High-efficiency quantum-nondemolition single-photon-number-resolving detector

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We discuss an approach to the problem of creating a photon-number-resolving detector using the giant Kerr nonlinearities available in electromagnetically induced transparency. Our scheme can implement a photon-number quantum-nondemolition measurement with high efficiency (~99%) using fewer than 1600 atoms embedded in a dielectric waveguide.

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In recent years we have seen signs of a new technological revolution, caused by a paradigm shift to information processing using the laws of quantum physics. One natural architecture for realizing quantum-information-processing (QIP) technology would be to use states of light as the information-processing medium. There have been significance developments in all-optical QIP following the recent discovery by Knill, Laflamme, and Milburn that passive linear optics, photodetectors, and single-photon sources can be used to create massive reversible nonlinearities [1]. Such nonlinearities are an essential requirement for optical QIP and many communication applications. These nonlinearities allow efficient gate operations to be performed. In principle, fundamental operations such as the nonlinear sign shift and controlled-NOT gates have been demonstrated experimentally [2–4]. However, such operations are relatively inefficient (they have a probability of success significantly less than 50%) and so are not scalable, due primarily to the current state of the art in single-photon sources and detectors. Good progress is being made on the development of single-photon sources [5,6]. Absorptive single-photon-resolution detection is possible [7–9], with efficiencies up to ~90% (visible spectrum) and ~30% (infrared, microwaves). However, *true* universal optical QIP will require significant further improvements in detector efficiencies, which will likely require a drastic change of approach to detection technology [9–11].

In this article, we propose an implementation of the quantum-nondemolition (QND) single-photon detection scheme originally described by Imoto, Haus, and Yamamoto [12], with the required optical nonlinearity provided by the giant Kerr effect achievable with ac Stark-shifted electromagnetically induced transparency (EIT) [13]. We show below that the scheme uses about 1600 EIT atoms and a weak pulse in the probe mode to achieve an error probability less than 1%. The effect of the QND measurement in turn means that signal photons are not destroyed and can be reused if required [14]. Furthermore, for a signal mode in a superposition state (such as a weak coherent state) the number-resolving QND measurement projects the signal mode into a definite number state [15], so the detector can be used as a heralded source of number eigenstates. We focus on EIT as an example here since it has already been used successfully to demonstrate cross-Kerr nonlinearities at low light levels in rubidium [16,17]. From the perspective of realizing our detector application, we concentrate on potential condensed-matter mechanisms of EIT. However, clearly any system capable of producing a comparable form and strength of Kerr interaction can be used, including optical fibers [18], silica whispering-gallery microresonators [19], and cavity QED systems [14,20].

Before we begin our detailed discussion of the EIT detection scheme, we first consider the photon-number QND measurement using a cross-Kerr nonlinearity [12,21]. The Kerr Hamiltonian has the canonical form

$$H_{\mathrm{QND}} = \hbar \chi a^{\dagger} a c^{\dagger} c, \tag{1}$$

where the signal (probe) mode has creation and destruction operators given by $a^{\dagger}, a(c^{\dagger}, c)$, respectively, and χ is the strength of the nonlinearity. If the signal field contains n_a photons and the probe field is in an initial coherent state with amplitude α_c, the cross-Kerr optical nonlinearity causes the combined system to evolve as

$$|\Psi(t)\rangle_{\mathrm{out}} = e^{i\chi t a^{\dagger} a c^{\dagger} c} |n_a\rangle |\alpha_c\rangle = |n_a\rangle |\alpha_c e^{i n_a \chi t}\rangle. \tag{2}$$

