Radiation properties of a Kerr nonlinear blackbody

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In a Kerr nonlinear blackbody, bare photons with opposite wave vectors and helicities are bound into pairs and unpaired photons are transformed into a different kind of quasiparticle, the nonpolariton. The nonpolariton system constitutes free thermal radiation in the blackbody. The present paper investigates the radiation properties of a Kerr nonlinear blackbody. We found that the spectral energy density and radiation pressure of a Kerr nonlinear blackbody are larger than those of a normal blackbody and that these two quantities are monotonically decreasing functions of the Kerr nonlinear coefficient. Above the transition temperature the photon system is in a normal thermal radiation state, but below the transition temperature it is in a squeezed thermal radiation state. In the transition from the normal to the squeezed thermal radiation state, the phase symmetry of the photon system is spontaneously broken.

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I. INTRODUCTION

Recently, quantum nonlinearity has gained extensive study. The statistics of polaritons in the nonlinear regime has been discussed [1]. In the presence of a second-order nonlinearity, the effects of a Kerr nonlinearity in second-harmonic generation have been investigated by using the positive P representation and a linearized fluctuation analysis [2]. A scheme for the quantum teleportation of the polarization state of a photon employing a cross-Kerr medium was presented and the experimental feasibility of the scheme discussed [3]. Within the framework of quantum-field theory, it has been shown that the photon system in a Kerr nonlinear blackbody is in a squeezed thermal radiation state [4,5].

Since the electromagnetic field is a quantum system of photons, the electromagnetic field in certain nonlinear media can exhibit some unusual quantum effects. Now consider a blackbody whose interior is filled by a Kerr nonlinear crystal. The crystal constitutes a Kerr nonlinear medium for the electromagnetic field. Further, the crystal is in thermal equilibrium with the electromagnetic field. The crystal and the thermal radiation constitute a system. We call this system a Kerr nonlinear blackbody. Such a Kerr nonlinear blackbody can be regarded as a rectangular crystal that has perfectly conducting walls and is kept at a constant temperature T. As shown in Fig. 1, there is a small hole in a wall through which thermal radiation can pass. In recent work [4,5], we have shown that the photon blackbody field in a Kerr nonlinear crystal is a squeezed thermal radiation state. In the present paper, we shall study the radiation properties of a Kerr nonlinear blackbody. We shall also investigate the symmetry of the squeezed thermal radiation state. Additional features that are worthy of exploration are pointed out here.

The electromagnetic field in thermal equilibrium is called blackbody radiation or thermal radiation. The bare photons in blackbody radiation can sense an attractive effective interaction by exchange of virtual nonpolar phonons. Such an interaction leads to a squeezed thermal radiation state, in which bare photons with opposite wave vectors and helicities are bound into pairs and unpaired bare photons are transformed into a different kind of quasiparticle, the nonpolariton. A nonpolariton is a condensate of virtual nonpolar phonons in momentum space below a transition temperature formed through a nonlinear photon-phonon interaction, with a bare photon acting as the nucleus of condensation. The vacuum for nonpolaritons is a condensate consisting of photon pairs and single nonpolaritons are elementary excitations from such a condensate. The squeezed thermal radiation state possesses some peculiar properties. First, the spectral energy density and radiation pressure of a Kerr nonlinear blackbody are larger than those of a normal blackbody. Second, the spectral energy density and radiation pressure are monotonically decreasing functions of the Kerr nonlinear coefficient. Third, in the transition from the normal to the squeezed thermal radiation state, the phase symmetry of the photon system is spontaneously broken. The predicted properties of the squeezed thermal radiation state might to be verified in present-day physics laboratories.

The remainder of this paper is organized as follows. Section II describes some properties of a normal blackbody. In Sec. III, we diagonalize the Hamiltonian of the photon system in a Kerr nonlinear blackbody. Section IV exposes the radiation properties of a Kerr nonlinear blackbody and gives the numerical calculation of physical quantities concerned. In Sec. V, we describe the symmetry change on phase transition. A comprehensive discussion is given in Sec. VI.

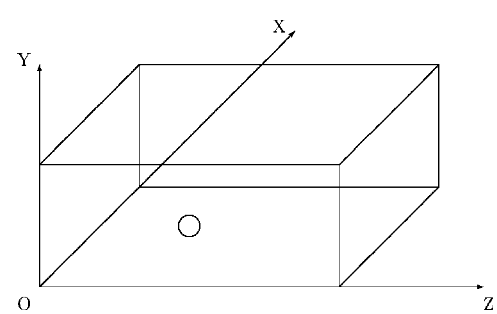

FIG. 1. A Kerr nonlinear blackbody: a rectangular Kerr nonlinear crystal enclosed by perfectly conducting walls and kept at a constant temperature; there is a very small hole in a wall.

II. NORMAL BLACKBODY

A. Quantization procedure

At the beginning of the 20th century the interpretation of the blackbody radiation spectrum revealed the dual character of electromagnetic radiation and became one of the origins of quantum theory. By definition, a blackbody absorbs 100% of all thermal radiation falling upon it. A close approximation to the blackbody is a small hole in a cavity in a solid that is maintained at some steady absolute temperature T. We shall call this system a normal blackbody. The electromagnetic field is composed of mutually exciting electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. The electromagnetic field is a transverse field, propagates in vacuum with the speed c of light, and satisfies the Maxwell equations. Since there are no free charges in the blackbody, we can set the scalar potential of the electromagnetic field to be zero. Hence, the electromagnetic field can be characterized by a single vector potential $\mathbf{A}$, which satisfies the Coulomb gauge $\nabla\cdot\mathbf{A}=0$. Consequently, the electric and magnetic fields are given by

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}. \tag{1}$$

The Hamiltonian of the electromagnetic field reads

$$H_{\text{em}} = \int d\mathbf{r}\left(\frac{\epsilon_0}{2}\mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2\right), \tag{2}$$

where ϵ_0 and μ_0 are the permittivity and the permeability of vacuum, respectively, with $\epsilon_0\mu_0=c^{-2}$.

