Double-dot chain as a macroscopic quantum bit

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We consider an array of N quantum dot pairs interacting via Coulomb interaction between adjacent dots and hopping inside each pair. We show that at the first order in the ratio of hopping and interaction amplitudes, the array maps in an effective two-level system with energy separation becoming exponentially small in the macroscopic (large-N) limit. Decoherence at zero temperature is studied in the limit of weak coupling with phonons. In this case, the macroscopic limit is robust with respect to decoherence. Some possible applications in quantum information processing are discussed.

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I. INTRODUCTION

In recent years, a lot of attention has been devoted to the existence of quantum superposition states in macroscopic systems. The first suggestion to understand this phenomenon is due to Schrödinger [1], who introduced the paradox of the cat in quantum superposition between life and death, strongly stressing the different behavior of the quantum world with respect to the human experience. It is commonly accepted that quantum behavior vanishes as the system size increases. Remarkable exceptions are quantum systems which undergo a phase transition, such as superconductors and superfluids. A quantum superposition of mesoscopic states has been observed in SQUID devices [2] and seems to be a promising tool for the realization of a quantum computer.

Quantum macroscopic states are expected to be robust with respect to decoherence and thus ideal candidates for quantum information storage. Moreover, it is well known that a quantum system which undergoes a phase transition lives in one of a particular set of states, for a time which becomes infinitely large in the limit of large system size. In particular, if the ground state is twofold degenerate, one can associate these states to a macroscopic quantum bit. The availability of macroscopic quantum bits is relevant for quantum information processing, as shown, for instance, in Ref. [3], in the case of spin clusters. Here a new application in teleportation processes is also shown.

Decoherence of a single qubit has been extensively studied [4–10]. One of the most relevant causes of decoherence is the coupling with a bosonic bath [11–17] whose effects are relevant also at zero temperature.

In the present paper, we investigate the coherence of an array of N double quantum dots coupled through Coulomb interaction in order to show that such system is a suitable candidate as a macroscopic qubit. The first step is to show that in the long time limit the array behaves as a two-level system with energy separation which vanishes for large N. The model is exactly equivalent to a one-dimensional antiferromagnetic Ising model in a transverse field. The system is characterized by two equivalent charge configurations and in the macroscopic limit the ordering at zero temperature implies a separation of phase space in two regions around each of the degenerate configurations. However, for a finite-size system, hopping induces oscillations between the two configurations. We associate to this behavior a macroscopic quantum bit. The antiferromagnetic Ising model in a transverse field has been studied since the pioneering work of Bethe [18–20], and has received recently renewed attention as a model for quantum computation [21–23]. We found it convenient to introduce a simple approximation which makes transparent how the two-level behavior appears asymptotically.

The study of decoherence in such a system is analogous to decoherence in a quantum register [24]. We show that, at least in the weak-coupling and zero-temperature limit and for a three-dimensional environment, the system exhibits a robustness growing with the size of the array.

Decoherence with respect to phonons of a single two-level system has been studied with various methods (see reviews by Leggett *et al.* [16] and Weiss [17]). We found, however, the resolvent method [25,26], already introduced in the discussion of electron-phonon interaction problems [27], convenient to obtain results at zero temperature in the double-dot chain.

The paper is organized as follows. In Sec. II, we introduce the model of the double-dot chain and its interaction with a phonon bath. The introduction of the resolvent method to discuss decoherence will be the argument of Sec. III. In Sec. IV, we shall apply the same method to show that the double-dot chain, in the limit $w/U \ll 1$, being w the hopping amplitude between dots inside each pair and U the Coulomb interaction between different pairs, behaves as an effective two-level system with energy separation decreasing exponentially with N. In Sec. V, we study decoherence in our system in the approximation introduced above. Finally, Sec. VI is devoted to conclusions. In the Appendix we review, inside the present approximation, decoherence effects for a single-dot pair.

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II. THE MODEL

In a previous work [28], we proposed an array of few coupled quantum dot pairs as a channel for teleportation. We want to show here the extension of the model to a case of N pairs and discuss the robustness of the system with respect to decoherence due to interaction with an external phonon bath at zero temperature. We expect that the extensive character of the interaction will increase decoherence while the macroscopic nature of the first two energy states will enhance the robustness of the system. It will be shown that the latter feature prevails. Neglecting spin effects, the double-dot array is characterized by the Hamiltonian

$$H_S = U\sum_{l=1}^{N-1}\sum_{\alpha=1}^{2} n_{l,\alpha}n_{l+1,\alpha} - w\sum_{l=1}^{N}\left(c_{l,1}^{\dagger}c_{l,2} + \text{H.c.}\right), \tag{1}$$

where $c_{l,\alpha}^{\dagger}$ creates an electron on the l(th) dot on the α(th) row of the array and $n_{l,\alpha}=c_{l,\alpha}^{\dagger}c_{l,\alpha}$.