We observe immediately that the Fock state $|n_a\rangle$ is unaffected by the interaction, but the coherent state $|\alpha_c\rangle$ picks up a phase shift directly proportional to the number of photons n_a in the $|n_a\rangle$ state. If we measure this phase shift using a homodyne measurement (depicted schematically in Fig. 1), we can infer the number of photons in the signal mode a. The homodyne apparatus allows measurement of the quadrature operator $\hat{x}(\phi) \equiv c e^{i\phi} + c^{\dagger} e^{-i\phi}$, with an expected result $\langle \hat{x}(\phi) \rangle = 2\,\mathrm{Re}[\alpha_c]\cos\delta + i 2\,\mathrm{Im}[\alpha_c]\sin\delta$, where $\delta = \phi + n_a \chi t$. For a real initial α_c, a highly efficient homodyne measurement of the momentum quadrature $Y \equiv \hat{x}(\pi/2)$ would yield the signal $\langle Y \rangle = 2\alpha_c \sin(n_a \chi t)$ with a variance of 1, thus giving a signal-to-noise ratio (SNR) of $\mathcal{R}_Y = 2\alpha_c \sin(n_a \chi t)$. If the input in mode a is the Fock state either $|0\rangle$ or $|1\rangle$, the corresponding output state of the probe mode c is the coherent state $|\alpha_c\rangle$ or $|\alpha_c e^{i\chi t}\rangle$. Using a momentum quadrature measurement, the probability of misidentifying one of these states for the other

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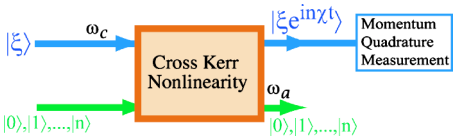

FIG. 1. Schematic diagram of a photon-number quantum-nondemolition detector based on a cross-Kerr optical nonlinearity [12]. The two inputs are a Fock state $|n_a\rangle$ (with $n_a=0, 1,\ldots$ in the signal mode a and a coherent state with real amplitude α_c in the probe mode c. The presence of photons in mode a causes a phase shift on the coherent state $|\alpha_c\rangle$ directly proportional to n_a which can be determined with a momentum quadrature measurement.

is then $P_{\text{error}}=\frac{1}{2}\text{erfc}(\mathcal{R}_Y/2\sqrt{2})$. A signal-to-noise ratio of $\mathcal{R}_Y=4.6$ would thus give $P_{\text{error}}\sim 10^{-2}$. To achieve the necessary phase shift we require $\alpha_c \sin(\chi t)\approx 2.3$, which can be achieved in a number of ways dependent upon the range of values available for α_c and χt. For example, we could choose $\alpha_c \gg 2.3$ with χt small and satisfy the above inequality; alternatively, we could choose $\chi t=\pi/2$ with $\alpha_c=2.3$. The particular regime chosen depends on the strength of the Kerr nonlinearity achievable in the physical system.

Figure 2 generalizes the detector shown schematically in Fig. 1 to the case where the polarization of the input state is resolved into different paths by a polarizing beam splitter. In general we may wish to apply different phase shifts to the two distinct polarizations, but in the case shown in Fig. 2 an identical phase shift is applied to each path, so that the detector is insensitive to the polarization of the input state. This is a particularly useful approach when the efficiency of the EIT system and/or the optical propagation path (e.g., as provided by a photonic crystal waveguide optimized for either TE or TM modes) is polarization dependent.

We now address the generation of the cross-Kerr nonlinearity required to perform the QND measurement. We consider a model (depicted in Fig. 3) of the nonlinear electric dipole interaction between three quantum electromagnetic radiation fields with angular frequencies $\omega_a, \omega_b, \omega_c$ and a corresponding four-level $\mathcal{N}$ atomic system [22]. Mode b should be thought of as a pump or coupling field: by choosing the correct conditions, we can factor both the coupling field and the atom out of the evolution of the atom-field system, creating an effective cross-Kerr nonlinear interaction between modes a and c. The effective vacuum Rabi frequency for each mode is defined as $|\Omega_k|^2=(\sigma_k/\eta_k\mathcal{A})A_k\Delta\omega_k/8\pi$, where $\sigma_k\equiv 3\lambda_k^2/2\pi$ is the resonant atomic absorption cross section at wavelength $\lambda_k\cong 2\pi c/\omega_k$ [23], η_k is the refractive index of the waveguide material, $\mathcal{A}$ is the effective laser mode cross-sectional area, $A_k\equiv f_k e^2\omega_k^2/2\pi\epsilon_0 m_e c^2$ for a transition with oscillator strength f_k, and $\Delta\omega_k$ is the bandwidth of the profile function describing the adiabatic interaction of a pulsed laser field with a stationary atom [24–26].