Now we need to quantize the electromagnetic field. Since plane-wave modes constitute a complete orthonormal set, they can be used for the expansion of the electromagnetic field in any arbitrary geometry. The blackbody occupies a volume V. In terms of the creation and annihilation operators $a^\dagger_{\mathbf{k}\sigma}$ and $a_{\mathbf{k}\sigma}$ of circularly polarized photons with wave vector $\mathbf{k}$ and helicity $\sigma=\pm1$, the vector potential of the electromagnetic field is expanded as

$$\mathbf{A}(\mathbf{r},t) = \sum_{k\sigma}\left(\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}\right)^{1/2}[a_{\mathbf{k}\sigma}(t)\mathbf{e}_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{\mathbf{k}\sigma}(t)\mathbf{e}^*_{\mathbf{k}\sigma}e^{-i\mathbf{k}\cdot\mathbf{r}}], \tag{3}$$

where $\hbar$ is the reduced Planck's constant, $\omega_{\mathbf{k}}=c|\mathbf{k}|$ is the angular frequency of a photon, and $\mathbf{e}_{\mathbf{k},\pm1}$ are two orthonormal circular polarization vectors perpendicular to $\mathbf{k}$. The photon operators obey the Bose equal-time commutation relations:

$$[a_{\mathbf{k}\sigma}(t),a^\dagger_{\mathbf{k}'\sigma'}(t)]_- = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma\sigma'}, \quad [a_{\mathbf{k}\sigma}(t),a_{\mathbf{k}'\sigma'}(t)]_- = 0. \tag{4}$$

They have the time dependence: $a_{\mathbf{k}\sigma}(t)=a_{\mathbf{k}\sigma}(0)\exp(-i\omega_{\mathbf{k}}t)$ and $a^\dagger_{\mathbf{k}\sigma}(t)=a^\dagger_{\mathbf{k}\sigma}(0)\exp(i\omega_{\mathbf{k}}t)$. On substituting Eqs. (1) and (3) into Eq. (2), the Hamiltonian of the electromagnetic field is quantized as

$$H_{\text{em}} = \sum_{\mathbf{k}\sigma}\hbar\omega_{\mathbf{k}}a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}, \tag{5}$$

where the zero-point energy terms are dropped. Equation (5) represents the Hamiltonian of the system of noninteracting photons in a normal blackbody.

$N_{\mathbf{k}\sigma}=a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}$ are known as the number operators of photons. The number operators have the eigenvalues $n_{\mathbf{k}\sigma}=0,1,2,\ldots$. Since the number operators commute with H_{em}, the number of photons in each mode $\mathbf{k}\sigma$ is constant in time. The number operators form a complete commuting set and simultaneous eigenstates of this set are given by

$$|\{n_{\mathbf{k}\sigma}\}\rangle = \prod_{\mathbf{k}\sigma}\left[\frac{1}{\sqrt{n_{\mathbf{k}\sigma}!}}(a^\dagger_{\mathbf{k}\sigma})^{n_{\mathbf{k}\sigma}}\right]|0\rangle, \tag{6}$$

where $|0\rangle$ is the vacuum state of the electromagnetic field. The state vector (6) is symmetric under the interchange of any two creation operators, consistent with the Bose-Einstein statistics. Because the number of photons is variable, the chemical potential of the photon system is null. Consequently, H_{em} is a grand canonical Hamiltonian.

B. Thermal radiation state

The state vector (6) signifies a multimode number state of photons, which is a pure state and therefore far from thermal equilibrium. However, the electromagnetic field within a blackbody is in thermal equilibrium [6]. Such equilibrium is established via the continual absorption and emission of photons by matter. The electromagnetic field in thermal equilibrium is called blackbody radiation and characterized by a definite temperature T. The photons in blackbody radiation are in a thermal radiation state, which is called a normal state. In order to characterize the thermal radiation state, we need to conceive a grand canonical ensemble of photons. Some identical systems of the ensemble may be in an eigenstate of the Hamiltonian H_{em} given by Eq. (5), while the distribution of the ensemble over the eigenstates is described by the density operator of the thermal radiation state

$$\rho = \frac{\exp(-H_{\text{em}}/k_BT)}{\text{Tr}\exp(-H_{\text{em}}/k_BT)}, \tag{7}$$

where k_B is Boltzmann's constant. The basis states used in the trace are the eigenstates of the Hamiltonian H_{em}, which are given by Eq. (6). The main thermodynamic quantity in normal blackbody radiation is the total energy E_n or the energy density $u_n=E_n/V$, which is the ensemble average of the corresponding microscopic quantity,

$$E_n = \sum_{\mathbf{k}\sigma}\hbar\omega_{\mathbf{k}}\langle N_{\mathbf{k}\sigma}\rangle. \tag{8}$$

Here we have utilized the average notation $\langle N_{\mathbf{k}\sigma}\rangle=\text{Tr}(\rho N_{\mathbf{k}\sigma})$.

It is easily found that the ensemble average of the number operator of photons in a mode $\mathbf{k}\sigma$ satisfies the well-known Bose-Einstein distribution,

$$\langle N_{\mathbf{k}\sigma}\rangle = \frac{1}{e^{\hbar\omega_{\mathbf{k}}/k_BT}-1}. \tag{9}$$

Putting Eq. (9) into Eq. (8) and in the usual way altering the summation to an integration, we obtain

$$E_n = V\int_0^\infty \rho_n(\omega,T)d\omega, \tag{10}$$

$$\rho_n(\omega,T) = \frac{\hbar}{\pi^2c^3}\frac{\omega^3}{e^{\hbar\omega/k_BT}-1}. \tag{11}$$