To extend the teleportation scheme described in [28], we introduce an initial superposition of two spin configurations of zero potential energy of N pairs,

$$|S\rangle = \alpha|\Phi\rangle + \beta|\Psi\rangle, \tag{2}$$

where $|\Phi\rangle=|\downarrow,\uparrow,\downarrow,\uparrow,\ldots,\uparrow\rangle$ and $|\Psi\rangle=|\uparrow,\downarrow,\uparrow,\downarrow,\ldots,\downarrow\rangle$. Let us consider an initial system of $N-1$ double quantum dots with hopping inside any pair and without Coulomb interaction. The ground state of this system is represented by the tensor product of $(|\uparrow\rangle+|\downarrow\rangle)/\sqrt{2}$ for each pair. By an adiabatic switching of electrostatic repulsion between adjacent pairs, the system is driven in its new ground state, which, for $w/U\ll 1$, is well approximated by $(|\Phi_{N-1}\rangle+|\Psi_{N-1}\rangle)/\sqrt{2}$. $|\Phi_{N-1}\rangle(|\Psi_{N-1}\rangle)$ is the same state as $|\Phi\rangle(|\Psi\rangle)$, defined on $N-1$ sites. The state $|S\rangle$ is obtained considering an extra double dot (as usual called Alice) in a superposition state $\alpha|\uparrow\rangle+\beta|\downarrow\rangle$ and its interaction with the first pair of $(|\Phi_{N-1}\rangle+|\Psi_{N-1}\rangle)$. If the interaction is adiabatically switched on again, then the system is driven in a state close to $|S\rangle$. A proper manipulation of system parameters permits us to transfer the information, i.e., α and β, encoded previously by Alice, to the last double dot (Bob) [28].

The model described above is suitable to be represented by a spin Hamiltonian through the mapping $\sigma_l^z=(n_{l,1}-n_{l,2})$ and $\sigma_l^x=(c_{l,1}^{\dagger}c_{l,2}+\text{H.c.})$. This picture is useful to study the decoherence effects induced by the interaction with a phonon bath. In the spin representation, the overall Hamiltonian becomes

$$H = H_S + H_B + H_{SB}, \tag{3}$$

$$H_S = -w\sum_l \sigma_l^x + \frac{U}{2}\sum_l\left(\sigma_l^z\sigma_{l+1}^z + 1\right), \tag{4}$$

$$H_B = \sum_{\mathbf{q}} \omega_{\mathbf{q}}a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}}, \tag{5}$$

$$H_{SB} = \sum_{\mathbf{q},l} g_{\mathbf{q}}n_l e^{iq\cos\theta l}\left(a_{\mathbf{q}}^{\dagger} + a_{-\mathbf{q}}\right), \tag{6}$$

where a mapping between the spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ and the charge states $|1,0\rangle$ and $|0,1\rangle$ has been performed and $n_l=(\sigma_l^z+1)/2$. We indicate with θ the angle between the phonon mode $\mathbf{q}$ and the dot chain direction. This notation is useful for describing a generic d-dimensional environment coupled with a one-dimensional system. The constant $g_{\mathbf{q}}$ represents the coupling of the dot charge with the mode $\mathbf{q}$. The explicit mathematical expression for $g_{\mathbf{q}}$ depends on the specific configuration of the system and the type of interaction. In Ref. [29], the explicit form of g_q in some remarkable case is given.

III. RESOLVENT METHOD FOR WEAK-COUPLING DECOHERENCE

Decoherence at zero temperature is studied using the resolvent method. At the initial time $t=0$, the system and bath are decoupled: $|\Xi(t=0)\rangle=|S\rangle\otimes|0\rangle$, where $|0\rangle$ is the vacuum phonon state.

