Feshbach resonance cooling of trapped atom pairs

Josh W. Dunn,[1] D. Blume,[2] Bogdan Borca,[1] B. E. Granger,[3] and Chris H. Greene[1]
[1]*Department of Physics and JILA, University of Colorado, Boulder, Colorado 80309-0440, USA*
[2]*Department of Physics, Washington State University, Pullman, Washington 99164-2814, USA*
[3]*Department of Physics, Santa Clara University, Santa Clara, California 95053, USA*
(Received 24 June 2004; published 4 March 2005)

Spectroscopic studies of few-body systems at ultracold temperatures provide valuable information that often cannot be extracted in a hot environment. Considering a pair of atoms, we propose a cooling mechanism that makes use of a scattering Feshbach resonance. Application of a series of time-dependent magnetic field ramps results in either zero, one, or two atoms remaining trapped. If two atoms remain in the trap after the field ramps are completed, then they have been cooled. Application of the proposed cooling mechanism to optical traps or lattices is considered.

DOI: 10.1103/PhysRevA.71.033402 PACS number(s): 32.80.Pj, 42.50.Vk

A Feshbach resonance [1–3] occurs for two atoms when their collision energy becomes degenerate with a bound state in a closed collision channel, producing brief transitions into and out of this state. In recent years, these resonances have been used extensively to control the interaction strength in dilute atomic gases [4–8]. Here we make use of the characteristics of a Feshbach resonance to reduce the energy of pairs of externally confined atoms. We show how this method can be used to cool pairs of atoms taken from a thermal distribution.

The basis of this method lies in the observation that the quantum-mechanical energy levels of two atoms in a harmonic trap shift by an energy corresponding to approximately two trap quanta as a control parameter is swept in one direction across the resonance. Throughout this article, we refer to this control parameter as the magnetic field B used to manipulate the atom-atom scattering length a in the vicinity of a pole. In other contexts, the shift of the energy levels could be introduced by varying the detuning of an off-resonant dressing laser, or by varying an electric field strength. The ideas presented here in terms of the control parameter B can be straightforwardly extended to those other contexts. In the following, we first develop the basic mechanism of the Feshbach resonance cooling process. The feasibility and effectiveness of the proposed scheme are then illustrated through an application to a realistic system of two atoms in a trap. Finally, possible applications to optical traps are discussed.

The Schrödinger equation for two interacting identical mass m atoms under spherical harmonic confinement with trapping frequency ν decouples into two equations: one involving the three relative coordinates of the pair, and another involving the three center-of-mass (c.m.) coordinates [9,10]. We consider the Schrödinger equation in the relative coordinate for two trapped atoms interacting through a central potential and assume for the time being that the c.m. coordinate is translationally cold. Accounting for an applied external magnetic field B through a B-dependent quantum defect $\beta_{El}(B)$, the energies $E_{nl}(B)$ associated with the relative motion of an atom pair are given by [10]

$$E_{nl}(B) = [2n - 2\beta_{El}(B) + l + 3/2]\hbar\omega, \tag{1}$$

where $\omega = 2\pi\nu$. Here, the quantum defect $\beta_{El}(B)$ depends strongly on the relative orbital angular momentum l of the pair, while it depends only weakly on the radial oscillator quantum number n. The dependence of $\beta_{El}(B)$ on the energy is weak on the scale of an oscillator quantum, i.e., $|d\beta_{El}(B)/dE_{nl}| \ll 1/\hbar\omega$.

As will become clear later, the quantum defect for one relative partial wave l for an atom pair, e.g., the s wave, p wave, or d wave, must rise by unity across the energy range $k_B\Delta T$ of interest, and across the accessible range of the control parameter ΔB. In fact, this variation of $\beta_{El}(B)$ by unity corresponds to the Feshbach resonance, which causes the scattering phase shift to rise by π. A simple closed-form expression exists for β_{El} [10,11], which simplifies at energies higher than a few trap quanta to $\beta_{El}(B) \approx \arctan[a(E_{nl},B)\hbar\omega/2L_{\rm osc}E_{nl}\sqrt{e}]$ where $a(E_{nl},B)$ is the energy- and field-dependent scattering length and $L_{\rm osc} = \sqrt{\hbar/(\mu\omega)}$ with $\mu = m/2$ denotes the characteristic oscillator length.