Equation (11) for the spectral energy density of blackbody radiation is called Planck's formula. The spectral energy density of blackbody radiation has a maximum at a frequency ω_m defined by the equation,

$$3 - \frac{xe^x}{e^x-1} = 0, \tag{12}$$

where $x=\hbar\omega_m/k_BT$. The numerical solution of Eq. (12) gives

$$\hbar\omega_m/k_BT = 2.821\,44. \tag{13}$$

With the new variable of integration $x=\hbar\omega/k_BT$, the resulting integral in Eq. (10) is equal to $\pi^4/15$. Equation (10) yields

$$E_n = 4\sigma VT^4/c, \tag{14}$$

where $\sigma=\pi^2k_B^4/60\hbar^3c^2$ is called the Stefan-Boltzmann constant. Thus the total energy of blackbody radiation is proportional to the fourth power of the temperature. This is the Stefan-Boltzmann law. For the future study, we need to write the energy density of normal blackbody radiation,

$$u_n(T) = 4\sigma T^4/c. \tag{15}$$

Thereby the pressure of normal blackbody radiation is given by

$$P_n(T) = \tfrac{1}{3}u_n(T). \tag{16}$$

III. KERR NONLINEAR BLACKBODY

The model of a Kerr nonlinear blackbody was described in I. The crystal under study is a covalent one. The optical vibration modes of a covalent crystal are all nonpolar modes that carry no electric dipole moments, so they are infrared inactive. For convenience the crystal is taken to be of cubic symmetry, so it is optically isotropic. A Kerr nonlinear crystal must be centrosymmetric. By "nonlinearity" we mean that the crystal is first-order Raman active. Nonpolar modes in a centrosymmetric crystal have even parity and are Raman active [7]. In the cubic system, the common covalent crystals that are both centrosymmetric and Raman active have a diamond structure. At this point, the crystal studied is determined as a specific crystal with a diamond structure, such as C. In a diamond-structure crystal a primitive cell contains two identical atoms that exhibit a triply degenerate nonpolar mode at zero wave vector, which is Raman active. For the Raman-active mode the two atoms in the primitive cell move in antiphase. Because the following treatment has no relation to acoustic modes, the vibrational modes of the crystal are limited to the Raman-active mode, whose zero-wave-vector frequency is denoted by ω_R.

In Ref. [5] we have known that the interaction between photons and phonons can lead to an attractive effective interaction among the photons themselves. The attractive effective interaction leads to bound photon pairs. The physical background for pairing is simple. A photon can emit or absorb a virtual nonpolar phonon. The emission of virtual nonpolar phonons by photons means that the photon is clothed with a cloud of virtual nonpolar phonons. If a second photon is near this cloud, it experiences a force of attraction. In the standing-wave configuration a photon pair is stable only if the two photons have opposite wave vectors and helicities. The pair Hamiltonian of the photon system is

$$H'_{\rm em} = \frac{1}{2}\sum_{\mathbf{k}\sigma}\hbar\omega_{\mathbf{k}}(a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma} + a^\dagger_{-\mathbf{k},-\sigma}a_{-\mathbf{k},-\sigma}) + \sum_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'}a^\dagger_{\mathbf{k}'\sigma'}a^\dagger_{-\mathbf{k}',-\sigma'}a_{-\mathbf{k},-\sigma}a_{\mathbf{k}\sigma}, \tag{17}$$

where the photons have the pair potential

$$V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} = \begin{cases} -V_0\hbar\omega_{\mathbf{k}}\hbar\omega_{\mathbf{k}'} & \text{if } \omega_{\mathbf{k}} \text{ and } \omega_{\mathbf{k}'} < \omega_R, \\ 0 & \text{otherwise}, \end{cases} \tag{18}$$

where V_0 is a positive constant. We shall assume that the crystal has a dispersion-free refractive index n, so that the photonic frequency is given by $\omega_{\mathbf{k}}=c|\mathbf{k}|/n$.

Unpaired bare photons in the photon system are transformed into a different kind of quasiparticle, the nonpolariton. A nonpolariton is a condensate of virtual nonpolar phonons in momentum space, with a bare photon acting as the nucleus of condensation. The diagonalization of the pair Hamiltonian (17) can be performed by the Bogoliubov transformation:

$$c_{\mathbf{k}\sigma} = Ua_{\mathbf{k}\sigma}U^\dagger = a_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} - a^\dagger_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma},$$

$$c^\dagger_{\mathbf{k}\sigma} = Ua^\dagger_{\mathbf{k}\sigma}U^\dagger = a^\dagger_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} - a_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma}, \tag{19}$$

where the parameter $\varphi_{\mathbf{k}\sigma}$ is assumed to be real and spherically symmetric: $\varphi_{-\mathbf{k},-\sigma}=\varphi_{\mathbf{k}\sigma}$. $c^\dagger_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ are the creation and annihilation operators, respectively, of nonpolaritons in the photon system; they also obey Bose equal-time commutation relations such as Eq. (4). The transition from the operators of bare photons to those of nonpolaritons can be effected by a unitary transformation:

$$U = \exp\left[\frac{1}{2}\sum_{\mathbf{k}\sigma}\varphi_{\mathbf{k}\sigma}(a^\dagger_{\mathbf{k}\sigma}a^\dagger_{-\mathbf{k},-\sigma} - a_{-\mathbf{k},-\sigma}a_{\mathbf{k}\sigma})\right]. \tag{20}$$

It is well known that the unitary transformation does not change the energy spectrum of the photon system. The normalized state vector of photon pairs in the photon system may be constructed as $|G\rangle=U|0\rangle$, such that $c_{\mathbf{k}\sigma}|G\rangle=0$.

As we know, the pair Hamiltonian (17) can be solved only when the pair potential $V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'}$ is negative. Under the mean-field approximation [5], the pair Hamiltonian of the photon system is diagonalized into

$$H'_{\rm em} = E_p + \sum_{\mathbf{k}\sigma} \hbar\tilde{\omega}_{\mathbf{k}}(T) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}. \quad (21)$$

The idea of a mean-field approximation was introduced by the Weiss theory of ferromagnetism to deal with phase transitions [8]. Here the idea is that individual nonpolaritons move independently in a mean field caused by all the other photons, which includes parts of the photon-photon interaction. The frequency of nonpolaritons is acquired as $\tilde{\omega}_{\mathbf{k}}(T) = v(T)|\mathbf{k}|$ where $v(T)$ is the velocity of nonpolaritons determined by the equation

$$v(T) = 2(c/n)V_0 {\sum_{\mathbf{k}}}' \hbar\omega_{\mathbf{k}} \coth\frac{\hbar v(T)|\mathbf{k}|}{2k_B T}, \quad (22)$$

where the prefactor 2 arises from the summation over helicities and the prime on the summation symbol means that $\omega_{\mathbf{k}} < \omega_R$. E_p is the energy of the system of photon pairs, as given by

$$E_p = \sum_{\mathbf{k}\sigma} \Bigg[\hbar\omega_{\mathbf{k}} \sinh^2\varphi_{\mathbf{k}\sigma} + \frac{1}{4}\sinh 2\varphi_{\mathbf{k}\sigma} \sum_{\mathbf{k}'\sigma'} V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} \sinh 2\varphi_{\mathbf{k}'\sigma'} \Bigg], \quad (23)$$

where the parameter $\varphi_{\mathbf{k}\sigma}$ is determined by the relations

$$\tanh 2\varphi_{\mathbf{k}\sigma} = \Delta(T), \quad v(T) = (c/n)\sqrt{1-\Delta^2(T)}. \quad (24)$$

$\Delta(T)$ is the order parameter for pairing of photons.

