave regime and the pulse regime. In both cases a suppression to  $-6$  dB is reliably achievable. A squeezing near  $-10$  dB could be achieved once all the components and material are optimized. Efficient implementation of the feedforward loops can be attained by employing high-quantum-efficiency photodiodes in the detectors and simultaneously using a circuitry with low electronic dark noise. Electronic noise is taken by the detector as classical noise of the ancillary modes, which is uncorrelated with the output signal modes and therefore

introduces noise in the correction procedure. A first generation of such detectors was used in previous feedforward experiments [4,5] and the development of the next generation of detectors with low electronic noise is one recent experimental task.

*Note added in proof.* Similar methods are used in the recent experimental demonstration of quantum cloning of light to accomplish a phase insensitive interaction [13].

#### ACKNOWLEDGMENTS

We would like to thank Ladislav Mišta, Jr., and J. Fiurášek for stimulating and helpful discussions. R.F. acknowledges the hospitality of Professor G. Leuchs from Universität Erlangen-Nürnberg. The work was supported by Project No. 202/03/D239 of The Grant Agency of the Czech Republic, the Projects No. LN00A015, No. CEZ:J14/98, and No. MSM6198959213 and “Measurement and Information in Optics” of the Ministry of Education of Czech Republic, and by the COVAQUIAL Grant No. FP6-511004 of the sixth framework program of the European Union. U.L.A. acknowledges the Alexander von Humboldt Foundation for support.

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