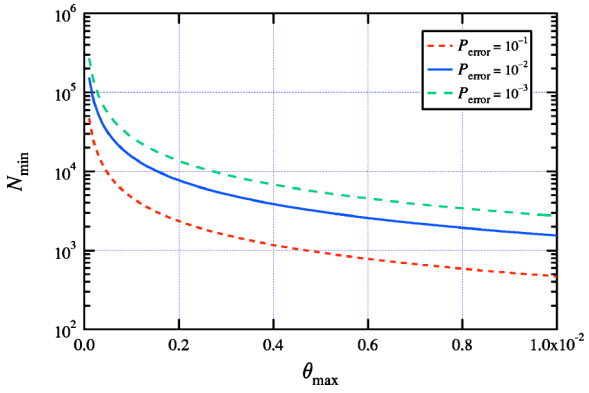

FIG. 2. Schematic diagram of a polarization-preserving photon-number quantum-nondemolition detector based on a pair of identical cross-Kerr optical nonlinearities. The signal mode is a Fock state with an unknown polarization, which is resolved into orthogonal polarization states by a polarizing beam splitter. The phase shift applied to the probe mode is proportional to n_a, independent of the polarization of the signal mode.

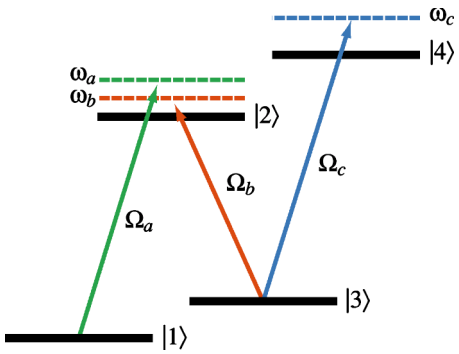

FIG. 3. Schematic diagram of the interaction between a four-level $\mathcal{N}$ atom and a nearly resonant three-frequency electromagnetic field. We note that the annihilation of a photon of frequency ω_k is represented by the complex number Ω_k.

It is difficult to achieve a substantial vacuum Rabi frequency using free-space fields [27], but encapsulating one or more atoms in a waveguide (such as a line defect in a photonic crystal structure) allows field transversality to be maintained at mode cross-sectional areas on the order of $\mathcal{A}\approx(\lambda/3\eta)^2$. Consider, then, a two-dimensional photonic crystal waveguide constructed from diamond thin film ($\eta=2.4$), with nitrogen-vacancy (NV) color centers fabricated in the center of the waveguide channel [30,31]. The optical transition at 637 nm in NV-diamond has an oscillator strength of approximately 0.12, within a factor of 3 of that of rubidium, which has been used successfully to demonstrate cross-Kerr nonlinearities at low light levels [16,17]. An EIT transmission window of about 8 MHz has been observed experimentally [30], so a pulse with $\Delta\omega_k/2\pi\lesssim 5$ MHz should propagate through this window with negligible loss. The corresponding vacuum Rabi frequency is therefore $\Omega\approx 3.6$ MHz.

We consider a number N of $\mathcal{N}$ atoms, fixed and stationary within a cylinder that is narrow but long compared to the optical wavelengths, with the three frequency modes of the system driven by Fock states containing n_a, n_b, and n_c photons, respectively. If the durations of the three pulse envelope functions are long compared to the lifetime of the atomic level $|2\rangle$, the evolution of the amplitude where all of the atoms are in the ground state $|1\rangle$ is simply given by $|1,n_a,n_b,n_c\rangle\rightarrow e^{-iWt}|1,n_a,n_b,n_c\rangle$. In general, W is complex [see Eq. (139) in Ref. [22]]; other states contribute to the full

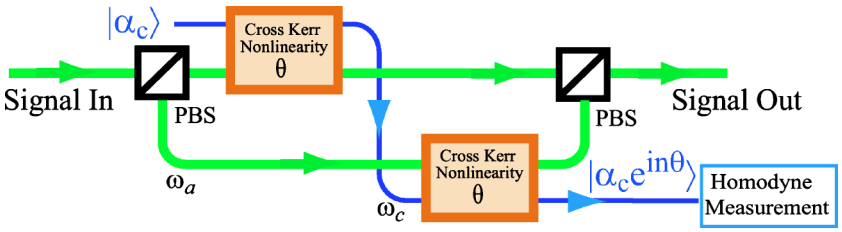

FIG. 4. Plot of the minimum number of NV-diamond color centers needed to generate the phase shift $\theta_{\max}$ for three different values of the error probability P_{error} in the case $n_a=1$. We have chosen $\langle n_b\rangle=10\langle n_c\rangle$, requiring a minimum detuning $\nu_{c\min}/\gamma_2=1.1$.

