Scheme for implementing quantum dense coding in cavity QED

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An experimentally feasible scheme for implementing quantum dense coding in cavity QED is proposed. In the scheme the atoms interact simultaneously with a highly detuned cavity mode with the assistance of a classical field. The scheme is insensitive to the cavity decay and the thermal field. Bell states can be exactly distinguished via detecting the atomic state, and the quantum dense coding can be realized in a simple way.

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Quantum dense coding [1] is one of the important applications of quantum entanglement in quantum communication. Dense coding can transmit two bits of classical information by sending only one quantum bit with the help of entanglement if the sender (Alice) and the receiver (Bob) share a maximally entangled state (Bell state). Two-particle quantum dense coding has been experimentally demonstrated by using optical systems and techniques of nuclear magnetic resonance [2–4]. Some theoretical schemes for quantum dense coding have been proposed [5–10] by using GHZ state and the nonmaximally two-particle entangled state. As one of the possible candidates for engineering quantum entanglement, the cavity quantum electrodynamics system always attracts much attention [11]. This is due to the fact that cold and localized atoms are not only the important resource of entanglement, but also well suited for storing quantum information in long-lived internal states. Particularly, Riebe *et al.* [12] and Barrett *et al.* [13] have implemented quantum teleportation of atomic qubits by using Ca^+ ions in a linear Paul trap and using Be^+ ions in a segmented ion trap, respectively. It shows that the techniques of manipulating and transforming quantum states have made a striking progress in cavity QED. However, the decoherence of the cavity field is still one of the main obstacles for the implementation of quantum information in cavity QED. Recently, Zheng and Guo proposed a scheme [14], where atoms interact with a nonresonant cavity and the cavity is only virtually excited. Osnaghi *et al.* [15] have experimentally demonstrated this scheme by the use of two Rydberg atoms crossing a nonresonant cavity. Following this idea, Lin *et al.* [16] proposed a scheme to implement quantum dense coding for two three-level atoms via cavity QED. In their scheme the photon-number-dependent Stark shifts cannot be canceled; the scheme requires that the cavity remains in the vacuum state throughout the procedure. Thus it is insensitive to the cavity decay. However, it is sensitive to the thermal field. Here we propose an alternative scheme for realizing quantum dense coding in cavity QED. The distinct advantage of the scheme is that during the passage of the atoms through the cavity field, a strong classical field is accompanied so that the photon-number-dependent parts are canceled. Thus this scheme is insensitive to both the cavity decay and the thermal field. More importantly, four Bell-state measurements can be exactly discriminated only by one step with the probability of success 1.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. In the rotating-wave approximation, the Hamiltonian is [17,18]

$$H = \omega_0 S_z + \omega_a a^+ a + \sum_{j=1}^{2} [g(a^+ S_j^- + a S_j^+) + \Omega(S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t})], \qquad (1)$$

where $S_z = \frac{1}{2}\sum_{j=1,2}|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|$, $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, and $|e_j\rangle$, $|g_j\rangle$ are the excited and ground states of the jth atom, respectively. a^+, a are the creation and annihilation operators for the cavity mode, g is the atom-cavity coupling strength, Ω is the Rabi frequency, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Assuming $\omega_0 = \omega$, in the interaction picture, the interaction Hamiltonian is [17,18]

$$H_I = \Omega \sum_{j=1,2} (S_j^+ + S_j^-) + g \sum_{j=1}^{2} (e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+), \qquad (2)$$

where δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω_a.

In the strong driving regime $\Omega \gg \delta, g$ and in the case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. Then in the interaction picture, the effective interaction Hamiltonian reads [18]

$$H_e = \frac{\lambda}{2}\left[\sum_{j=1}^{2} (|e\rangle_{jj}\langle e| + |g\rangle_{jj}\langle g|) + \sum_{i,j=1, i\neq j}^{2} (S_i^+ S_j^- + S_i^+ S_j^+ + \text{H.c.})\right], \qquad (3)$$

where $\lambda = g^2/2\delta$. We note that the effective Hamiltonian is independent of the cavity field state, allowing it to be in a thermal state.

Then the evolution operator of the system is given by

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$$U(t) = e^{-iH_0 t} e^{-iH_e t}, \tag{4}$$

where $H_0 = \Omega\Sigma_{j=1,2}(S_j^+ + S_j^-)$.