The time evolution of the state $|\Xi(t)\rangle=\exp(-iHt)|\Xi(t=0)\rangle$ is studied in terms of the complex Laplace transform defined as

$$|\Xi(\omega)\rangle = i\lim_{\delta\to 0}\int_0^{\infty} e^{i\omega t-\delta t}|\Xi(t)\rangle dt. \tag{7}$$

The resolvent method allows us to write

$$|\Xi(\omega)\rangle = \frac{1}{\omega - H}|\Xi(t=0)\rangle. \tag{8}$$

Using the identity

$$\frac{1}{\omega - H} = \frac{1}{\omega - H_0} + \frac{1}{\omega - H_0}H_I\frac{1}{\omega - H} \tag{9}$$

and performing a projection on the vacuum phonon state, we define a new system state $|\Phi_S(\omega)\rangle=\langle 0|\Xi(\omega)\rangle$ that obeys to the evolution equation

$$|\Phi_S(\omega)\rangle = \frac{1}{\omega - H_S}|\Phi_S(t=0)\rangle + \langle 0|\frac{1}{\omega - H_S}H_{SB}\frac{1}{\omega - H}|\Xi(t=0)\rangle. \tag{10}$$

Here the bath ground-state energy is set to zero and $H_0=H_S+H_B$ and $H_I=H_{SB}$.

In the weak-coupling limit, only corrections to the imaginary part of $|\Phi_S(\omega)\rangle$ will be taken into account. We first perform an iteration inside Eq. (10) replacing $(\omega-H)^{-1}$ with the right-hand side of Eq. (9), and then introduce a complete set of intermediate phonon states,

$$|\Phi_S(\omega)\rangle = \frac{1}{\omega - H_S}|\Phi_S(t=0)\rangle + \langle 0|\frac{1}{\omega - H_S}H_{SB}\frac{1}{\omega - H_S - H_B}|\Xi(t=0)\rangle + \sum_k \langle 0|\frac{1}{\omega - H_S}H_{SB}\frac{1}{\omega - H_S - H_B}H_{SB}|k\rangle \times \langle k|\frac{1}{\omega - H}|\Xi(t=0)\rangle. \quad (11)$$

In a perturbative approach, terms involving powers of g_q are small and can be neglected, unless self-energy contributes appear. In the latter case, a not negligible imaginary part can arise performing the sum over q in the continuous limit.

The contribution involving self-energy in the sum corresponds to $k=0$, since $\langle 0|(\omega - H)^{-1}|\Xi(t=0)\rangle$ is exactly $|\Phi_S(\omega)\rangle$. Then, all other linear and quadratic contributions in H_{SB} will be neglected. Hence, Eq. (11) becomes

$$\left(1 - \frac{1}{\omega - H_S}G(H_S)\right)|\Phi_S(\omega)\rangle = \frac{1}{\omega - H_S}|\Phi_S(t=0)\rangle, \quad (12)$$

where

$$G(H_S) = \langle 0|H_{SB}\frac{1}{\omega - H_S - H_B}H_{SB}|0\rangle \quad (13)$$

is the self-energy operator acting on the system subspace. The right term of Eq. (12) describes the evolution of the macroscopic state isolated from phonons. As we will show in Sec. IV, in the limit of $w/U \ll 1$, the macroscopic dot chain behaves as a two-level system oscillating between the H_S asymptotic eigenstates $|\pm\rangle = 2^{-1/2}(|\Phi\rangle \pm |\Psi\rangle)$ with energies $E_\pm$. So, Eq. (12) becomes

$$\left(1 - \frac{1}{\omega - H_S}G(H_S)\right)|\Phi_S(\omega)\rangle = \frac{1}{\omega - E_+}|+\rangle\langle +|\Phi_S(t=0)\rangle + \frac{1}{\omega - E_-}|-\rangle\langle -|\Phi_S(t=0)\rangle. \quad (14)$$

Noting that the operator $G(H_S)$ maps the subspace spanned by $|\pm\rangle$ into itself, it is possible to reduce Eq. (14) in terms of two coupled equations,

$$(\omega - E_+ - G^{++})\langle +|\Phi_S(\omega)\rangle - G^{+-}\langle -|\Phi_S(\omega)\rangle = \langle +|\Phi_S(t=0)\rangle, \quad (15)$$

$$(\omega - E_- - G^{--})\langle -|\Phi_S(\omega)\rangle - G^{-+}\langle +|\Phi_S(\omega)\rangle = \langle -|\Phi_S(t=0)\rangle, \quad (16)$$

where $G^{\pm\pm} = \langle \pm|G|\pm\rangle$.