evolution and there is photon absorption and loss in the system. However, for our detector we require the probability of even single-photon loss from mode a to be very small, with a real W preserving the norm of $|1,n_a,n_b,n_c\rangle$. For this, we assume that the laser frequencies ω_a and ω_b are both precisely tuned to the corresponding atomic transition frequencies, so that the EIT Raman resonance condition is satisfied. Then, in the weak-signal regime, we assume that the decoherence rate γ_k is dominated by spontaneous emission from atomic level $|k\rangle$, and that $|\Omega_a|\lesssim\gamma_2$, so that W is given by

$$W=\frac{N|\Omega_a|^2|\Omega_c|^2n_an_c}{\nu_c|\Omega_b|^2n_b+i(\gamma_4|\Omega_b|^2n_b+\gamma_2|\Omega_c|^2n_c)}, \tag{3}$$

where $\nu_c\equiv\omega_c-\omega_{43}$. In principle, in this regime we must have $\nu_c|\Omega_b|^2n_b\gg\gamma_4|\Omega_b|^2n_b+\gamma_2|\Omega_c|^2n_c$ to obtain a nearly real W and a low residual absorption. As we shall show below, in practical cases where $|Wt|\ll1$, this constraint can be substantially relaxed. For NV-diamond, $\gamma_2^{-1}=2\times25$ ns [28,29], and the spin decoherence lifetime is 0.1 ms [30,31], so for $N\lesssim10\,000$, dephasing can be neglected, and $|\Omega_a|/\gamma_2\approx1$.

Under these conditions, the state $|1,n_a,n_b,n_c\rangle$ simply acquires a phase shift which is the basis for the emergence of the approximate cross-Kerr nonlinearity [22]. In general, when the pump and probe fields are intense coherent states (parametrized by α_b and α_c, respectively), the evolution of the state $|1,n_a,\alpha_b,\alpha_c\rangle$ can be approximated as $|1,n_a,\alpha_b,\alpha_ce^{-in_a(\theta-i\kappa)}\rangle$ [22], where the angle θ and the residual absorption κ is defined for the case $|\Omega_b|^2=|\Omega_c|^2$ by

$$\theta-i\kappa=\frac{N|\Omega_a|^2t}{\nu_c|\alpha_b|^2+i(\gamma_4|\alpha_b|^2+\gamma_2|\alpha_c|^2)}. \tag{4}$$

This is equivalent to a damped evolution generated by the cross-Kerr Hamiltonian of Eq. (1), with $\theta=\chi t$.

What values of θ-and, therefore, $\mathcal{R}_Y$-are achievable? To establish an estimate we need to make several assumptions about the physical system and its geometry. We assume that the interaction region (where the light and $\mathcal{N}$ atoms interact) is encapsulated within the photonic crystal waveguide described above, and that the pulses have weakly super-Gaussian profiles so that the bandwidth-interaction time product is $\Delta\omega_kt\approx3\pi$, giving $|\Omega_a|^2t\approx81\eta\gamma_2/8\pi$. Suppose now that the largest phase shift that can be applied in practice (without significantly distorting the signal pulse) is $\theta_{\max}$, and that the corresponding value of α_c needed to obtain a given signal-to-noise ratio is therefore $\alpha_c=\mathcal{R}_Y/2\theta_{\max}$ (if $\theta_{\max}\ll1$). Using Eq. (4), we can calculate the minimum number of atoms and the corresponding minimum detuning ν_c needed to generate this phase shift, by taking the real part and solving for ν_c explicitly. As ν_c is required to be real we find

$$N_{\min}=\frac{2\theta_{\max}}{|\Omega_a|^2t}(\gamma_4|\alpha_b|^2+\gamma_2|\alpha_c|^2)=\frac{\gamma_2\mathcal{R}_Y^2}{2\theta_{\max}|\Omega_a|^2t}\left(\frac{\gamma_4|\alpha_b|^2}{\gamma_2|\alpha_c|^2}+1\right) \tag{5}$$

with

$$\nu_{c\min}=\frac{N_{\min}|\Omega_a|^2t}{2|\alpha_b|^2\theta_{\max}}=\frac{\gamma_4|\alpha_b|^2+\gamma_2|\alpha_c|^2}{|\alpha_b|^2}. \tag{6}$$