The velocity $v(T)$ determined by Eq. (22) is a monotonically increasing function of temperature T, which is equal to c/n at the transition temperature T_c. In other words, the order parameter $\Delta(T)$ is a monotonically decreasing function of temperature T, which vanishes at the transition temperature T_c. In Ref. [5] we showed that below T_c the photon system is in a squeezed thermal radiation state, in which the photons with opposite wave vectors and helicities are bound into pairs and unpaired photons are transformed into nonpolaritons. At T_c, both photon pairs and nonpolaritons become single bare photons. Above T_c, a Kerr nonlinear blackbody behaves like a normal blackbody.

IV. RADIATION PROPERTIES

A. Formulas

For future study it will be convenient to define the number operators $N_{\mathbf{k}\sigma} = c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$ for nonpolaritons. The number operators have the eigenvalues $n_{\mathbf{k}\sigma} = 0, 1, 2, \ldots$. The eigenstates of number operators $N_{\mathbf{k}\sigma}$ are given by

$$|\{n_{\mathbf{k}\sigma}\}\rangle = \prod_{\mathbf{k}\sigma} \left[\frac{1}{\sqrt{n_{\mathbf{k}\sigma}!}} (c^\dagger_{\mathbf{k}\sigma})^{n_{\mathbf{k}\sigma}} \right] |G\rangle. \quad (25)$$

The Hilbert space of the photon system is spanned by the complete orthonormal basis vectors $|\{n_{\mathbf{k}\sigma}\}\rangle$. We have shown that a Kerr nonlinear blackbody below the transition temperature T_c is in the squeezed thermal radiation state. In order to characterize the squeezed thermal radiation state, we need to conceive a grand canonical ensemble of nonpolaritons. Some identical systems of the ensemble may be in an eigenstate of the Hamiltonian $H'_{\rm em}$ given by Eq. (21), while the distribution of the ensemble over the eigenstates is described by the density operator of the squeezed thermal radiation state,

$$\rho = \frac{\exp(-H'_{\rm em}/k_B T)}{\mathrm{Tr}\exp(-H'_{\rm em}/k_B T)}, \quad (26)$$

where the basis states used in the trace are the eigenstates of the Hamiltonian $H'_{\rm em}$, which are given by Eq. (25).

The gas of free nonpolaritons constitutes a thermal radiation in the Kerr nonlinear blackbody. In what follows we shall study the properties of such thermal radiation. A main thermodynamic quantity in the Kerr nonlinear blackbody is the energy E_r of the thermal radiation, as given by

$$E_r = \sum_{\mathbf{k}\sigma} \hbar\tilde{\omega}_{\mathbf{k}}(T)\langle N_{\mathbf{k}\sigma}\rangle. \quad (27)$$

It is useful to introduce a dimensionless constant γ. The constant γ is meaningful only if $\gamma < 1$ and signifies the coupling strength between a bare photon and virtual nonpolar phonons. The dimensionless parameter γ is directly proportional to the Kerr nonlinear coefficient. The requirement of $\gamma < 1$ shows that our theory applies only to the weak nonlinear case. Inasmuch as most materials are weakly nonlinear, the requirement of $\gamma < 1$ holds usually. Both the transition temperature T_c and velocity $v(T)$ depend on the parameter γ. For example, the zero-temperature velocity has a value $v(0) = \gamma c/n$.

Our main task is to calculate the energy E_r of the thermal radiation defined by Eq. (27). It is easily found that the ensemble average of the number operator of nonpolaritons in a mode $\mathbf{k}\sigma$ satisfies the well-known Bose-Einstein distribution

$$\langle N_{\mathbf{k}\sigma}\rangle = \frac{1}{e^{\hbar\tilde{\omega}_{\mathbf{k}}(T)/k_B T} - 1}. \quad (28)$$

Putting Eq. (28) into Eq. (27) and in the usual way altering the summation to an integration, we obtain

$$E_r = V\int_0^\infty \rho_r(\tilde{\omega}, T)\, d\tilde{\omega}, \quad (29)$$

$$\rho_r(\tilde{\omega}, T) = \frac{\hbar}{\pi^2 v^3(T)} \frac{\tilde{\omega}^3(T)}{e^{\hbar\tilde{\omega}(T)/k_B T} - 1}, \quad (30)$$

where $\tilde{\omega}(T) = v(T)|\mathbf{k}|$. With the new variable of integration $x = \hbar\tilde{\omega}/k_B T$, the resulting integral in Eq. (29) is equal to $\pi^4/15$. Then Eq. (29) yields the following result: $E_r(T) = Vu_r(T)$ and

$$u_r(T) = 4\sigma(T)T^4/v(T), \quad (31)$$

where $u_r(T)$ is the energy density of the thermal radiation and $\sigma(T) = \pi^2 k_B^4/60\hbar^3 v^2(T)$ is the temperature-dependent Stefan-Boltzmann constant. $u_r(T)$ is a monotonically increasing function of temperature T and at zero temperature it is