To the leading order in the system-bath coupling, we obtain

$$\langle +|\Phi_S(\omega)\rangle = \frac{1}{\omega - E_+ - G^{++}}\langle +|\Phi_S(t=0)\rangle, \quad (17)$$

$$\langle -|\Phi_S(\omega)\rangle = \frac{1}{\omega - E_- - G^{--}}\langle -|\Phi_S(t=0)\rangle. \quad (18)$$

The solution in the time domain is obtained assuming first the correction introduced by the matrix elements of G as negligible, and then calculating the latter in $\omega = E_+$ or $\omega = E_-$.

For instance, the integral

$$\int_C \frac{e^{-i\omega t}}{\omega - E_+ - G^{++}}d\omega$$

is calculated assuming first $G^{++}=0$, obtaining for the pole $\omega = E_+$, and then substituting this value inside G^{++}, which depends on ω. Afterwards, the principal value of G^{++} will be ignored, and only the imaginary part will matter. We compare, in the Appendix, the results of our approximation with those known for single quantum dot pairs.

IV. DOUBLE-DOT ARRAY EVOLUTION

As first step, we calculate the evolution of the system when it is decoupled from the bath. We find it convenient to explicitly solve the evolution from an initial state corresponding, respectively, to $|\Phi\rangle$ or $|\Psi\rangle$ introduced in Eq. (2).

We distinguish in the system Hamiltonian the hopping term $H_I = -w\Sigma_l \sigma_l^x$ from the potential energy $H_0 = (U/2)[\Sigma_l(\sigma_l^z\sigma_{l+1}^z + 1)]$. This is the antiferromagnetic version of the well known one-dimensional Ising model in a transverse field [31]. It is worth noting that the absence of periodic boundary conditions implies a relaxation mechanism of an initially ordered state where a single domain wall propagates between the two end points of the array. This feature makes a difference in the excitation spectrum which is relevant for an array of finite size.

Applying H_I on $|\Phi(t=0)\rangle$, the system is driven in a new configuration labeled as $|\Phi_1(t=0)\rangle$. The action of H_I generates a sum of states, each of which differentiates from $|\Phi(t=0)\rangle$ due to one spin flip in a different place along the array. Here it is important to note that flips on the first and the last qubit put the system in a state with Coulomb energy U, while all intermediate transitions lead to a state with a $2U$ electrostatic energy. In the limit of U large with respect to w, we shall neglect all configurations involving intermediate states with energy greater than U.

In each step of a repeated application of H_I, it is possible to go towards new configurations or to come back. Then, for $n>0$, we write

$$H_I|\Phi_n(t=0)\rangle = -w[|\Phi_{n-1}(t=0)\rangle + |\Phi_{n+1}(t=0)\rangle]. \quad (19)$$

After N steps the system reaches $|\Psi\rangle$, and after $2N$ steps it comes back to the initial configuration. Defining $|\Phi_N\rangle = |\Psi\rangle$ and $|\Phi_0\rangle = |\Phi_{2N}\rangle = |\Phi\rangle$, and taking into account the time evolution, we obtain

$$(\omega - U)|\Phi_n(\omega)\rangle = |\Phi_n(t=0)\rangle - w[|\Phi_{n-1}(\omega)\rangle + |\Phi_{n+1}(\omega)\rangle] - U(\delta_{n,0} + \delta_{n,N})|\Phi_n(\omega)\rangle. \quad (20)$$

The system is solved by means of the discrete Fourier transform defined as

$$\left|\tilde{\Phi}_k(\omega)\right\rangle = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} |\Phi_n(\omega)\rangle e^{ink},$$

$$|\Phi_n(\omega)\rangle = \frac{1}{\sqrt{2N}} \sum_{k=0}^{2N-1} \left|\tilde{\Phi}_k(\omega)\right\rangle e^{-ink}. \tag{21}$$

As a consequence of periodicity conditions, $k=(2\pi/2N)n$, where $n=0,1,2,\ldots,2N-1$.