In this paper, we focus on an s-wave resonance, though this formalism can be readily extended to higher partial wave resonances. When an s-wave Feshbach resonance occurs, the limiting low-energy scattering phase shift is proportional to the wave number $k = (2\mu E/\hbar^2)^{1/2}$. Omitting the subscript l, the E- and B-dependent scattering length is then given by

$$a(E_n,B) = a_{\rm bg} + \frac{\Gamma_E\sqrt{\hbar^2/(8\mu E_n)}}{E_n + (B - B_{\rm res})E'_{\rm res}(B)}, \tag{2}$$

where $a_{\rm bg}$ is the background scattering length. At the magnetic field strength $B_{\rm res}$ of the resonance a zero-energy bound state occurs. The resonance width Γ_E in energy is related to the width Δ in the control parameter by $\Gamma_E = 2ka_{\rm bg}E'_{\rm res}(B)\Delta$, where $E'_{\rm res}$ denotes the rate at which the resonance energy $E_{\rm res}$ varies with the control parameter [12]. Figure 1 illustrates the characteristic s-wave energy levels E_n appropriate for the relative motion of two atoms in a spherical harmonic oscillator trap, as functions of the applied magnetic field B for a magnetic Feshbach resonance in ^{85}Rb(2,−2)+^{85}Rb(2,

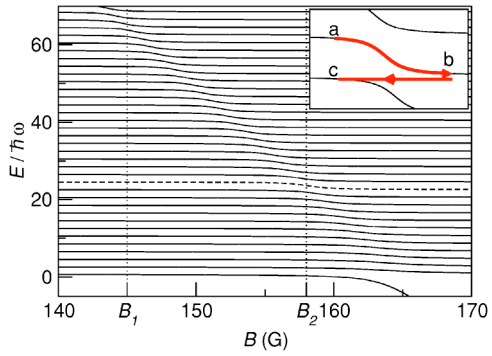

FIG. 1. (Color online) Energy levels E_n for the relative coordinate of a harmonically trapped ^{85}Rb atom pair near the $B_0 \approx 155.2$ G Feshbach resonance, as a function of the magnetic field B. A rather large trapping frequency of $\nu=1$ MHz is used in order to clarify the field dependence of the energy levels. Cooling is performed by ramping the magnetic field B slowly from B_1 to B_2 and then quickly back to B_1. A more realistic ramp would likely encompass more level curves (i.e., cover a larger field range). The state that undergoes a shift for $B=B_2$ (which we will label as $n=Q$ later) is indicated by a dashed line. Ideal cooling is described diagrammatically in the inset (same axes), where population transfer from point a to point b occurs during the slow field ramp and from b to c during the fast ramp. See the text for details.

-2). In Fig. 1, the parameters adopted are $B_{\rm res}=155.2$ G, $E'_{\rm res}=-3.5$ MHz/G, $\Gamma_B=10$ G, and $a_{\rm bg}=-380a_0$, where a_0 is the Bohr radius.

Feshbach resonance cooling entails ramping the magnetic field through the region where the energy levels shift by $\approx 2\hbar\omega$. Figure 1 denotes the internal energy eigenvalues for a pair of atoms as a function of magnetic field B. For a pair of atoms initially in an eigenstate at $B=B_1$, a sufficiently slow (adiabatic) field ramp from B_1 to B_2 will decrease the internal energy of the pair by $\approx 2\hbar\omega$ if the energy level undergoes a shift in that field range (see inset of Fig. 1). A fast (nonadiabatic) ramp from B_2 back to B_1 will ideally project the atom pair onto an eigenstate $|n(B_2)\rangle$ with the same energy as $|n(B_1)\rangle$, resulting in a net decrease in energy of $\approx 2\hbar\omega$. Further field ramps can then be performed.