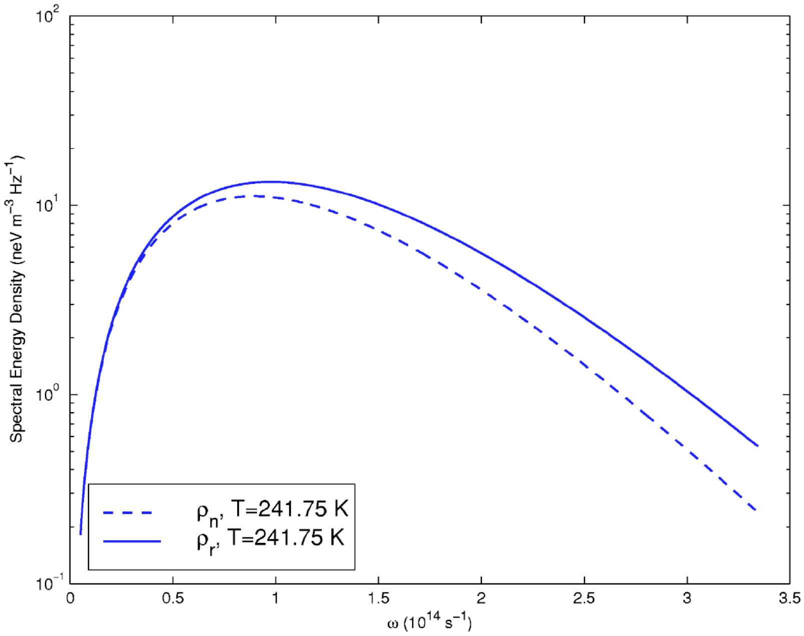

FIG. 2. Variation of spectral energy density with frequency ω; the dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively.

equal to zero. The radiation pressure of a Kerr nonlinear blackbody is immediately given by $P_r(T)=\frac{1}{3}u_r(T)$. Therefore, the radiation pressure is also a monotonically increasing function of temperature T.

B. Numerical calculation

$\rho_r(\tilde{\omega},T)$ given by Eq. (30) denotes the spectral energy density of the thermal radiation in a Kerr nonlinear blackbody and is an observable. The frequency of nonpolaritons can be written as $\tilde{\omega}_{\mathbf{k}}(T)=[nv(T)/c]\omega_{\mathbf{k}}$ in terms of the bare photon frequency $\omega_{\mathbf{k}}=c|\mathbf{k}|/n$. Therefore, we define the spectral energy density of the Kerr nonlinear blackbody radiation $\tilde{\rho}_r(\omega,T)$ by the transformation

$$\rho_r(\tilde{\omega},T)d\tilde{\omega}=\tilde{\rho}_r(\omega,T)d\omega. \tag{32}$$

Consequently, $\tilde{\rho}_r(\omega,T)$ is given by

$$\tilde{\rho}_r(\omega,T)=\frac{\hbar v(T)}{\pi^2(c/n)^4}\frac{\omega^3}{e^{[nv(T)/c]\hbar\omega/k_BT}-1}. \tag{33}$$

Similar to the spectral energy density of normal blackbody radiation discussed in Sec. II, the maximum of the spectral energy density of Kerr nonlinear blackbody radiation is located at

$$\hbar\omega_m/k_BT=2.821\,44c/nv(T). \tag{34}$$

At temperatures $T\geqslant T_c, v(T)=c/n$, such that the spectral energy density of Kerr nonlinear blackbody radiation is the same as that of normal blackbody radiation. At temperatures $T<T_c, v(T)<c/n$, such that the spectral energy density of Kerr nonlinear blackbody radiation is different from that of normal blackbody radiation. For example, the maximum of the spectral energy density of Kerr nonlinear blackbody radiation is located at a higher frequency in comparison with normal blackbody radiation.

For convenience, in Eq. (33) we set the refractive index $n=1$, such that Eq. (33) can be compared with Eq. (11). As known, both transition temperature T_c and velocity $v(T)$ depend on the dimensionless parameter γ. $T_c=464.9$ K at $\gamma=0.9$. We assume that the Kerr nonlinear blackbody is at temperature $T=241.75$ K, so it is in a squeezed thermal radiation state. $v(T)=0.916c$ at $\gamma=0.9$ and $T=241.75$ K. The spectral energy density of blackbodies at temperature $T=241.75$ K is plotted in Fig. 2 as a function of its frequency, where the dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively. There are the two features: (1) the spectral energy density of the Kerr nonlinear blackbody is larger than that of the normal blackbody; (2) the peak frequency of the Kerr nonlinear blackbody is at $\omega_m=9.7517\times10^{13}$ s^{-1} while the peak frequency of the normal blackbody is at $\omega_m=8.9297\times10^{13}$ s^{-1}. The two features are due to the fact that in the squeezed thermal radiation state the velocity $v(T)$ of nonpolaritons is smaller than the speed c of light. The spectral energy density of blackbodies at frequency $\omega=\omega_R=2.51\times10^{14}$ s^{-1} is plotted in Fig. 3 as a function of relative temperature $x=k_BT/\hbar\omega$, where the dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively. There are the two features: (1) the spectral energy densities are monotonically increasing functions of T as temperature increases from zero to transition temperature T_c; (2) the spectral energy density of the Kerr nonlinear blackbody is equal to that of the normal blackbody at $T=T_c$, because of $v(T_c)=c$.

The energy density $u_r(T)$ and radiation pressure $P_r(T)$ of a Kerr nonlinear blackbody depend on dimensionless parameter γ and thus we set $\gamma=0.9$. The variation of $u_n(T)$ and $u_r(T)$ with relative temperature $x=k_BT/\hbar\omega_R$ is shown in Fig. 4, where the dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively, and temperature T varies from zero to transition temperature T_c. There are the three features: (1) $u_n(T)$ and $u_r(T)$ are monotonically increasing functions of temperature T; (2) at zero temperature $u_n(0)=u_r(0)=0$ and at transition temperature $T_c u_n(T_c)=u_r(T_c)=22.016$ Mev m^{-3}; (3) as $0<T<T_c, u_n(T)<u_r(T)$.