Assume two atoms are initially in the state $|ge\rangle_{12}$. The two atoms interact simultaneously with a single-mode cavity, at the same time the atoms are driven by a classical field, the state evolution of the system is

$$\begin{aligned}|\psi\rangle_{12} = e^{-i\lambda t}\{&\cos(\lambda t)[\cos(\Omega t)|g\rangle_1 - i\sin(\Omega t)|e\rangle_1][\cos(\Omega t)|e\rangle_2 \\ &- i\sin(\Omega t)|g\rangle_2] - i\sin(\lambda t)[\cos(\Omega t)|e\rangle_1 \\ &- i\sin(\Omega t)|g\rangle_1][\cos(\Omega t)|g\rangle_2 - i\sin(\Omega t)|e\rangle_2]\}.\end{aligned} \tag{5}$$

With the choice of $\Omega t = \pi$, $\lambda t = \pi/4$, we obtain the maximally two-atom entangled state

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}(|ge\rangle_{12} - i|eg\rangle_{12}). \tag{6}$$

Suppose that atom 1 belongs to Alice and atom 2 belongs to Bob.

To realize the quantum dense coding, Alice performs on her atom 1 one of the four local operations $\{I, \sigma^x, i\sigma^y, \sigma^z\}$ (where I is the identity operator and σ^i are three Pauli operators), which will, respectively, transform the state of two atoms into one of the following four states:

$$|\psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|ge\rangle_{12} - i|eg\rangle_{12}), \tag{7}$$

$$|\phi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|ee\rangle_{12} - i|gg\rangle_{12}), \tag{8}$$

$$|\phi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|ee\rangle_{12} + i|gg\rangle_{12}), \tag{9}$$

$$|\psi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|ge\rangle_{12} + i|eg\rangle_{12}). \tag{10}$$

Then Alice sends her atom 1 to Bob. In this case Bob possesses two atoms 1 and 2. Now what Bob wants to do is know how to discriminate the four states.

Bob let atom 1 and atom 2 simultaneously interact with another single-mode cavity and at the same time two atoms are driven by a classical field. The system evolution is described by Eq. (4). After an interaction time τ the states $|\psi^-\rangle$ and $|\phi^-\rangle$ evolve to

$$\begin{aligned}|\psi^-\rangle_{12} = \frac{e^{-i\lambda\tau}}{\sqrt{2}}\{&\cos(\lambda\tau)[\cos(\Omega\tau)|g\rangle_1 - i\sin(\Omega\tau)|e\rangle_1][\cos(\Omega\tau)|e\rangle_2 - i\sin(\Omega\tau)|g\rangle_2] - i\sin(\lambda\tau)[\cos(\Omega\tau)|e\rangle_1 - i\sin(\Omega\tau)|g\rangle_1] \\ &\times[\cos(\Omega\tau)|g\rangle_2 - i\sin(\Omega\tau)|e\rangle_2] - i\cos(\lambda\tau)[\cos(\Omega t)|e\rangle_1 - i\sin(\Omega\tau)|g\rangle_1][\cos(\Omega\tau)|g\rangle_2 - i\sin(\Omega\tau)|e\rangle_2] - \sin(\lambda\tau) \\ &\times[\cos(\Omega\tau)|g\rangle_1 - i\sin(\Omega\tau)|e\rangle_1][\cos(\Omega\tau)|e\rangle_2 - i\sin(\Omega\tau)|g\rangle_2]\},\end{aligned} \tag{11}$$

$$\begin{aligned}|\phi^-\rangle_{12} = \frac{e^{-i\lambda\tau}}{\sqrt{2}}\{&\cos(\lambda\tau)[\cos(\Omega t)|e\rangle_1 - i\sin(\Omega\tau)|g\rangle_1][\cos(\Omega\tau)|e\rangle_2 - i\sin(\Omega\tau)|g\rangle_2] - i\sin(\lambda\tau)[\cos(\Omega\tau)|g\rangle_1 - i\sin(\Omega\tau)|e\rangle_1] \\ &\times[\cos(\Omega\tau)|g\rangle_2 - i\sin(\Omega\tau)|e\rangle_2] - i\cos(\lambda\tau)[\cos(\Omega t)|g\rangle_1 - i\sin(\Omega\tau)|e\rangle_1][\cos(\Omega\tau)|g\rangle_2 - i\sin(\Omega\tau)|e\rangle_2] - \sin(\lambda\tau) \\ &\times[\cos(\Omega\tau)|e\rangle_1 - i\sin(\Omega\tau)|g\rangle_1][\cos(\Omega\tau)|e\rangle_2 - i\sin(\Omega\tau)|g\rangle_2]\}.\end{aligned} \tag{12}$$