When we choose $N=N_{\min}$ and $\nu_c=\nu_{c\min}$, we find that $\kappa=\theta$, so that the state $|1,n_a,\alpha_b,\alpha_c\rangle$ evolves according to $|1,n_a,\alpha_b,\alpha_ce^{-(1+i)n_a\theta}\rangle$. Therefore, when $\theta\ll1$, the residual absorption of a signal photon in mode a can also be made intrinsically small, even though the detuning is not large compared to the absorption linewidth [32].

Figure 4 shows the minimum number of NV-diamond color centers as a function of the maximum single-photon phase angle for three different values of the error probability P_{error}. We note that $\gamma_4=\gamma_2$ for the optical transitions in NV-diamond, and for convenience we have chosen $\langle n_b\rangle=10\langle n_c\rangle$, requiring a minimum detuning $\nu_{c\min}/\gamma_2=1.1$ in all three cases. Note that the minimum number of atoms needed to obtain a given phase shift decreases as the phase shift increases, because the constraint that the signal-to-noise ratio remain constant allows the values of $|\alpha_b|$ and $|\alpha_c|$ to decrease as $\theta_{\max}$ increases. As an example, we choose $P_{\text{error}}=0.01$ and $\theta_{\max}=0.01$ rad, requiring $\langle n_c\rangle=5.6\times10^4$ to maintain $\mathcal{R}_Y=4.6$. Therefore, $N\approx1600$ is sufficient to achieve the desired phase shift, resulting in a residual absorption of less than 1%.

There is considerable flexibility in the engineering-design parameter space for this implementation of the QND detector. For example, if we choose $\theta_{\max}=0.1$, following the design procedure outlined above for $P_{\text{error}}=0.01$ leads to a residual absorption of almost 10% for $N\approx160$ and $\langle n_c\rangle\approx560$. However, if we increase the number of atoms to 800 and the detuning to $\nu_c=11\gamma_2$, then the absorption is reduced to 1%. Note also that the detector can perform a QND measurement on a Fock state with $n_a>1$ with single-photon resolution. For example, as n_a increases from 1 to 2, the phase shift $n_a\theta$ doubles, and the SNR also increases from 4.6 to 9.2 for constant α_c. The detector sensitivity improves until the phase shift becomes so large that one of two fundamental limits is reached: either the SNR decreases below the 1% error

threshold, or the strong nonlinear interaction begins to significantly distort the pulse profile of the signal Fock state.

In summary, we have presented a scheme for a highly efficient photon-number quantum-nondemolition detector (with single-photon resolution) based on the cross-Kerr nonlinearity produced by an EIT condensed-matter system with approximately 1600 color centers. We have explored several different operating regimes, and we have examined in detail the performance of the detector for a NV-diamond photonic crystal waveguide system. In particular, we have shown that efficient detection is possible with small phase shifts, which will likely be necessary to ensure that the EIT optical nonlinearity does not distort the pulse envelope of the signal state. Future modeling will address detector performance for pulsed Fock state profile functions, and much experimental work remains to be done to implement such a detector. For example, fabricating EIT atomic or molecular systems into a dielectric waveguide is challenging but feasible. A method for orienting the color-center spins uniformly in such a condensed-matter system must be found, and spatial hole-burning techniques will be needed to overcome the effects of inhomogeneous broadening on the transparency of a condensed-matter medium [33]. The overall efficiency of the detector is likely to be limited by the efficiency of the homodyne measurement of the phase shift, which will depend on the degree to which the homodyne detector can be spatiotemporally mode matched to a single-photon signal. Nevertheless, EIT provides us with the best known candidate mechanism for the implementation of the original QND proposal by Imoto, Haus, and Yamamoto [12], and even a weak nonlinearity could allow efficient reuse of resources in linear-optics quantum-computation schemes.

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