From Eq. (20), it follows that

$$[\omega - U + 2w\cos k]\left|\tilde{\Phi}_k(\omega)\right\rangle = \left|\tilde{\Phi}_k(t=0)\right\rangle - \frac{U}{\sqrt{2N}}\left[|\Phi_0(\omega)\rangle + e^{iNk}|\Phi_N(\omega)\rangle\right]. \tag{22}$$

It is now possible to extract two equations connecting $|\Phi_0\rangle$ to $|\Phi_N\rangle$,

$$|\Phi_0(\omega)\rangle = \frac{[1+B_0(\omega)]|A_0(\omega)\rangle - B_N(\omega)|A_N(\omega)\rangle}{[1+B_0(\omega)]^2 - B_N^2(\omega)}, \tag{23}$$

$$|\Phi_N(\omega)\rangle = \frac{[1+B_0(\omega)]|A_N(\omega)\rangle - B_N(\omega)|A_0(\omega)\rangle}{[1+B_0(\omega)]^2 - B_N^2(\omega)}, \tag{24}$$

where

$$|A_n(\omega)\rangle = \frac{1}{\sqrt{2N}} \sum_{k=0}^{2\pi[(2N-1)/2N]} \frac{e^{-ink}\left|\tilde{\Phi}_k(t=0)\right\rangle}{\omega - U + 2w\cos k}, \tag{25}$$

$$B_n(\omega) = \frac{1}{2N}\frac{U}{\omega - U} \sum_{q=0}^{2N-1} \frac{e^{-in(\pi/N)q}}{1 - a(\omega)\cos\frac{\pi}{N}q}, \tag{26}$$

with $k=(\pi/N)q$, $a(\omega)=2w/(U-\omega)$ and noting that $B_N=B_{-N}$.

The asymptotic behavior is determined by values of ω close to zero. Then $a(\omega)\ll 1$ and the denominator of $B_n(\omega)$ reads as geometric series,

$$B_n(\omega) = \frac{1}{2N}\frac{U}{\omega - U} \sum_{q=0}^{2N-1} e^{-in(\pi/N)q} \sum_{l=0}^{\infty} a^l(\omega)\cos^l\frac{\pi}{N}q \tag{27}$$

or

$$B_n(\omega) = \frac{1}{2N}\frac{U}{\omega - U} \sum_{q=0}^{2N-1} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \binom{l}{m} \left(\frac{a(\omega)}{2}\right)^l \times \exp\left[i\frac{\pi}{N}(l-2m-n)q\right]. \tag{28}$$

The sum over q gives

$$B_n(\omega) = \frac{1}{2N}\frac{U}{\omega - U} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \binom{l}{m} \left(\frac{a(\omega)}{2}\right)^l \frac{1 - e^{2i\pi(l-2m-n)}}{1 - e^{i(\pi/N)(l-2m-n)}}. \tag{29}$$

The condition for a nonvanishing $B_n(\omega)$ is $(l-2m-n) = 2N\,K$, where K is any integer between $-\infty$ and $+\infty$,

$$B_n(\omega) = \frac{U}{\omega - U} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{l!}{m!\,(l-m)!} \left(\frac{a(\omega)}{2}\right)^l \delta_{(l-2m-n),2N\,K}. \tag{30}$$

or, using the Kronecker δ function,

$$B_n(\omega) = \frac{U}{\omega - U} \sum_{l=0}^{\infty} \sum_{K=-\infty}^{\infty} \frac{l!}{\left(\frac{l+n+2N\,K}{2}\right)!\left(\frac{l-n-2N\,K}{2}\right)!} \left(\frac{a(\omega)}{2}\right)^l. \tag{31}$$

Since the coefficients of a Newton's binomial formula have to be real and positive, in the limit $a(\omega)\ll 1$ we obtain

$$B_0(\omega) \simeq \frac{U}{\omega - U}(1+M), \tag{32}$$

where

$$M = 1 - \frac{1}{2N} \sum_{q=0}^{2N-1} \frac{1}{1 - \frac{2w}{U}\cos q} \tag{33}$$

contains powers of w/U and has to be calculated at the desired order in q, and

$$B_N(\omega) \simeq -\frac{1}{2^N}\left(\frac{2w}{U}\right)^N. \tag{34}$$

Here we note that the last contribution cannot be ignored because it gives rise to the energy separation between $|\Phi_0\rangle$ and $|\Phi_N\rangle$.