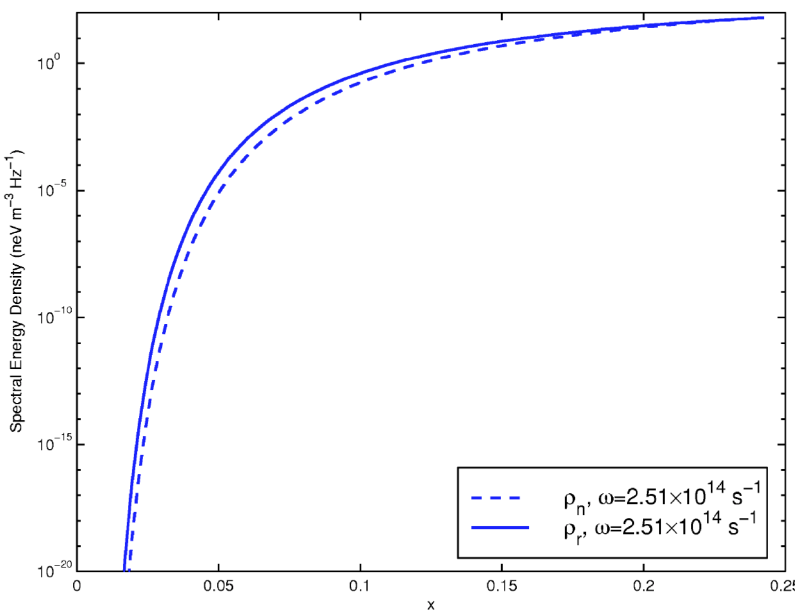

FIG. 3. Variation of spectral energy density with relative temperature $x=k_BT/\hbar\omega_R$, where the temperature T varies from zero to the transition temperature T_c. The dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively.

The radiation pressure of blackbodies is plotted in Fig. 5 as a function of relative temperature $x=k_BT/\hbar\omega_R$, where temperature T varies from zero to transition temperature T_c. The dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively. Figure 5 shows that the radiation pressure of a Kerr nonlinear blackbody can be larger than that of a normal blackbody and that both are equal at the transition temperature T_c.

We think that the value $\gamma=0.9$ used in Figs. 2–5 corresponds to a realistic material at least. For fixed temperature $T=241.75$ K and varying frequency ω, Fig. 6 shows variation of the spectral energy density ρ_r with the parameter γ. The spectral energy density is a monotonically decreasing function of the parameter γ at a fixed temperature and frequency. As known, transition temperature T_c depends on the dimensionless parameter γ, i.e., $T_c=T_c(\gamma)$. Now we need to introduce a new relative temperature $T/T_c(\gamma)$. For varying relative temperature $T/T_c(\gamma)$, Fig. 7 shows variation of the radiation pressure P_r with the parameter γ. The radiation pressure is also a monotonically decreasing function of the parameter γ at a fixed relative temperature. The reason for these results is that the larger the parameter γ, the smaller is the energy of the nonpolariton system.

V. BROKEN SYMMETRY

The superconducting theory of solids established by Bardeen, Cooper, and Schrieffer is a striking success of the quantum field theory of solids [9,10]. The basic physical mechanism is that the electron-electron Coulomb repulsion is overcome by the attractive interaction via acoustic phonons, leading to massive electron pairs known as Cooper pairs, which incorporate the acoustic phonons and hence propagate without phonon scattering. The transition of the electron sys-

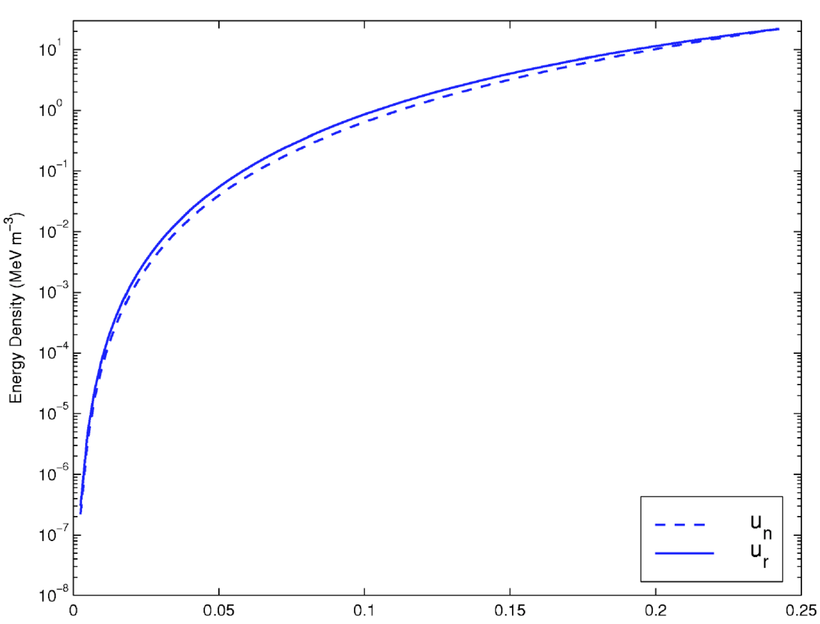

FIG. 4. Variation of energy density with relative temperature $x=k_BT/\hbar\omega_R$, where the temperature T varies from zero to the transition temperature T_c. u_n and u_r denote the energy densities of the normal and Kerr nonlinear blackbodies, respectively.

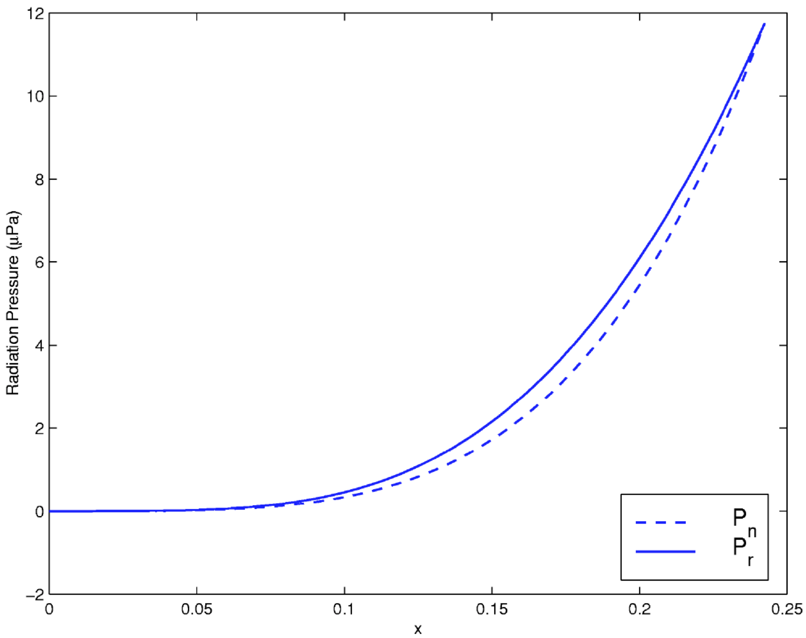

FIG. 5. Variation of radiation pressure with relative temperature $x = k_B T/\hbar\omega_R$, where the temperature T varies from zero to the transition temperature T_c. The dashed and solid lines correspond to the normal and Kerr nonlinear blackbodies, respectively.

tem from the normal to the superconducting state is connected with a change in the gauge symmetry of the system's state. Many important concepts in this theory have a certain generality and are sure to be applicable to the photon system. In the squeezing transition of the photon system, what symmetry is broken?

