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Entanglement swapping without joint measurement

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We propose an entanglement swapping scheme in cavity QED. In the scheme, the previously used joint measurement is not needed. The entanglement swapping in our proposal is a non-post-selection one, i.e., after the swapping is done, the swapped entanglement is still there.

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Entanglement plays an important role in quantum-information processing (QIP). Several schemes have been proposed to generate entangled states, such as the nonlinear interaction between optical pulse and nonlinear crystal [1], the interaction between different particles, and so on [2,3]. After the entanglement generation, the entangled particles must be distributed among distant users for quantum-communication purpose. During the distribution process, the entanglement of the particles will inevitably decrease. The longer the distance, the bigger the decrease. To avoid this problem, an alternative method has been proposed to generate entanglement between two distant particles that have never interacted before. That is the so-called entanglement swapping [4]. In this method, there are usually three spatially separate users, and two of them have shared one pair of entangled particles with the third. Then the third user will operate a joint measurement (such as Bell-state measurement) on the two particles he possesses. Corresponding to the measurement result, the two particles possessed by the two spatially separate users will collapse into an entangled state without any entanglement before the joint measurement. Recently, entanglement swapping schemes have been proposed using linear optical elements with post-selection measurement [5] or without post-selection measurement [6]. In addition to the entanglement generation, entanglement purification is another application of entanglement swapping [7]. So the realization of entanglement swapping is very important for the quantum communication. Because the Bell-state measurement or other types of joint measurement is the core of the previous entanglement swapping schemes, the realization of the Bell-state measurement is of the key importance for the entanglement swapping. Hitherto, we still can not discriminate the four Bell states totally and conclusively. At the same time, the realization of the Bell-state measurement is still difficult in experiment, especially for the atomic Bell states.

To overcome this difficulty, we propose a new entanglement swapping scheme, where we only need single measurement rather than joint measurement. Although, our scheme follows Zheng's teleportation scheme [8], where he realized the teleportation of unknown atomic state without a Bell-state measurement, we have improved the scheme. In this paper, we discuss a scheme for entanglement swapping using concepts of cavity QED. Initially, one party A shares an entangled pair of atoms with another party C. C also shares an entangled pair of cavities with a third party B. Next, the atom and the cavity of C are made to interact for a fixed time after which a state measurement is performed on atom or on cavity. For both cases it is shown that, with a certain finite probability, the final state of A's atom and B's cavity is a maximally entangled state.

Although the creation of entangled photons is relatively easy when compared with entangling cavities and atoms, the entangled states with photons are difficult to be stored for future use. We may choose to swap the entanglement to two atoms which can be easily stored for future use. This has potential value for real application in practice in the future.

Next, let us go into the detailed entanglement swapping scheme. Suppose there are three spatially separate users Alice, Bob, and Clare. Alice and Clare have shared a pair of atoms (1, 2) with atom 1 belonging to Alice and atom 2 belonging to Clare. These two atoms have been previously prepared in the following entangled state:

$$|\Phi\rangle_{12} = a|e\rangle_1|e\rangle_2 + b|g\rangle_1|g\rangle_2, \qquad (1)$$

where a and b are the normalization coefficients.

In addition to the atom 2, Clare also possesses one single mode cavity 3, which is entangled with another single mode cavity 4, and the cavity 4 belongs to Bob. Similarly, we also suppose the two cavities have been prepared in the following entangled state:

$$|\Phi\rangle_{34} = a|1\rangle_3|1\rangle_4 + b|0\rangle_3|0\rangle_4, \qquad (2)$$

where $|0\rangle$ and $|1\rangle$ denote the vacuum state and one photon state of the cavity mode, respectively, and the normalization coefficients are all the same to the atomic entangled state in Eq. (1). From the above two entangled states, we conclude that there is no correlation between the atom 1 and cavity 4 at this moment. After entanglement swapping, the atom 1 and cavity 4 will be left in a maximally entangled state.

To realize the entanglement swapping, Clare will let the atom 2 through the cavity 3. Suppose the atomic transition frequency is resonant with the cavity mode, then, in the interaction picture the interaction can be described as

$$H_I = g(aS^+ + a^+S^-), \qquad (3)$$

where g is the coupling constant between the atom and the cavity mode, a and a^+ are annihilation and creation operators

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of the cavity mode, respectively, and $S^{+}=|e\rangle\langle g|$, $S^{-}=|g\rangle\langle e|$ are raising and lowering operators for atom 2 with $|e\rangle$, $|g\rangle$ being the excited state and ground state of the atoms, respectively.