Furthermore, we obtain

$$|A_0(\omega)\rangle \simeq \frac{1}{U}|\Phi_0(t=0)\rangle \tag{35}$$

and

$$|A_N(\omega)\rangle \simeq \frac{1}{U}|\Phi_N(t=0)\rangle. \tag{36}$$

As a result, after an inverse Laplace transform, we get

$$|\Phi_0(t)\rangle = e^{iMUt}[|\Phi_0(t=0)\rangle\cos\Delta t + i|\Phi_N(t=0)\rangle\sin\Delta t] + O\left(\frac{w}{U}\right) \tag{37}$$

and

$$|\Phi_N(t)\rangle = e^{iMUt}[|\Phi_N(t=0)\rangle\cos\Delta t + i|\Phi_0(t=0)\rangle\sin\Delta t] + O\left(\frac{w}{U}\right), \tag{38}$$

having introduced the energy gap

$$\Delta = 2w(2w/U)^{N-1}. \tag{39}$$

We eventually obtain the long time behavior of a two-level system with energy separation exponentially vanishing in the large-N limit. Actually, in Ref. [19] [see Eq. (3.32c)], the eigenvalue of Eq. (39) was derived. On the basis of this

result, the phenomenon of asymptotic degeneracy was established and shown to be directly related to the appearance of the ordered phase in the large-N limit.

V. DOUBLE-DOT ARRAY DECOHERENCE IN THE LONG TIME LIMIT

According to the previous analysis, we, can limit ourselves to considering only the first two states $|\pm\rangle$ of the array. The decoherence rate however, will be modified by the extensive interaction with the bath.

We have to calculate the matrix elements of $G(H_S)$ in the subspace of $|+\rangle$ and $|-\rangle$ taking into account the particular system-bath interaction H_{SB} defined in Eq. (6). Here

$$G(H_S) = \sum_{\mathbf{q},l,l'} e^{iq\cos\theta(l-l')}|g_{\mathbf{q}}|^2 n_{l'} \frac{1}{\omega - H_S - \omega_{\mathbf{q}}} n_l, \quad (40)$$

where the sum over l,l' runs over the array sites where electrons are present.

We choose the basis elements $|+\rangle$ and $|-\rangle$ defined, respectively, as the sum and the difference of $|\Phi\rangle$ and $|\Psi\rangle$.

We introduce the form factor $\Lambda_{q\cos\theta}$ defined through

$$\sum_l n_l e^{iq\cos\theta l}|\Phi\rangle = \Lambda_{q\cos\theta}|\Phi\rangle \quad (41)$$

or

$$\sum_l n_l e^{iq\cos\theta l}|\Psi\rangle = e^{iq}\Lambda_{q\cos\theta}|\Psi\rangle. \quad (42)$$

Explicitly, $\Lambda_{q\cos\theta} = (1 - e^{i2q\cos\theta N})/(1 - e^{i2q\cos\theta})$.

The matrix elements of the self-energy operator are

$$G^{++} = \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 |\Lambda_{q\cos\theta}|^2 \left[\frac{\cos^2\frac{q\cos\theta}{2}}{\omega - E_+ - \omega_{\mathbf{q}}} + \frac{\sin^2\frac{q\cos\theta}{2}}{\omega - E_- - \omega_{\mathbf{q}}} \right], \quad (43)$$

$$G^{--} = \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 |\Lambda_{q\cos\theta}|^2 \left[\frac{\cos^2\frac{q\cos\theta}{2}}{\omega - E_- - \omega_{\mathbf{q}}} + \frac{\sin^2\frac{q\cos\theta}{2}}{\omega - E_+ - \omega_{\mathbf{q}}} \right], \quad (44)$$

$$G^{-+} = (G^{+-})^* = i\sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 |\Lambda_{q\cos\theta}|^2 \cos\frac{q\cos\theta}{2} \times \sin\frac{q\cos\theta}{2} \left[\frac{1}{\omega - E_+ - \omega_{\mathbf{q}}} - \frac{1}{\omega - E_- - \omega_{\mathbf{q}}} \right]. \quad (45)$$

We find convenient the introduction of the following generalized densities of states:

$$\rho^{+-}(\epsilon) = (\rho^{-+})^*(\epsilon) = -i\sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 |\Lambda_{q\cos\theta}|^2 \times \cos\frac{q\cos\theta}{2}\sin\frac{q\cos\theta}{2}\delta(\epsilon - \omega_{\mathbf{q}}), \quad (46)$$

$$\rho_1(\epsilon) = \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 |\Lambda_{q\cos\theta}|^2 \cos^2\frac{q\cos\theta}{2}\delta(\epsilon - \omega_{\mathbf{q}}), \quad (47)$$