To model the effects of the magnetic field ramps, we have developed a two-channel Feshbach resonance model, based on the single-channel model described in Ref. [13]. Both of these models describe a two-atom Feshbach resonance for a harmonic trap, and utilize a zero-range potential to describe the interaction between the two atoms. The two-channel model has the advantage of allowing for a field-dependent resonance state. In the two channels, the s-wave radial solutions for the relative coordinate r of the atom pair satisfy the equations

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{1}{2}\mu\omega^2 r^2\right)u_1(r)=Eu_1(r), \tag{3}$$

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{1}{2}\mu\omega^2 r^2\right)u_2(r)=(E-\varepsilon)u_2(r), \tag{4}$$

where ε is the energy shift of the second channel from the first channel. The zero-range potential imposes a boundary condition at the origin, which is parametrized as

$$\frac{d}{dr}\begin{pmatrix}u_1(r)\\ u_2(r)\end{pmatrix}_{r=0}=\begin{pmatrix}-1/a_1 & \beta\\ \beta & -1/a_2\end{pmatrix}\begin{pmatrix}u_1(r)\\ u_2(r)\end{pmatrix}_{r=0}. \tag{5}$$

A quantum-defect-theory treatment, similar to that of Ref. [13], can then be applied. The scattering length predicted by this model (when $\omega\to 0$) is

$$a(E,B)=\left(\frac{1}{a_1}+\frac{|\beta|^2}{\sqrt{2\mu\varepsilon(B)/\hbar^2-2\mu E/\hbar^2}-1/a_2}\right)^{-1}, \tag{6}$$

which can be compared to the measured scattering length to determine the values of the parameters a_1, a_2, and β. The parameters also affect the magnetic-field dependence of the adiabatic energy states, and their adjustment is able to provide satisfactory agreement with experimental data in the regions of interest to us. For example, for ^{85}Rb, we find $a_1=-435a_0$, $a_2=1.49a_0$, and $\beta=0.001\,16a_0^{-1}$. Simulations can then be performed by specifying an initial state of the system and numerically solving the Schrödinger equation.

The simulations reveal the effect of the adiabatic and nonadiabatic field ramps. Assume first that the atom pair is in a pure state at $B=B_1$. As expected, the adiabatic field ramp (B_1 to B_2 in Fig. 1) decreases the energy of the atom pair irrespective of the initial eigenstate chosen. A nonadiabatic ramp (B_2 to B_1 in Fig. 1) causes a state at B_2 that is not degenerate with the resonance state to project onto a state at B_1 with approximately the same energy as the initial state at B_2. However, if the pair is initially in the state at B_2 that is degenerate with the resonance state, the fast ramp results in a strong projection onto the resonance state. In this case, the atom pair gains energy since the resonance state at B_1 has a higher energy than the initial state at B_2.

We now generalize our scheme to accommodate a mixed initial state, and show how cooling can be performed for an ensemble of atom pairs. For an atom pair taken from a thermal distribution, the occupation probability of a level with energy E_n in the relative motion is determined in terms of a Boltzmann factor by $e^{-E_n/\tau}/Z(\tau)$ with $\tau=k_{\rm B}T$, where $k_{\rm B}$ is Boltzmann's constant, $Z(\tau)=\Sigma_i e^{-E_i/\tau}$ is the partition function with the sum running over all states of the system, and T is the temperature of the source of the two atoms. Based on the results of field ramps for pure states discussed above, we see that application of a cooling cycle (slow ramp from B_1 to B_2 plus fast ramp back to B_1) for a mixed state will do two things: (a) decrease by $2\hbar\omega$ the energy of the population in states which undergo a full energy shift between B_1 and B_2, and (b) increase the energy of the population in the state that is degenerate with the resonance (i.e., undergoing an energy shift) at B_2 by moving it to the state (or states) degenerate with the resonance at B_1. We will denote this state from which the heated population originates as $n=Q$.