If we choose the interaction time τ and Rabi frequency Ω appropriately so that $\lambda\tau = \pi/4$, $\Omega t = \pi$, the above quantum states will be

$$|\psi^-\rangle \to -i|e\rangle_1|g\rangle_2, \tag{13}$$

$$|\phi^-\rangle \to -i|g\rangle_1|g\rangle_2. \tag{14}$$

In this case the other two states evolve to

$$|\phi^+\rangle \to |e\rangle_1|e\rangle_2, \tag{15}$$

$$|\psi^+\rangle \to |g\rangle_1|e\rangle_2. \tag{16}$$

Hence, the Bell-state measurement can be achieved by detecting atoms 1 and 2 separately. With the outcome of the measurement on Bob's two atoms, Bob can distinguish Alice's operation on atom 1. So he receives two bits of classical information from Alice. That is, the dense coding is achieved.

Next we give a brief discussion on the experimental matters. For the Rydberg atoms with principal quantum numbers 49, 50, and 51, the radiative time is about $T_r = 3\times10^{-2}$ s, and the coupling constant is $g = 2\pi\times24$ kHz [14,15]. The required atom-cavity-field interaction time is on the order of $T\approx10^{-4}$ s. Then the time needed to complete the whole procedure is much shorter than T_r. Meanwhile it is noted that the atomic state evolution is independent of the cavity field state, thus based on cavity QED techniques presently the proposed scheme might be realizable.

In summary, we have proposed a simple scheme to realize quantum dense coding using the interaction of two two-level atoms with a single-mode nonresonant cavity with the assistance of a strong classical driving field. Four states codified by Alice can be completely discriminated only via the atom-cavity-field interaction. The success probability of this scheme is equal to 1. In the scheme we provide a way to achieve all operations of dense coding from generation of entangled state to Bell-state measurement. During the preparation of the entangled state and the Bell-state measurement, our scheme only involves atom-field interaction with a large detuning and does not require the transfer of quantum information between the atoms and cavity. In addition, with the help of a strong classical driving field the photon-number-dependent parts in the evolution operator are canceled. Thus the scheme is insensitive to the thermal field and the cavity decay. Our scheme may offer the identification of a convenient and reliable Bell analyzer in cavity QED, which could also be used for realizing teleportation and other purposes.

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[1] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[2] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).
[3] X. Li, Q. Pan, J. Jing, J. Zhang, C. Xie, and K. Peng, Phys. Rev. Lett. **88**, 047904 (2002).
[4] X. Fang, X. Zhu, M. Feng, X. Mao, and F. Du, Phys. Rev. A **61**, 022307 (2000).
[5] J. C. Hao, C. F. Li, and G. C. Guo, Phys. Rev. A **63**, 054301 (2001).
[6] J. L. Cereceda, e-print quant-ph/0105096.
[7] A. Barenco and A. K. Ekert, J. Mod. Opt. **42**, 1253 (1995).
[8] P. Hausladen, R. Jozsa, B. Schumacher, M. Westmoreland, and W. K. Wootters, Phys. Rev. A **54**, 1869 (1996).
[9] S. Bose, M. B. Plenio, and V. Vedral, J. Mod. Opt. **47**, 291 (2000).
[10] J. C. Hao, C. F. Li, and G. C. Guo, Phys. Lett. A **278**, 113 (2000).
[11] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, Science **288**, 2024 (2000); J. M. Raimond *et al.*, Rev. Mod. Phys. **73**, 565 (2001).
[12] M. Riebe *et al.*, Nature (London) **429**, 734 (2004).
[13] M. D. Barrett *et al.*, Nature (London) **429**, 737 (2004).
[14] S. B. Zheng and G. C. Guo, Phys. Rev. Lett. **85**, 2392 (2000).
[15] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001).
[16] X. M. Lin *et al.*, Phys. Lett. A **313**, 351 (2003).
[17] E. Solano, G. S. Agarwal, and H. Walther, Phys. Rev. Lett. **90**, 027903 (2003).
[18] S. B. Zheng, Phys. Rev. A **68**, 035801 (2003).