Before swapping, the state of the total system is

$$|\Psi\rangle_{1234}=(a|e\rangle_1|e\rangle_2+b|g\rangle_1|g\rangle_2)\otimes(a|1\rangle_3|1\rangle_4+b|0\rangle_3|0\rangle_4). \tag{4}$$

After interaction time t, the state of the total system will evolve into the following state:

$$\begin{aligned}|\Psi\rangle'_{1234}=&\,a^2|e\rangle_1|1\rangle_4[\cos(\sqrt{2}gt)|e\rangle_2|1\rangle_3-i\sin(\sqrt{2}gt)|g\rangle_2|2\rangle_3]\\&+ab|e\rangle_1|0\rangle_4[\cos(gt)|e\rangle_2|0\rangle_3-i\sin(gt)|g\rangle_2|1\rangle_3]\\&+ab|g\rangle_1|1\rangle_4[\cos(gt)|g\rangle_2|1\rangle_3-i\sin(gt)|e\rangle_2|0\rangle_3]\\&+b^2|g\rangle_1|0\rangle_4|g\rangle_2|0\rangle_3.\end{aligned} \tag{5}$$

After the atom flying out of the cavity, Clare will detect the atom 2. If the atom 2 is detected in excited state, the atom 1 and cavities 3, 4 will collapse into

$$\begin{aligned}|\Psi\rangle'_{134}=&\,N\{ab[\cos(gt)|e\rangle_1|0\rangle_4-i\sin(gt)|g\rangle_1|1\rangle_4]|0\rangle_3\\&+a^2\cos(\sqrt{2}gt)|e\rangle_1|1\rangle_3|1\rangle_4\},\end{aligned} \tag{6}$$

where N is the normalization factor. If Clare chooses the interaction time to satisfy $gt=7\pi/4$, we get that $\cos\sqrt{2}gt=0.079\approx 0$. So the third term in Eq. (6) can be eliminated. Then the atom 1 and cavity 4 collapsed approximately into a maximally entangled state without detection on the cavity 3:

$$|\Psi\rangle_{14}=\frac{1}{\sqrt{2}}(|e\rangle_1|0\rangle_4+i|g\rangle_1|1\rangle_4), \tag{7}$$

with probability $P=|b|^2(1-|b|^2)$. After a rotation operation, the entangled state can be transformed into the standard form with a zero relative phase factor. The fidelity of the output state relative to a perfect maximally entangled state is $F=|b|^2/[|b|^2+(1-|b|^2)\cos^2(\sqrt{2}gt)]$. From Fig. 1, we conclude that the fidelity is bigger than 0.9 unless $b<0.25$, and the fidelity reaches 0.99 when $b=0.6$. The biggest successful probability can reach 0.25. After swapping, Bob can send a resonant atom through the cavity 4. If Bob sets the interaction time appropriately, the interaction can swap the atom and cavity excitations. Then the two atoms, which belong to Alice and Bob, and never interact before, are in a maximally entangled state.

From the above process, we found that the measurement on atomic state is still needed in the above scheme. Next, we will prove that this scheme also can realize the entanglement swapping without the measurement on atoms. From the evolution result in Eq. (5), if the coefficient b of the initial state is relative small, the last term of Eq. (5) can be eliminated from the total state approximately. Then Clare can detect the cavity 3 rather than the atom 2 to complete the entanglement swapping process. Further more, Clare only need an ordinary photon detector, which only distinguishes the vacuum and nonvacuum Fock states, rather than a sophisticated photon detector that can distinguish one and two photons. If cavity 3 is detected in the vacuum state, atoms 1, 2 and cavity 4 will collapse into

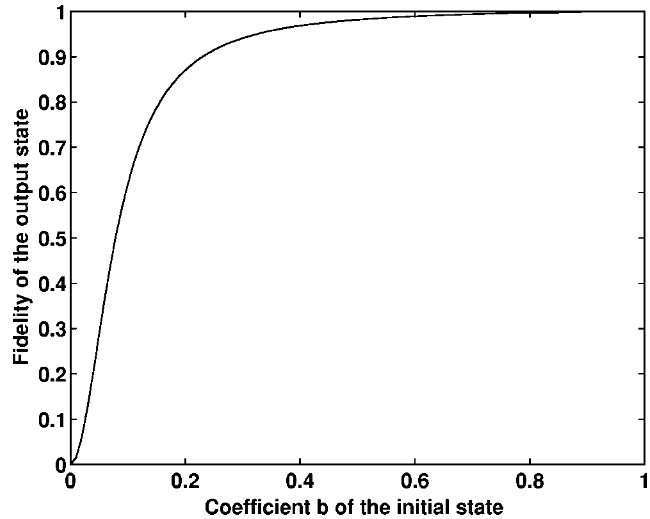

FIG. 1. The fidelity of the output state relative to a perfect maximally entangled state varies with the coefficient b of the initial state. Here $gt=7\pi/4$.

$$\begin{aligned}|\Psi\rangle'_{124}=&\,N'\{ab[\cos(gt)|e\rangle_1|0\rangle_4-i\sin(gt)|g\rangle_1|1\rangle_4]|e\rangle_2\\&+b^2|g\rangle_1|0\rangle_4|g\rangle_2\},\end{aligned} \tag{8}$$

where N' is the normalization factor. If Clare chooses the interaction time to satisfy $gt=7\pi/4$, the atom 1 and cavity 4 will collapse approximately into a maximally entangled state expressed in Eq. (7) without detection on the atom 2. The probability of obtaining this state is still $P'=|b|^2\times(1-|b|^2)$, and the fidelity of the output state relative to the perfect maximally entangled state is $F'=1-|b|^2$. To achieve $F'=0.96$, the coefficient b must be 0.2, which still guarantee that the fidelity is bigger than 0.95. Here, the successful probability is 0.04.