Figure 2 illustrates the effect of a single cooling cycle on a mixed state, using the same Feshbach resonance as shown in Fig. 1. The upper curve (black line) represents an initial s-wave probability distribution for the states associated with the relative coordinate of an atom pair in a harmonic trap with $\nu=1$ MHz and source temperature $T=1$ mK. The lower curve on the left side of the figure and the spike on the right

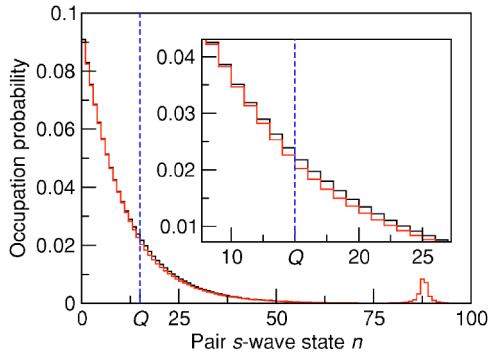

FIG. 2. (Color online) Illustration of the effect of a single Feshbach resonance cooling cycle for an atom pair taken from a thermal distribution with $T=1$ mK and trapped with frequency $\nu=1$ MHz. The upper curve (black line) represents the initial thermal population distribution, and the lower curve on the left side of the figure combined with the spike on the right side of the figure (red line) represents the population distribution after application of one slow and one fast magnetic ramp. The state Q (here $Q=15$) is indicated. Population initially in a state with $n>Q$ is decreased in energy by moving it to the next-lowest state (see the inset closeup). However, population from $n=Q$ and nearby states is increased in energy by being moved to higher states with $n\approx 88$.

side of the figure (red line) represents the same probability distribution after application of a slow and a fast magnetic field ramp, for $Q=15$. Application of one cooling cycle moves the population of the state Q to states with much higher energy, here $n\approx 88$, evidenced by the spike in Fig. 2. At the same time, the field ramps move the population of each state with $n>Q$ to the next-lowest state, which has $\approx 2\hbar\omega$ less energy. Our numerical simulations indicate that the net result of these two processes is to increase the average energy. The few cases where the pair gains a large amount of energy overcomes the many cases where the pair loses a small amount of energy. (Actually, this behavior is not unexpected; see Ref. [14].) However, since the atoms that gain energy can be displaced to an arbitrarily high energy state (determined by the extremum of the field ramp; B_1 in Fig. 1), it should be possible to remove them using methods identical to those used for evaporative cooling.

It is interesting to note that such a process is similar to a conceptual "fractional" evaporative cooling experiment, whereby Q becomes the evaporative-cooling cutoff parameter and, instead of all the population with $n>Q$ being removed, only a fraction of this population is skimmed off and removed from the trap. This similarity can be seen in Fig. 2. Note, however, that Feshbach resonance cooling is distinguished from evaporative cooling in that it is not fundamentally statistical: the more that is known about the initial state of an atom pair, the more effectively they can be cooled. In fact, if the initial state of the pair is known precisely, a field-ramp sequence can be designed to move the atom pair to the ground state of the trap in principle 100% of the time.

As with evaporative cooling, the requirement of atom removal leads to considerations of efficiency. An estimate of the efficiency can be made by assuming that the population of the state Q is removed from the trap, while the population of all states with $n>Q$ are moved to the next-lowest state, that is, to states with $n-1$. This assumes that the range of the field ramps is such that the heated fraction ($n=Q$) ends up at an energy corresponding to negligible thermal population (as in Fig. 2), and that all population above a specified energy can be removed. If we approximate the level energies at $B=B_1$ by $E_n(B_1)\approx 2n\hbar\omega$, the probability to remove an atom pair during a cycle is

$$P_{\mathrm{rem}}(Q,\tau)=\frac{e^{-2\hbar\omega Q/\tau}}{Z(\tau)}. \tag{7}$$

The average energy decrease in a cooling cycle is due to the energy of the $n\approx Q$ population removed from the trap, plus the energy loss for states $n>Q$:

$$\Delta E(Q,\tau)=(2\hbar\omega Q+\langle E_{\mathrm{c.m.}}\rangle)\frac{e^{-2\hbar\omega Q/\tau}}{Z(\tau)}+\sum_{n=Q+1}^{\infty}2\hbar\omega\frac{e^{-2\hbar\omega n/\tau}}{Z(\tau)}. \tag{8}$$