$$\rho_2(\epsilon) = \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 |\Lambda_{q\cos\theta}|^2 \sin^2\frac{q\cos\theta}{2}\delta(\epsilon - \omega_{\mathbf{q}}), \quad (48)$$

from which follows

$$G^{++} = \int d\epsilon \left[\frac{\rho_1(\epsilon)}{\omega - E_+ - \epsilon} + \frac{\rho_2(\epsilon)}{\omega - E_- - \epsilon} \right], \quad (49)$$

$$G^{+-} = -i\int d\epsilon \rho^{+-}(\epsilon) \left[\frac{1}{\omega - E_+ - \epsilon} - \frac{1}{\omega - E_- - \epsilon} \right], \quad (50)$$

$$G^{--} = \int d\epsilon \left[\frac{\rho_1(\epsilon)}{\omega - E_- - \epsilon} + \frac{\rho_2(\epsilon)}{\omega - E_+ - \epsilon} \right]. \quad (51)$$

The real part of G gives a negligible contribution to the pole location if compared with E_- and E_+. Thus, assuming a density of state different from zero only for positive ϵ, as in Ref. [12], the only nonvanishing contribution is $\gamma = -\mathrm{Im}\, G^{--}$,

$$\gamma = -\pi\rho_2(\Delta), \quad (52)$$

where $\Delta = E_- - E_+$ is the energy gap of the two-level system and is positive ($|+\rangle$ being the ground state).

Then the solution for $\langle +|\Phi_S(t)\rangle$ and $\langle -|\Phi_S(t)\rangle$ is

$$\langle +|\Phi_S(t)\rangle = e^{iE_+ t}\langle +|\Phi_S(t=0)\rangle, \quad (53)$$

$$\langle -|\Phi_S(t)\rangle = e^{iE_- t - \gamma t}\langle -|\Phi_S(t=0)\rangle. \quad (54)$$

As expected, the ground state is not affected by decoherence, while the excited state relaxes. Damping is proportional to the density of states calculated at the energy gap. The density of states is, however, quite different from that of a single-dot pair. Two competitive effects appear. The first one is represented by the presence of the form factor $\Lambda_{q\cos\theta}$ inside ρ_2, which, in the large-N limit, increases the dephasing rate by a factor proportional to N^2. The second, predominant, effect to be considered is the expontential reduction with N of the energy separation.

For instance, in the simple case of $|g_{\mathbf{q}}|^2 = 1/N$ and $\omega_{\mathbf{q}} = cq$ (longitudinal phonons),

$$\gamma(\Delta) \propto \int d\cos\theta d^d q \frac{\sin^2 qN}{\sin^2 q\cos\theta}\sin^2\frac{q\cos\theta}{2}\delta(\Delta - c^2q^2), \quad (55)$$

where d is the dimension of the bath and c is the speed of sound. If we compare this quantity with the system oscillation frequency, we obtain

$$\frac{\gamma(\Delta)}{\Delta} \propto N^2\Delta^{d/2-1}. \quad (56)$$

This result indicates that, for a phonon bath in three dimensions, the macroscopic limit involves a growth of the robustness with respect to decoherence.

VI. CONCLUSIONS

The existence of a macroscopic degenerate ground state is a general feature of a system exhibiting a phase transition. It would be of interest to exploit such degenerate states as elements of a macroscopic qubit. Here we considered an array of N interacting dot pairs. We showed that, in the long-time limit and neglecting correction of order w/U, the system oscillates between the two configurations characterized by zero electrostatic energy. Such a system has a robustness which increases with the size as shown calculating the decoherence due to a phonon bath at zero temperature. Decoherence calculation has been performed in the framework of an application of the resolvent method. It is, however, important to note that a general conclusion about the robustness of macroscopic qubits should consider nonzero temperature effects and other mechanisms for dephasing in semiconductors (such as cotunneling and background charge fluctuations).

As said, coherent manipulation of $|S\rangle$ is useful in order to realize a support for quantum information transfer allowing a solid-state teleportation. The basic information processing steps, i.e., initialization of the system, local gates, and readout, are, respectively, obtained by adiabatic variation of system parameters, oscillations between $|\Phi\rangle$ and $|\Psi\rangle$, and local charge measurement. Moreover, it is worth noting that, because of the analogy with spin clusters behavior, most of the considerations on quantum computing developed in Ref. [3] should be valid also for our system, with the advantage that instead of measuring spin states, we need to detect local charges. Due to the asymptotic behavior of the effective hopping amplitude, such a qubit requires gate times exponentially increasing with the size of the system, and the optimal chain length will be determined by the specific application.