In order to answer the question, we first discuss the symmetry of the normal thermal radiation state of the photon system above the transition temperature T_c. With a single mode index $\mu = \mathbf{k}\sigma$, one can introduce the coherent state of the μth mode: $|\alpha_\mu\rangle = D(\alpha_\mu)|0\rangle$ where $D(\alpha_\mu)$ is the so-called displacement operator

$$D(\alpha_\mu) = \exp(\alpha_\mu a_\mu^\dagger - \alpha_\mu^* a_\mu), \tag{35}$$

with α_μ being a complex number. Further one can introduce the density operator of the normal thermal radiation state of the μth mode:

$$\rho_\mu = \frac{\exp(-\hbar\omega_\mu N_\mu/k_B T)}{\mathrm{Tr}\,\exp(-\hbar\omega_\mu N_\mu/k_B T)}, \tag{36}$$

where $N_\mu = a_\mu^\dagger a_\mu$ is the number operator of photons. In quantum optics, it is well known that the probability of finding the μth mode in the state $|\alpha_\mu\rangle$ is defined by the Q representation:

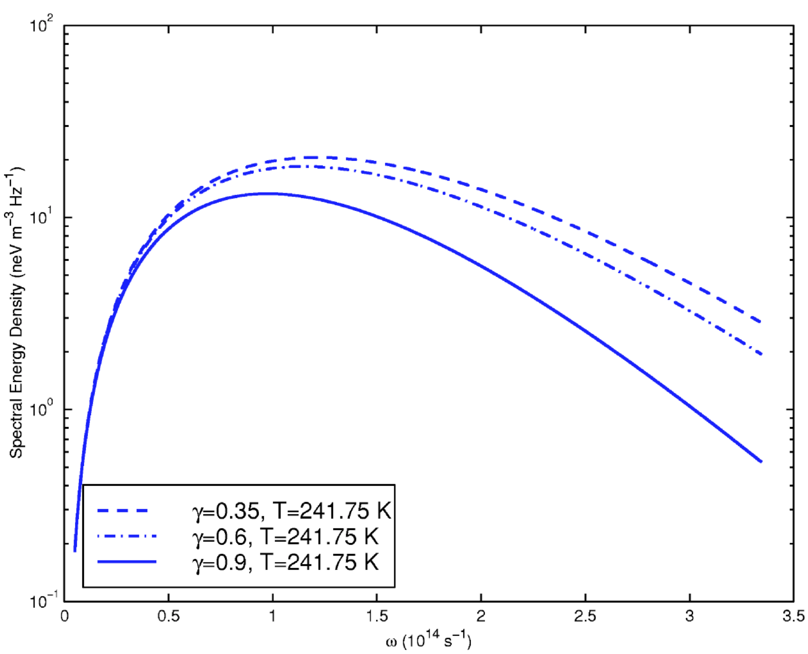

FIG. 6. For three values of γ, variation of the spectral energy density of a Kerr nonlinear blackbody with frequency ω.

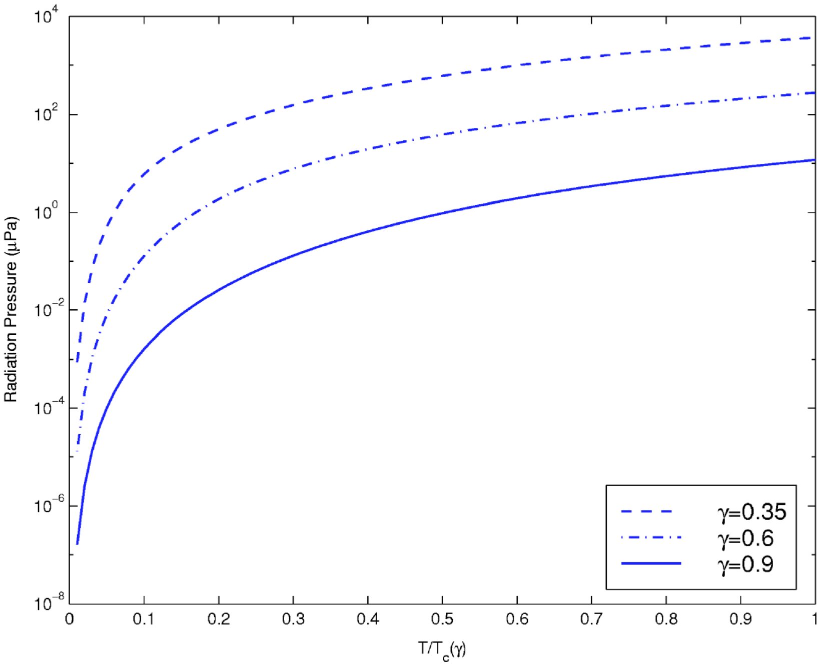

FIG. 7. For three values of γ, variation of the radiation pressure of a Kerr nonlinear blackbody with relative temperature $T/T_c(\gamma)$.

$$Q(\alpha_\mu) = \frac{1}{\pi}\langle\alpha_\mu|\rho_\mu|\alpha_\mu\rangle. \tag{37}$$

It is easily found that the Q representation of the normal thermal radiation state is given by a Gaussian distribution [11],

$$Q(\alpha_\mu) = \frac{1}{\pi(1+\langle N_\mu\rangle)}\exp\left(-\frac{|\alpha_\mu|^2}{1+\langle N_\mu\rangle}\right), \tag{38}$$

where $\langle N_\mu\rangle$ is given by Eq. (9). The real and imaginary parts of α_μ represent two quadrature phase variables; hence there is an equipartition of $Q(\alpha_\mu)$ in the phase space. Therefore, the Q representation of the normal thermal radiation state has phase symmetry.