Noting that $\langle E_{\mathrm{c.m.}}\rangle=3\tau$ (since $\langle E_{\mathrm{tot}}\rangle=3\tau$ for a single atom in a harmonic trap), and with $\Sigma_{n=Q+1}^{\infty}e^{-2\hbar\omega n/\tau}\approx e^{-2\hbar\omega Q/\tau}\tau/2\hbar\omega$, Eq. (8) becomes

$$\Delta E(Q,\tau)=\frac{e^{-2\hbar\omega Q/\tau}}{Z(\tau)}(2\hbar\omega Q+4\tau). \tag{9}$$

The energy efficiency E_{eff}, defined as the amount of energy removed per atom removed, is then given by

$$E_{\mathrm{eff}}(Q,\tau)=2\hbar\omega Q+4\tau. \tag{10}$$

Since Q determines the efficiency of the cooling process, it is referred to as the cooling parameter. Results from our numerical model indicate that Eq. (10) provides a good estimate of the efficiency.

The time scale for one cooling cycle is determined by the speed of the adiabatic field ramp. This speed in turn is determined by the strength of the coupling between the resonance state and the trap states. The smaller the coupling for an avoided crossing, the slower is the field ramp required to maintain adiabaticity. The coupling between the resonance state and the trap states is related to the resonance width parameter Γ_E, which can be used in a Landau-Zener estimate of the transition probability [12],

$$P_{\mathrm{tr}}\cong\exp\left(-\frac{2}{|dB/dt|}\frac{\omega\Gamma_E}{|dE/dB|}\right). \tag{11}$$

Motivated by the possibility of experimentally trapping a small, deterministic number of atoms [15,16], we now explore the experimental feasibility of our cooling scheme. A Feshbach resonance cooling experiment involves a sequence of cooling cycles. As discussed above, a single experiment could result in a heated atom pair, which in turn would be lost from the trap. To see the effect of multiple field ramps, Eqs. (7) and (8) can be iterated. For a variety of cooling efficiency parameters Q, Fig. 3 shows the probability for an atom pair to remain trapped vs. the average total kinetic energy (the energy of both the relative and the c.m. degrees of freedom) of the two atoms in oscillator units. Included in this calculation is the probability that the atom pair is in an s-wave state to begin with, because the field ramp has no

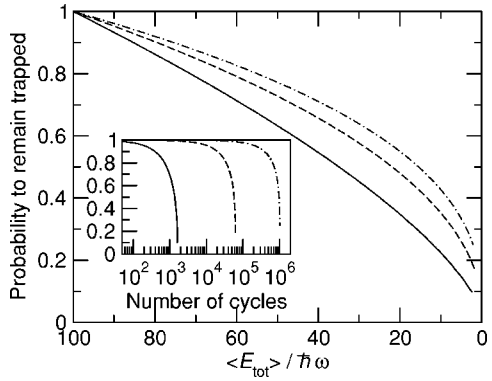

FIG. 3. Probability that a pair of atoms remains trapped vs the average total kinetic energy of the two atoms in oscillator units (note that $k_B T = \langle E_{\mathrm{tot}} \rangle / 6$ for two harmonically trapped atoms). Three different cooling parameters are used: $2\hbar\omega Q = 5\tau$ (solid line), 9τ (dashed line), and 12τ (dot-dashed line). It is assumed that rethermalization occurs between cooling cycles (see text), although this scheme does not necessarily require it. Inset: probability to remain trapped vs the number of cooling cycles for the same three cooling parameters.

effect on other partial waves. We assume that rethermalization occurs between cooling cycles, which could be ensured by, for example, introducing a slight anharmonicity into the trapping potential. (In the absence of anharmonicity, the relative and c.m. degrees of freedom would remain uncoupled.)