APPENDIX: DECOHERENCE RATE IN A DOUBLE QUANTUM DOT

We introduce a double quantum dot in contact with a bosonic bath with a Hamiltonian,

$$H = H_S + H_B + H_{SB},$$

$$H_S = \frac{\varepsilon}{2}\sigma_z + T\sigma_x,$$

$$H_B = \sum_q \omega_q a_q^\dagger a_q,$$

$$H_{SB} = \frac{1}{2}\sigma_z \sum_q g_q\left(a_q^\dagger + a_q\right), \tag{A1}$$

discussed in Ref. [12]. Here a one-dimensional bath is considered for simplicity. Labeling with $|L\rangle$ and $|R\rangle$ the eigenstates of σ_z with respective eigenvalues +1 and −1, the eigenstates of H_S are

$$|\pm\rangle = \frac{1}{N_\pm}[\pm 2T|L\rangle + (\Delta \mp \varepsilon)|R\rangle], \tag{A2}$$

where $\Delta = \sqrt{\varepsilon^2 + 4T^2}$ and $N_\pm = \sqrt{(\Delta \mp \varepsilon)^2 + 4T^2}$ while the respective eigenvalues are $\varepsilon_\pm = \pm\frac{1}{2}\Delta$.

By inversion, we obtain

$$|L\rangle = N_+\frac{\Delta + \varepsilon}{4T\Delta}|+\rangle - N_-\frac{\Delta - \varepsilon}{4T\Delta}|-\rangle, \tag{A3}$$

$$|R\rangle = \frac{N_+}{2\Delta}|+\rangle + \frac{N_-}{2\Delta}|-\rangle. \tag{A4}$$

Equation (12) now has to be solved using

$$G(H_S) = \frac{1}{4}\sum_q |g_q|^2 \sigma_z \frac{1}{\omega - \omega_q - H_S}\sigma_z. \tag{A5}$$

We need to calculate $\langle +|G(H_S)|+\rangle$ and $\langle -|G(H_S)|-\rangle$. Actually, obtaining G^{++} will be enough, due to the intrinsic robustness of the ground state $|-\rangle$ [30], which implies that G^{--} has to be zero (this feature is easily checked in the present formalism). To do it, first we write $|+\rangle$ in the $|L,R\rangle$ basis, then apply σ_z, come back in the $|\pm\rangle$ basis in order to apply $(\omega - \omega_q - H_S)^{-1}$, rewrite the new state through $|L,R\rangle$ to apply the second σ_z operator, and finally reexpress the result in terms of $|+\rangle$ and $|-\rangle$. The result is

$$G^{++} = \frac{1}{4}\sum_q |g_q|^2\left[\frac{1}{\omega - \omega_q - \frac{\Delta}{2}}\left(\frac{\varepsilon}{\Delta}\right)^2 + \frac{1}{\omega - \omega_q + \frac{\Delta}{2}}\left(\frac{\Delta - \varepsilon}{\Delta}\right)^2\right]. \tag{A6}$$

The sum over q is performed as an integral through the introduction of the density of states ρ, which is assumed to be different from zero only for positive values of its argument [12]. The second term inside the square brackets gives the contribution to the imaginary part, which is $\gamma = -[\pi T^2 \rho(\Delta)]/\Delta^2$. The evolution is thus

$$\langle + |\Phi_S(t)\rangle = \langle + |\Phi_S(t=0)\rangle e^{-i(\Delta/2)t} e^{-\pi(T^2/\Delta^2)\rho(\Delta)t}, \tag{A7}$$

$$\langle - |\Phi_S(t)\rangle = \langle - |\Phi_S(t=0)\rangle e^{i(\Delta/2)t}. \tag{A8}$$

The density matrix in the basis $|\pm\rangle$ is then

$$\rho(t) = \begin{pmatrix} \rho^{++}(0)e^{-2\gamma t} & \rho^{-+}(0)e^{-\gamma t}e^{i\Delta t} \\ \rho^{-+}(0)e^{-\gamma t}e^{i\Delta t} & 1 - \rho^{++}(0)e^{-2\gamma t} \end{pmatrix} \tag{A9}$$

with the same dephasing rate obtained in [12], in the regime of zero temperature, using Markovian assumptions.

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