The coefficients in Eqs. (1) and (2) are chosen to be identical in this paper (in amplitude and phase). If there exists an error between the coefficients in Eqs. (1) and (2), what will the result become? Through analysis, we find that the current scheme allows the existence of a small error between the coefficients in Eqs. (1) and (2). Suppose the states in Eqs. (1) and (2) can be reexpressed as

$$|\Phi\rangle_{12}=\sqrt{1-b^2}|e\rangle_1|e\rangle_2+b|g\rangle_1|g\rangle_2, \tag{9}$$

$$|\Phi\rangle_{34}=\sqrt{1-b^2(1+k)^2}|1\rangle_3|1\rangle_4+b(1+k)|0\rangle_3|0\rangle_4, \tag{10}$$

where k is the small error rate constant of the coefficient b, and k, b must satisfy $|b|<1$, $|b(1+k)|<1$. To deduce the success probability and the fidelity of the output state, we will consider the first case as example. If the above mentioned error exists, the success probability and the fidelity of the output state will become

$$P_{\mathrm{new}}=\frac{1}{2}\{(1-b^2)b^2(1+k)^2+b^2[1-b^2(1+k)^2]\}, \tag{11}$$

$$F_{\text{new}} = \frac{\frac{1}{2}[\sqrt{1-b^2}b(1+k) + b\sqrt{1-b^2(1+k)^2}]^2}{(1-b^2)b^2(1+k)^2 + b^2[1-b^2(1+k)^2] + 2(1-b^2)[1-b^2(1+k)^2]\cos^2\sqrt{2}gt}. \tag{12}$$

If we let the error rate constant $k=0.1$, $b=0.6$, $P_{\text{new}}=0.24098$, $F_{\text{new}}=0.98463$. That is to say, this kind of error will affect the probability of success and the fidelity very slightly. The same analysis applies to the second case.

In addition, the current scheme requires the entangled pairs of atoms (cavities) be distributed between A and C (B and C), which should be separated by large distances. This point cannot easily be realized within the current cavity QED technology. Unlike the photon-based schemes, where optical fibers provide a low-loss communication channel, sending single atoms (or entire cavities) over large distances while maintaining their quantum state is still beyond present technology. This may narrow the application of the current scheme. But swapping and teleportation are not only supposed to make quantum communication between two remote parties, it has broad applications in quantum information processing, e.g., in quantum computation [9]. We mean that, even though the two cavities are not separated far away, the swapping is still useful in QIP. For another example, the entangled states with photons are difficult to be stored for future use. We may choose to swap the entanglement to two atoms which can be easily stored for future use.

Next, we will discuss the feasibility of the scheme. From Refs. [8,10], we get that, we should make use of Rydberg atoms with principal quantum numbers 50 and 51, because their radiative time are $T_r=3\times10^{-2}$ s. For a normal cavity, the decay time can reach $T_c=1.0\times10^{-3}$ s. The coupling constant is $g=2\pi\times25$ kHz. Then we get that the interaction time of atom and cavity is $7\pi/4g=3.5\times10^{-5}$ s, so we can evaluate that the total time for the whole scheme is about $T=3.5\times10^{-4}$ s, which is much shorter than T_r, T_c. Hence, the current scheme might be realizable in the near future.

In the paper, we considered the initial states of the form $a|e\rangle_1|e\rangle_2+b|g\rangle_1|g\rangle_2$ for atoms and $a|1\rangle_3|1\rangle_4+b|0\rangle_3|0\rangle_4$ for cavities just for clarifying the principle of the current scheme. Considering the feasibility of the scheme, initial states of the form $a|g\rangle_1|e\rangle_2+b|e\rangle_1|g\rangle_2$ for atoms and $a|1\rangle_3|0\rangle_4+b|0\rangle_3|1\rangle_4$ for cavities might be more realistic, and this kind of entangled states for Rydberg atoms and microwave cavities have been prepared by Haroche's group in experiment [10,11]. Further calculation suggests that initial states of the form $a|g\rangle_1|e\rangle_2+b|e\rangle_1|g\rangle_2$ for atoms and $a|1\rangle_3|0\rangle_4+b|0\rangle_3|1\rangle_4$ for cavities also can lead to the same results as derived in the current scheme.

Seemingly only one measurement is needed in the schemes, but a coincidence measurement (of one atom and one cavity) is essentially required to obtain a maximally entangled state as in Ref. [5]. Generally, a single measurement never yields ideal output. Here because the state to be measured can be approximately factorized from the total state of the system, we replace the coincidence measurement with a single measurement.

Based on cavity QED, we have presented an entanglement swapping scheme. The most distinct advantage of it is that it does not need a joint measurement needed by the previous entanglement swapping schemes. It only needs a resonant interaction between an atom and a cavity mode and a measurement on cavity (or atom). Our proposal is for non-post-selection, i.e., after the swapping is done, the swapped entanglement is still there. This has potential value for real application in practice in the future.

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