To be more specific, we consider a crossed-beam optical dipole trap [17] which offers a good blend of large trap frequency (for a large *s*-wave fraction), isotropy, and anharmonicity (for rethermalization between the relative and c.m. degrees of freedom). Assuming the dipole trap has an average frequency of $\nu = 10$ kHz and contains two atoms taken from a source with temperature $T = 8\ \mu$K ($\langle E_{\mathrm{tot}} \rangle / \hbar\omega = 100$), we see from Fig. 3, solid line, that a temperature of $0.16\ \mu$K ($\langle E_{\mathrm{tot}} \rangle / \hbar\omega = 2$, both atoms in the ground state) could be reached 10% of the time by performing fewer than 2000 cooling cycles. For a range in magnetic field for the ramps of $\Delta B \approx 1$ G, and using Eq. (11) with $P_{\mathrm{tr}} = 0.1$, we see that such a series of field ramps could take place in under 1 s. A perturbative calculation accounting for the trap anharmonicity indicates that rethermalization between the relative and c.m. degrees of freedom should occur on a time scale comparable to a single ramp time for a crossed-beam dipole trap. This will ensure that the relative *s*-wave distribution will rethermalize with each ramp and that the cooling of the relative coordinate will also cool the c.m. coordinate (both of which we have assumed up to this point).

Feshbach resonance cooling could also be applied to atom pairs in an optical lattice. In this case, field ramps could be performed on the lattice ensemble of atom pairs, with a certain percentage of sites resulting in cooled pairs, while other sites will have either zero atoms or one (uncooled) atom. It may also be possible to prepare the optical lattice by some other means to have a high probability of exactly double occupancy at each lattice site (see, for example, Ref. [18]). From such an initial state, a Feshbach resonance cooling scheme could be used to efficiently cool atom pairs to low-lying trap states.

In summary, we have developed a two-body theory that describes how the energy of an atom pair can be reduced. The resulting cooling scheme, which makes use of Feshbach resonances, offers a viable means to manipulate small, deterministic numbers of trapped atoms [15,16] with present-day technology. Since it is not clear at present how efficiently other cooling methods such as evaporation can be applied to small atom samples, our proposal may prove quite useful. Extension of our scheme to atom clouds is possible. This is, however, beyond the scope of this paper.

This work was supported in part by the National Science Foundation. We thank M. Holland, C. Wieman, E. Cornell, P. Julienne, D. Jin, and M. Baertschy for helpful discussions.

[1] H. Feshbach, *Theoretical Nuclear Physics* (Wiley, New York, 1992).
[2] W. C. Stwalley, Phys. Rev. Lett. **37**, 1628 (1976).
[3] E. Tiesinga, B. J. Verhaar, and H. T. C. Stoof, Phys. Rev. A **47**, 4114 (1993).
[4] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Nature (London) **424**, 47 (2003).
[5] J. Herbig, T. Kraemer, M. Mark, T. Weber, C. Chin, H.-S. Nägerl, and R. Grimm, Science **301**, 1510 (2003).
[6] S. Durr, T. Volz, A. Marte, and G. Rempe, Phys. Rev. Lett. **92**, 020406 (2004).
[7] K. E. Strecker, G. B. Partridge, and R. G. Hulet, Phys. Rev. Lett. **91**, 080406 (2003).
[8] K. Xu, T. Mukaiyama, J. R. Abo-Shaeer, J. K. Chin, D. E. Miller, and W. Ketterle, Phys. Rev. Lett. **91**, 210402 (2003).
[9] T. Busch, B.-G. Englert, K. Rzazewski, and M. Wilkins, Found. Phys. **28**, 549 (1998).
[10] D. Blume and C. H. Greene, Phys. Rev. A **65**, 043613 (2002).
[11] E. L. Bolda, E. Tiesinga, and P. S. Julienne, Phys. Rev. A **66**, 013403 (2002).
[12] F. H. Mies, E. Tiesinga, and P. S. Julienne, Phys. Rev. A **61**, 022721 (2000).
[13] B. Borca, D. Blume, and C. H. Greene, New J. Phys. **5**, 111.1 (2003).
[14] W. Ketterle and D. E. Pritchard, Phys. Rev. A **46**, 4051 (1992).
[15] D. Frese, B. Ueberholz, S. Kuhr, W. Alt, D. Schrader, V. Gomer, and D. Meschede, Phys. Rev. Lett. **85**, 3777 (2000).
[16] N. Schlosser, G. Reymond, I. Protsenko, and P. Grangier, Nature (London) **411**, 1024 (2001).
[17] M. Barrett, J. Sauer, and M. S. Chapman, Phys. Rev. Lett. **87**, 010404 (2001).
[18] P. Rabl, A. J. Daley, P. O. Fedichev, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **91**, 110403 (2003).