Next we inspect the symmetry of the squeezed thermal radiation state of the photon system at zero temperature. The ground state is a many-mode squeezed vacuum state $|G\rangle = U|0\rangle$ where the many-mode squeeze operator U is given by Eq. (20). Now we concentrate on a single-mode squeezed vacuum state $|r_\mu\rangle = S(r_\mu)|0\rangle$ where the single-mode squeeze operator $S(r_\mu)$ is given by

$$S(r_\mu) = \exp[\tfrac{1}{2}r_\mu(a_\mu^2 - a_\mu^{\dagger 2})], \tag{39}$$

with $r_\mu = -\varphi_\mu$ being a real number. Because the state $|r_\mu\rangle$ is a pure state, the density operator of the state takes the form $\rho_\mu = |r_\mu\rangle\langle r_\mu|$. In this case, the probability of finding the μth mode in the coherent state $|\alpha_\mu\rangle$ is given by the Q representation:

$$Q(\alpha_\mu) = \frac{1}{\pi}\langle\alpha_\mu|\rho_\mu|\alpha_\mu\rangle = \frac{1}{\pi}|\langle\alpha_\mu|r_\mu\rangle|^2. \tag{40}$$

In terms of the quadrature phase variables $x_1 = (\alpha_\mu + \alpha_\mu^*)/2$ and $x_2 = (\alpha_\mu - \alpha_\mu^*)/2i$, it is easily found that the Q representation of the squeezed vacuum state is given by [12]

$$Q(x_1,x_2) = \frac{\mathrm{sech}\, r_\mu}{\pi}\exp[-(x_1^2+x_2^2) - (x_1^2 - x_2^2)\tanh r_\mu]. \tag{41}$$

Hence there is an unequal partition of $Q(x_1,x_2)$ in the phase space. Therefore, the Q representation of the squeezed vacuum state apparently lacks phase symmetry. We conclude that in the transition from the normal to the squeezed thermal radiation state, the phase symmetry is spontaneously broken.

VI. DISCUSSION

In this paper we expose the radiation properties of the photon system in a Kerr nonlinear blackbody. The photon system below a transition temperature is in a squeezed thermal radiation state. The photon system in the squeezed thermal radiation state consists of two parts: photon pairs and individual nonpolaritons. The nonpolariton system constitutes free thermal radiation in the Kerr nonlinear blackbody. The spectral energy density $\rho_r(T)$ and radiation pressure $P_r(T)$ of the Kerr nonlinear blackbody are monotonically increasing functions of temperature T. It is easy to understand why $\rho(T)$ and $P_r(T)$ increase with increasing of temperature. Both $\rho_r(T)$ and $P_r(T)$ measure the nonpolariton number. As the temperature is increased, the nonpolariton number increases correspondingly, so $\rho_r(T)$ and $P_r(T)$ become larger.

It has been found that the spectral energy density and radiation pressure of a Kerr nonlinear blackbody are larger than those of a normal blackbody. The reason for this is as follows. The thermal radiation of a Kerr nonlinear blackbody manifests itself in the form of the nonpolariton system, whereas the thermal radiation of a normal blackbody manifests itself in the form of the bare photon system. A nonpolariton is the condensate of virtual nonpolar phonons with a bare photon acting as the nucleus of condensation, and so the energy of the nonpolariton system is the sum of the energies of the nonpolar phonon system and the bare photon system. Consequently, the energy of the nonpolariton system is larger than that of the bare photon system. We have observed a peculiar property that the spectral energy density and radiation pressure of a Kerr nonlinear blackbody are monotonically decreasing functions of the Kerr nonlinear coefficient γ. The physical mechanism for it is simple. The parameter γ signifies the coupling strength between a bare photon and virtual nonpolar phonons. The larger the parameter γ, the smaller is the coupling strength. When the parameter γ is increased, the nonpolar phonon weight in a nonpolariton is decreased, so that the energy of the nonpolariton system becomes smaller. In the strong-coupling limit $0<\gamma\ll 1$, the energy of the nonpolariton system is largest. This case shows that arbitrarily weak nonlinearity can destroy a linear photon system. Now we realize that weak nonlinearity can cause a major changing of states from normal to squeezed thermal radiation states. Inasmuch as the Kerr nonlinearity in our theory originates from the Raman nonlinearity of phonons, we conclude that the Raman nonlinearity of phonons is weak. However, in the weak-coupling limit $\gamma=1$, all nonpolaritons become bare photons.

Now we discuss how to measure the spectral energy density. There is a small hole in one wall of a Kerr nonlinear blackbody. Below T_c a small number of nonpolaritons escapes through the hole at a spectral energy density $\rho_r(\omega,T)$. By using a spectroscope and adjusting temperature T, one can draw the curves $\rho_r(\omega,T)$, which should be identical with Figs. 2 and 3. Further, by using a pressure gauge and adjusting temperature T, one can find the temperature dependence of the radiation pressure $P_r(T)$ predicted in Fig. 5. Finally, by using different Kerr nonlinear materials, one can find the dependence of the spectral energy density and radiation pressure on the parameter γ predicted in Figs. 6 and 7.

To sum up, we have investigated the radiation properties for a blackbody whose interior is filled by a Kerr nonlinear crystal. The photon system below the transition temperature consists of photon pairs and individual nonpolaritons. The nonpolariton system constitutes thermal radiation in the Kerr nonlinear blackbody. It has been found that the spectral energy density and radiation pressure of a Kerr nonlinear blackbody are larger than those of a normal blackbody. The spectral energy density and radiation pressure are monotonically decreasing functions of the Kerr nonlinear coefficient. Above the transition temperature the photon system is in a normal thermal radiation state, but below the transition temperature it is in a squeezed thermal radiation state. In the transition from the normal to the squeezed thermal radiation state, the phase symmetry of the photon system is spontaneously broken. The predicted properties might be verified in present-day physics laboratories.

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